COLORADO SCHOOL OF MINES
ELECTRICAL ENGINEERING DEPARTMENT

EENG 577

ADVANCED ELECTRICAL MACHINE DYNAMICS FOR SMART-GRID SYSTEMS

Advanced Topics
M4-1 Synchronous Machine Park's Transformation

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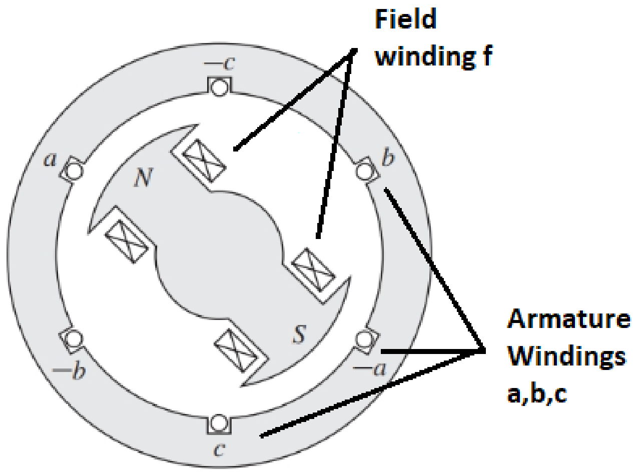
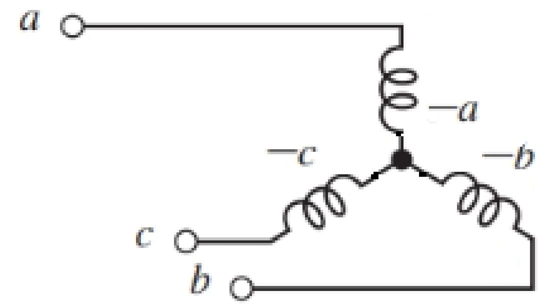


(a)



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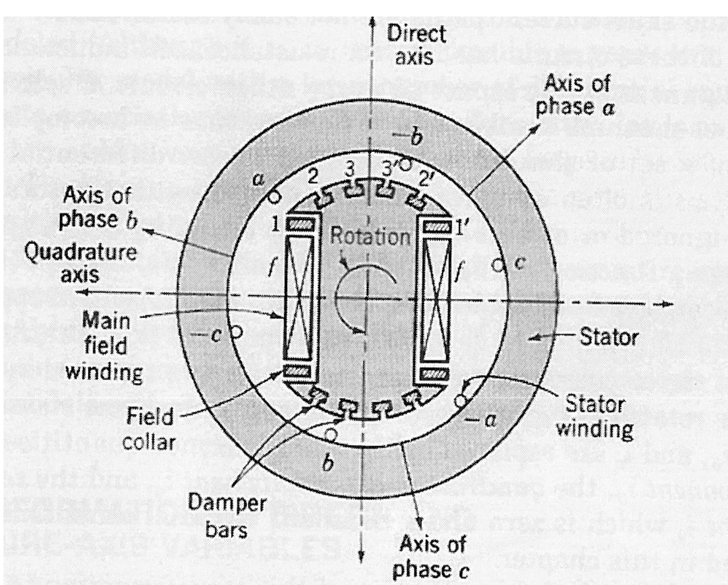

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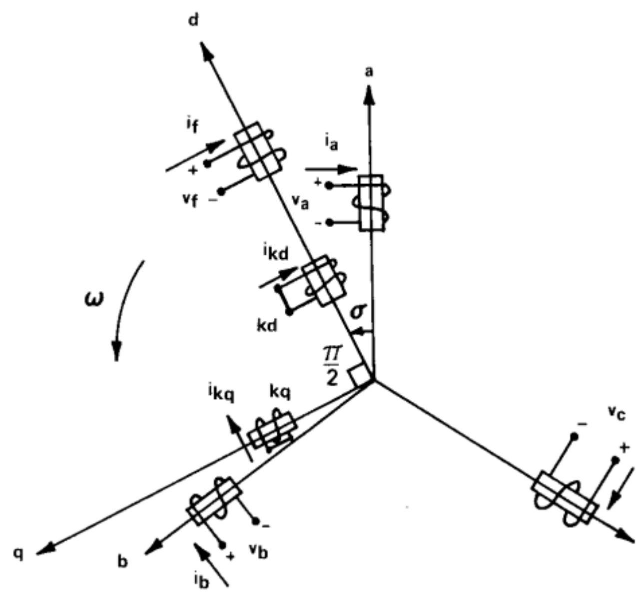


(d)

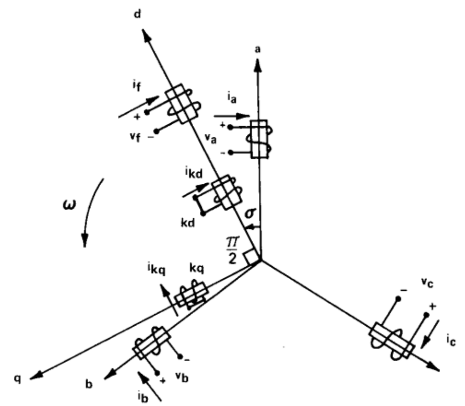
Schematic of the Generator Windings

- The State Space (SS) model of the synchronous generator can be expressed using compact matrix notation as follows:

$$\underline{V} = \underline{RI} + \frac{d}{dt}\{\underline{\Lambda}\} \text{ ; where } \underline{\Lambda} = \underline{LI} \text{ thus resulting in:}$$

$$\underline{V} = \underline{RI} + \frac{d}{dt}\{\underline{LI}\}$$

Accordingly, the synchronous generator with three armature phases a, b, c, the field winding f, and rotor equivalent damping circuits kd and kq can be represented by a 6th order SS model as follows:



Schematic of the Generator Windings

$$\begin{bmatrix} v_a \\ v_b \\ v_c \\ v_f \\ v_{kd} \\ v_{kq} \end{bmatrix} = \begin{bmatrix} r_s & 0 & 0 & 0 & 0 & 0 \\ 0 & r_s & 0 & 0 & 0 & 0 \\ 0 & 0 & r_s & 0 & 0 & 0 \\ 0 & 0 & 0 & r_f & 0 & 0 \\ 0 & 0 & 0 & 0 & r_{kd} & 0 \\ 0 & 0 & 0 & 0 & 0 & r_{kq} \end{bmatrix} \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_f \\ i_{kd} \\ i_{kq} \end{bmatrix}$$

$$+ \frac{d}{dt}\left\{ \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} & L_{af} & L_{akd} & L_{akq} \\ L_{ba} & L_{bb} & L_{bc} & L_{bf} & L_{bkd} & L_{bkq} \\ L_{ca} & L_{cb} & L_{cc} & L_{cf} & L_{ckd} & L_{ckq} \\ L_{fa} & L_{fb} & L_{fc} & L_{ff} & L_{fkd} & L_{fkq} \\ L_{kda} & L_{kdb} & L_{kdc} & L_{kdf} & L_{kdkd} & L_{kdkq} \\ L_{kqa} & L_{kqb} & L_{kqc} & L_{kqf} & L_{kqkd} & L_{kqkq} \end{bmatrix} \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_f \\ i_{kd} \\ i_{kq} \end{bmatrix} \right\}$$

Typically, most of the corresponding machine inductances have the following relationships:

$$L_{aa} = L_{sa} + L_{sv}\cos(2\theta)$$

$$L_{bb} = L_{sa} + L_{sv}\cos(2\theta - \frac{4\pi}{3})$$

$$L_{cc} = L_{sa} + L_{sv}\cos(2\theta - \frac{2\pi}{3})$$

$$L_{ab} = L_{ba} = -L_{ma} + L_{mv}\cos(2\theta - \frac{2\pi}{3})$$

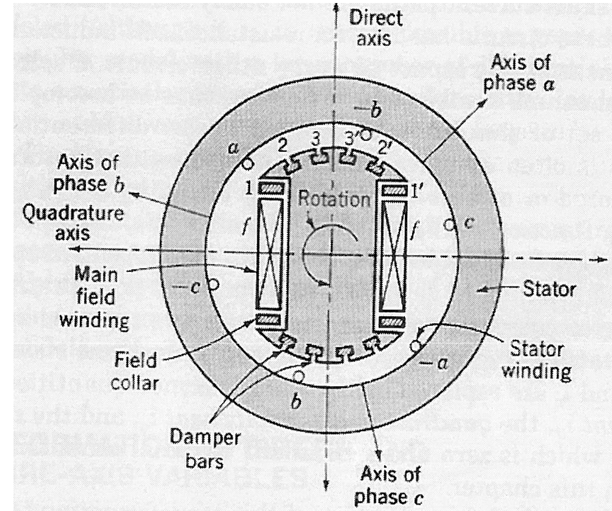
$$L_{cb} = L_{bc} = -L_{ma} + L_{mv}\cos(2\theta)$$

$$L_{ac} = L_{ca} = -L_{ma} + L_{mv}\cos(2\theta - \frac{4\pi}{3})$$

$$L_{af} = L_{fa} = L_{afm}\cos(2\theta)$$

$$L_{bf} = L_{fb} = L_{afm}\cos(2\theta - \frac{2\pi}{3})$$

$$L_{cf} = L_{fc} = L_{afm}\cos(2\theta - \frac{4\pi}{3})$$



$$L_{akd} = L_{kda} = L_{akdm}\cos(2\theta)$$

$$L_{bkd} = L_{kdb} = L_{akdm}\cos(2\theta - \frac{2\pi}{3})$$

$$L_{ckd} = L_{kdc} = L_{akdm}\cos(2\theta - \frac{4\pi}{3})$$

$$L_{akq} = L_{kqa} = -L_{akqm}\cos(2\theta)$$

$$L_{bkq} = L_{kqb} = -L_{bkqm}\cos(2\theta - \frac{2\pi}{3})$$

$$L_{ckq} = L_{kqc} = -L_{ckqm}\cos(2\theta - \frac{4\pi}{3})$$

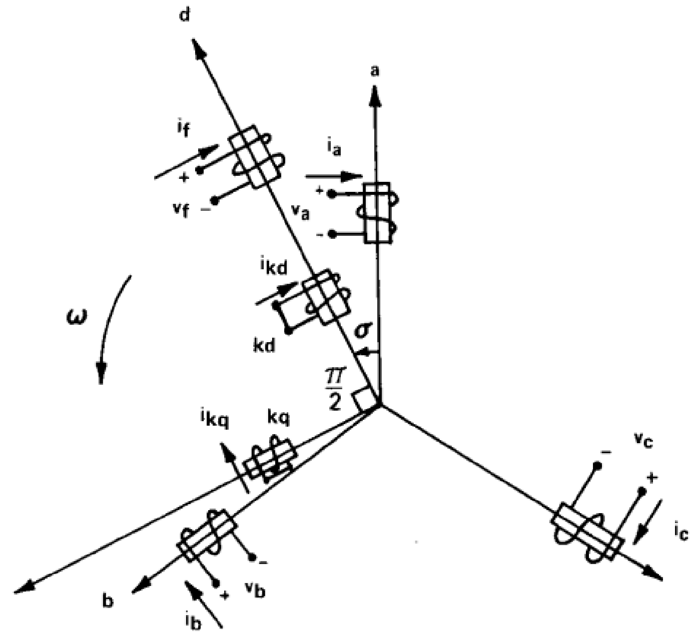
<u>Note</u>

These expressions have $\theta = \omega t$ which is a function of time t.

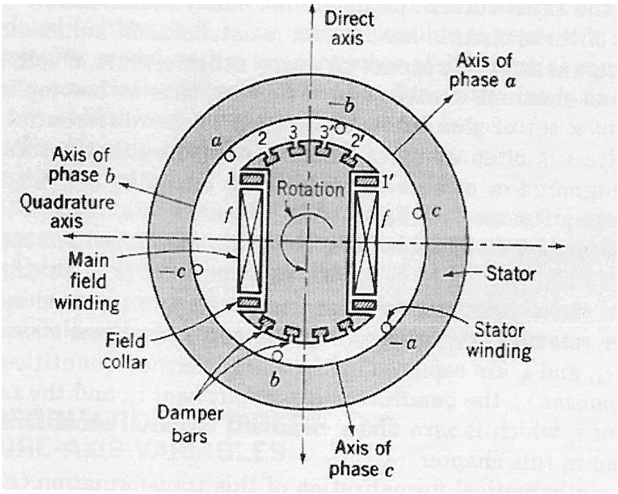
Advanced Topics

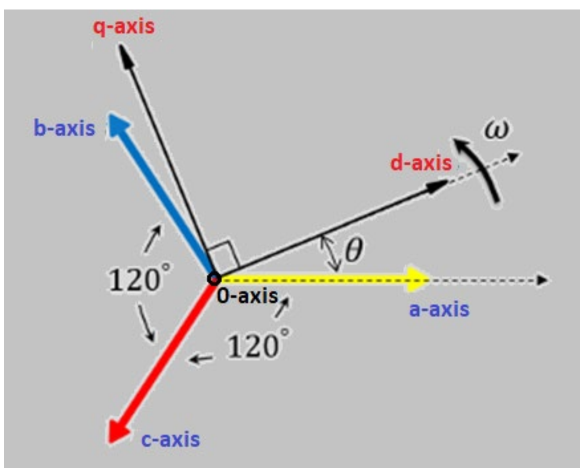
Synchronous Generator Park's Transformation

Park's Transformation is basically a transformation of machine quantities from a stationary (a,b,c) reference frame to a rotating (d,q,0) reference frame.

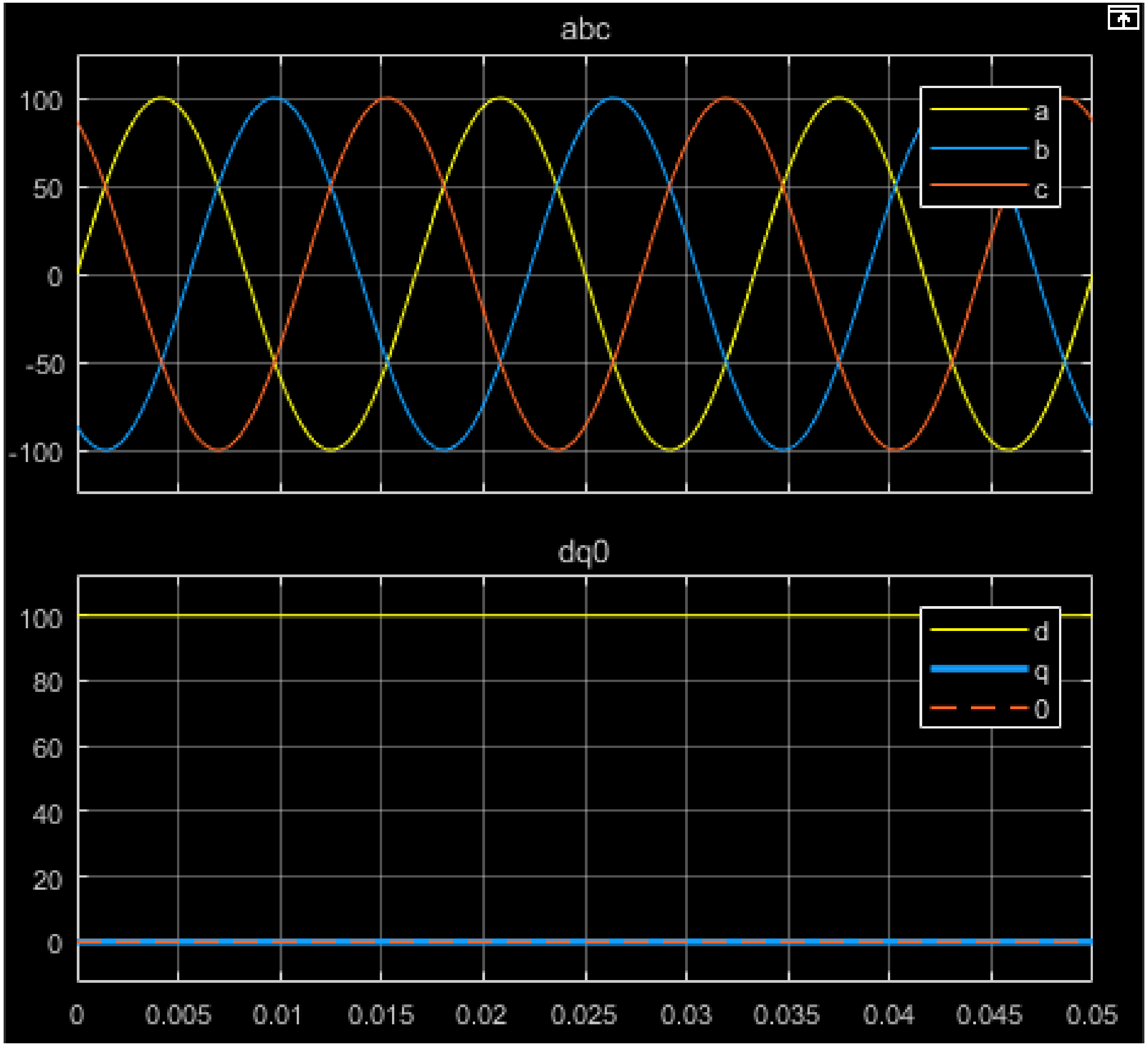


Schematic of the Generator Windings





The *a*-axis and the *d*-axis are initially aligned.



Alignment of the *a*-phase vector to the *d*-axis

Park's Transformation Application To Synchronous Machine

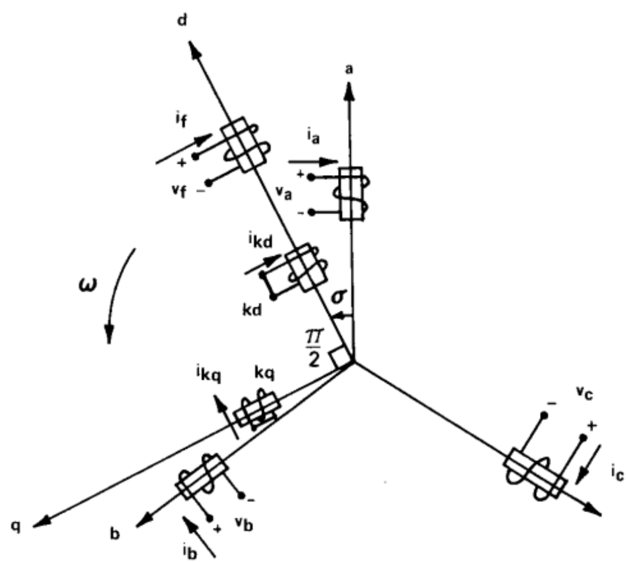
Note: one can use σ=θ

$$\begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} = \underbrace{\frac{2}{3}\begin{bmatrix} \cos\sigma & \cos(\sigma-2\pi/3) & \cos(\sigma-4\pi/3) \\ -\sin\sigma & -\sin(\sigma-2\pi/3) & -\sin(\sigma-4\pi/3) \\ 1/2 & 1/2 & 1/2 \end{bmatrix}}_{\underline{T}\ \text{Park's Transformation}} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \quad (1)$$

In matrix notation

$$\underline{I}_{dq0} = \underline{T} \cdot \underline{I}_{abc} \quad (2)$$

or $$\underline{I}_{abc} = \underline{T}^{-1} \cdot \underline{I}_{dq0} \quad (3)$$



Schematic of the Generator Windings

where

$$\underline{T}^{-1} = \begin{bmatrix} \cos\sigma & -\sin\sigma & 1 \\ \cos(\sigma-2\pi/3) & -\sin(\sigma-2\pi/3) & 1 \\ \cos(\sigma-4\pi/3) & -\sin(\sigma-4\pi/3) & 1 \end{bmatrix} \quad (4)$$

One also can write

$$\underline{V}_{dq0} = \underline{T} \cdot \underline{V}_{abc} \qquad (5)$$

and $$\underline{V}_{abc} = \underline{T}^{-1} \underline{V}_{dq0} \qquad (6)$$

The definitions can be applied to flux linkages as well:

$$\underline{\Lambda}_{dq0} = \underline{T} \, \underline{\Lambda}_{abc} \qquad (7)$$

and $$\underline{\Lambda}_{abc} = T^{-1} \underline{\Lambda}_{dq0} \qquad (8)$$

Consider the synchronous machine state model which was derived earlier in class

$$\begin{bmatrix} \underline{V}_{abc} \\ \underline{V}_{fkdkq} \end{bmatrix} = \begin{bmatrix} \underline{R}_{ss} & \underline{0} \\ \underline{0} & \underline{R}_{rr} \end{bmatrix} \begin{bmatrix} \underline{I}_{abc} \\ \underline{I}_{fkdkq} \end{bmatrix} + \frac{d}{dt}\left\{ \begin{bmatrix} \underline{\mathcal{L}}_{ss} & \underline{\mathcal{L}}_{sr} \\ \underline{\mathcal{L}}_{sr}^{t} & \underline{\mathcal{L}}_{rr} \end{bmatrix} \begin{bmatrix} \underline{I}_{abc} \\ \underline{I}_{fkdkq} \end{bmatrix} \right\} \quad (9)$$

Or equation (9) can be written as:

$$\begin{bmatrix} \underline{V}_{abc} \\ \underline{V}_{fkdkq} \end{bmatrix} = \begin{bmatrix} \underline{R}_{ss} & \underline{0} \\ \underline{0} & \underline{R}_{rr} \end{bmatrix} \begin{bmatrix} \underline{I}_{abc} \\ \underline{I}_{fkdkq} \end{bmatrix} + \frac{d}{dt}\left\{ \begin{bmatrix} \underline{\Lambda}_{abc} \\ \underline{\Lambda}_{fkdkq} \end{bmatrix} \right\} \quad (10)$$

Applying Park's Transformation, equation (10) becomes:

$$\begin{bmatrix} \underline{T} \cdot \underline{V}_{abc} \\ \underline{V}_{fkdkq} \end{bmatrix} = \begin{bmatrix} \underline{T}\,\underline{R}_{ss}\,\underline{T}^{-1} & \underline{0} \\ \underline{0} & \underline{R}_{rr} \end{bmatrix} \begin{bmatrix} \underline{T} \cdot \underline{I}_{abc} \\ \underline{I}_{fkdkq} \end{bmatrix} +$$

$$\begin{bmatrix} \underline{T} & \underline{0} \\ \underline{0} & \underline{U} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} \underline{\Lambda}_{abc} \\ \underline{\Lambda}_{fkdkq} \end{bmatrix} \qquad (11)$$

where $\underline{U}$ is the identity matrix

note $\underline{\Lambda}_{abc} = T^{-1}\,\underline{\Lambda}_{dqo}$ (12)

and from equations (9) and (10)

$$\underline{\Lambda}_{abc} = \underline{\mathcal{L}}_{ss} \cdot \underline{I}_{abc} + \underline{\mathcal{L}}_{sr} \cdot \underline{I}_{fkdkq} \tag{13}$$

$$\underline{T}^{-1}\,\underline{\Lambda}_{dqo} = \underline{\mathcal{L}}_{ss}\,\underline{T}^{-1}\,\underline{I}_{dqo} + \underline{\mathcal{L}}_{sr} \cdot \underline{I}_{fkdkq} \tag{14}$$

$$\therefore\ \underline{\Lambda}_{dqo} = \underline{T}\,\underline{\mathcal{L}}_{ss}\,\underline{T}^{-1} \cdot \underline{I}_{dqo} + \underline{T}\,\underline{\mathcal{L}}_{sr}\,\underline{I}_{fkdkq} \tag{15}$$

Hence by using equations (11) and (12) one can writhe the following:

$$\underline{V}_{dq0} = \left(\underline{T}\,\underline{R}_{ss}\,\underline{T}^{-1} \right) \underline{I}_{dq0} + \underline{T} \frac{d}{dt} \left(\underline{T}^{-1} \underline{\Lambda}_{dq0} \right) \tag{16}$$

$$\underline{V}_{fkdkq} = \left(\underline{R}_{rr} \right) \underline{I}_{fkdkq} + \frac{d}{dt} \left(\underline{\Lambda}_{fkdkq} \right) \tag{17}$$

* The next step is to rewrite equations (16) and (17) in terms of the synchronous machine inductance submatrices $\underline{\mathcal{L}}_{ss}$, $\underline{\mathcal{L}}_{sr}$, $\underline{\mathcal{L}}_{sr}^{t}$ and $\underline{\mathcal{L}}_{rr}$ and the resistance submatrices $\underline{R}_{ss}$ and $\underline{R}_{rr}$

(a) First consider equation (17). From equations (9) and (10) the following can be written for $\underline{\Lambda}_{fkdkq}$:

$$\underline{\Lambda}_{fkdkq} = \left(\underline{\mathcal{L}}_{sr}^{t} \right) \cdot \underline{I}_{abc} + \left(\underline{\mathcal{L}}_{rr} \right) \cdot \underline{I}_{fkdkq} \tag{18}$$

Applying Park's Transformation to equation (18) results in the following:

$$\underline{\Lambda}_{fKdKq} = \left(\underline{\mathcal{L}}_{sr}^{t} \cdot \underline{T}^{-1} \right) \underline{I}_{dq0} + \left(\underline{\mathcal{L}}_{rr} \right) \cdot \underline{I}_{fKdKq} \qquad (19)$$

Next consider $\frac{d}{dt} \underline{\Lambda}_{fKdKq}$. From equation (19) one can write the following:

$$\frac{d}{dt}\left\{ \underline{\Lambda}_{fKdKq} \right\} = \frac{d}{dt}\left\{ \left(\underline{\mathcal{L}}_{sr}^{t} \cdot \underline{T}^{-1} \right) \right\} \underline{I}_{dq0} + \left(\underline{\mathcal{L}}_{sr}^{t} \cdot \underline{T}^{-1} \right) \frac{d}{dt}\left\{ \underline{I}_{dq0} \right\} +$$

$$\frac{d}{dt}\left\{ \left(\underline{\mathcal{L}}_{rr} \right) \right\} \underline{I}_{fKKq} + \left(\underline{\mathcal{L}}_{rr} \right) \frac{d}{dt}\left\{ \underline{I}_{fKdKq} \right\} \qquad (20)$$

However it was shown earlier in class when we derived the state model for the synchronous machine that the matrix $\underline{\mathcal{L}}_{rr}$ is time independent.

Furthermore, one can show that the matrix $\left(\underline{\mathcal{L}}_{sr}^{t} \cdot \underline{T}^{-1}\right)$ is as follows:

$$\left(\underline{\mathcal{L}}_{sr}^{t} \cdot \underline{T}^{-1}\right) = \begin{bmatrix} \frac{3}{2} L_{afm} & 0 & 0 \\ \frac{3}{2} L_{akdm} & 0 & 0 \\ 0 & \frac{3}{2} L_{akqm} & 0 \end{bmatrix} \qquad (21)$$

where L_{afm}, L_{akdm}, and L_{akqm} are constants as was shown earlier in class during the development of the synchronous machine state model.

From the above discussion, one can rewrite equation (20) as follows:

$$\boxed{\frac{d}{dt}\left\{\underline{\Lambda}_{fkdkq}\right\} = \left(\underline{\mathcal{L}}_{sr}^{t}\,\underline{T}^{-1}\right)\frac{d}{dt}\left(\underline{I}_{dq0}\right) + \left(\underline{\mathcal{L}}_{rr}\right)\frac{d}{dt}\left(\underline{I}_{fkdkq}\right) \qquad (22)}$$

(b) Next consider the term $(\underline{T}\,\underline{R}_{ss}\,\underline{T}^{-1})$ in equation (16):

$$\underline{T}\cdot\underline{R}_{ss}\cdot\underline{T}^{-1} = \underline{T}\cdot r_s\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\cdot\underline{T}^{-1} = r_s\,\underline{T}\cdot\underline{U}\cdot\underline{T}^{-1}$$

$$= r_s\,\underline{T}\,\underline{T}^{-1} = r_s\,\underline{U} = \begin{bmatrix}r_s & 0 & 0\\ 0 & r_s & 0\\ 0 & 0 & r_s\end{bmatrix} \qquad (23)$$

(c) Next Consider the term $\underline{T}\frac{d}{dt}\left(\underline{T}^{-1}\underline{\Lambda}_{dq0}\right)$ of equation (16)

It can be written as follows:

$$\underline{T}\cdot\frac{d}{dt}\left(\underline{T}^{-1}\underline{\Lambda}_{dq0}\right) = \underline{T}\frac{d}{dt}\left(\underline{\Lambda}_{abc}\right) \tag{24}$$

Note also:

$$\frac{d}{dt}\left(\underline{\Lambda}_{dq0}\right) = \frac{d}{dt}\left(\underline{T}\,\underline{\Lambda}_{abc}\right) = \underline{T}\cdot\frac{d}{dt}\left(\underline{\Lambda}_{abc}\right) + \left(\frac{d}{dt}\underline{T}\right)\underline{\Lambda}_{abc} \tag{25}$$

$$\therefore\ \underline{T}\cdot\frac{d}{dt}\left(\underline{\Lambda}_{abc}\right) = \frac{d}{dt}\left(\underline{\Lambda}_{dq0}\right) - \left(\frac{d}{dt}\underline{T}\right)\underline{T}^{-1}\underline{\Lambda}_{dq0} \tag{26}$$

From equation (24) and (26)

$$\underline{T} \cdot \frac{d}{dt}\left(\underline{T}^{-1} \underline{\Lambda}_{dq0}\right) = \frac{d}{dt}\left(\underline{\Lambda}_{dq0}\right) - \left(\frac{d}{dt}\underline{T}\right)\underline{T}^{-1}\underline{\Lambda}_{dq0} \qquad (27)$$

It can be shown that:

$$\left(\frac{d}{dt}\underline{T}\right) = \omega\, \underline{G} \cdot \underline{T} \qquad (28)$$

where $$\underline{G} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (29)$$

Accordingly equation (28) can be written as follows:

$$\underline{T}\left(\frac{d}{dt}\underline{T}^{-1}\underline{\Lambda}_{dq0}\right) = \left(\frac{d}{dt}\underline{\Lambda}_{dq0}\right) - \left(\omega\,\underline{G}\,\underline{T}\,\underline{T}^{-1}\Lambda_{dq0}\right)$$

$$= \left(\frac{d}{dt}\underline{\Lambda}_{dq0}\right) - \left(\omega\,\underline{G}\,\Lambda_{dq0}\right) \qquad (30)$$

Next consider the term $\frac{d}{dt}\underline{\Lambda}_{dqo}$ of equation (30). From equation (15) one can write the following:

$$\frac{d}{dt}\underline{\Lambda}_{dqo} = \frac{d}{dt}\left\{\left(\underline{T}\,\underline{\mathcal{L}}_{ss}\,\underline{T}^{-1}\right)\underline{I}_{dqo} + \left(\underline{T}\,\underline{\mathcal{L}}_{sr}\right)\underline{I}_{fkdkq}\right\}$$

$$= \frac{d}{dt}\left\{\left(\underline{T}\,\underline{\mathcal{L}}_{ss}\,\underline{T}^{-1}\right)\right\}\underline{I}_{dqo} + \left(\underline{T}\,\underline{\mathcal{L}}_{ss}\,\underline{T}^{-1}\right)\frac{d}{dt}\left\{\underline{I}_{dqo}\right\}$$

$$+ \frac{d}{dt}\left\{\left(\underline{T}\,\underline{\mathcal{L}}_{sr}\right)\right\}\underline{I}_{fkdkq} + \left(\underline{T}\,\underline{\mathcal{L}}_{sr}\right)\frac{d}{dt}\left\{\underline{I}_{fkdkq}\right\} \qquad (31)$$

But it can be shown that:

$$\left(\underline{T}\,\underline{\mathcal{L}}_{ss}\,\underline{T}^{-1}\right) = \begin{bmatrix} (L_{sa}+L_{ma}+\frac{3}{2}L_v) & 0 & 0 \\ 0 & (L_{sa}+L_{ma}-\frac{3}{2}L_v) & 0 \\ 0 & 0 & (L_{sa}-2L_{ma}) \end{bmatrix} \quad (32)$$

where L_{sa}, L_{ma}, and L_v are constants as was shown earlier when the synchronous machine state model was developed in class.

Also, it can be shown that:

$$\left(\underline{T}\,\underline{\mathcal{L}}_{sr}\right) = \begin{bmatrix} L_{afm} & L_{akdm} & 0 \\ 0 & 0 & L_{akqm} \\ 0 & 0 & 0 \end{bmatrix} \quad (33)$$

where L_{afm}, L_{akdm}, and L_{akqm} are constants as was shown earlier when the synchronous machine state model was developed in class.

Hence from equations (32) and (33) one can conclude that the matrices $(\underline{T}\,\underline{\underline{\mathcal{L}}}_{ss}\,\underline{T}^{-1})$ and $(\underline{T}\,\underline{\underline{\mathcal{L}}}_{sr})$ are time independent.

Accordingly, equation (31) reduces to the following:

$$\frac{d}{dt}\left(\underline{\Lambda}_{dq0}\right) = \left(\underline{T}\cdot\underline{\underline{\mathcal{L}}}_{ss}\cdot\underline{T}^{-1}\right)\cdot\frac{d}{dt}\left(\underline{I}_{dq0}\right) + \left(\underline{T}\cdot\underline{\underline{\mathcal{L}}}_{sr}\right)\frac{d}{dt}\left(\underline{I}_{fkdkq}\right) \quad (34)$$

Now by using equations (15) and (34) in equation (30) one obtains the following:

$$\boxed{\begin{aligned}\underline{T}\left(\frac{d}{dt}\underline{T}^{-1}\underline{\Lambda}_{dq0}\right) &= \left(\underline{T}\,\underline{\underline{\mathcal{L}}}_{ss}\,\underline{T}^{-1}\right)\dot{\underline{I}}_{dq0} + \left(\underline{T}\cdot\underline{\underline{\mathcal{L}}}_{sr}\right)\cdot\dot{\underline{I}}_{fkdkq} \\ &\quad - \omega\underline{G}\left(\underline{T}\cdot\underline{\underline{\mathcal{L}}}_{ss}\cdot\underline{T}^{-1}\right)\cdot\underline{I}_{dq0} - \omega\underline{G}\cdot\left(\underline{T}\cdot\underline{\underline{\mathcal{L}}}_{sr}\right)\cdot\underline{I}_{fkdkq} \qquad (35)\end{aligned}}$$

Where the dot $(\dot{})$ in a vector represents the operator $\left(\frac{d}{dt}\right)$

Now one is ready the use equations (22), (23), and (35) in equations (16) and (17).

Doing so, one arrives at the following equation:

$$\begin{bmatrix} \underline{V}_{dq0} \\ \underline{V}_{fkdkq} \end{bmatrix} = \begin{bmatrix} \underline{R}_{ss} & \underline{0} \\ \underline{0} & \underline{R}_{rr} \end{bmatrix} \begin{bmatrix} \underline{I}_{dq0} \\ \underline{I}_{fkdkq} \end{bmatrix} +$$

$$\begin{bmatrix} (\underline{T} \cdot \underline{\mathcal{L}}_{ss} \cdot \underline{T}^{-1}) & (\underline{T} \cdot \underline{\mathcal{L}}_{sr}) \\ (\underline{\mathcal{L}}_{sr}^{t} \cdot \underline{T}^{-1}) & (\underline{\mathcal{L}}_{rr}) \end{bmatrix} \begin{bmatrix} \underline{\dot{I}}_{dq0} \\ \underline{\dot{I}}_{fkdkq} \end{bmatrix} -$$

$$\omega \begin{bmatrix} \underline{G} \cdot (\underline{T} \cdot \underline{\mathcal{L}}_{ss} \cdot \underline{T}^{-1}) & \underline{G} \cdot (\underline{T} \cdot \underline{\mathcal{L}}_{sr}) \\ \underline{0} & \underline{0} \end{bmatrix} \begin{bmatrix} \underline{I}_{dq0} \\ \underline{I}_{fkdkq} \end{bmatrix} \qquad (36)$$

Now one needs to evaluate $\underline{G} \cdot (\underline{T} \cdot \underline{\mathcal{L}}_{ss} \cdot \underline{T}^{-1})$ and $\underline{G}(\underline{T} \cdot \underline{\mathcal{L}}_{sr})$.

From equations (29) and (32) one obtains the following:

$$\underline{G} \cdot (\underline{T}\, \underline{\mathcal{L}}_{ss} \cdot \underline{T}^{-1}) = \begin{bmatrix} 0 & (L_{sa} + L_{ma} - \frac{3}{2} L_v) & 0 \\ -(L_{sa} + L_{ma} + \frac{3}{2} L_v) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (37)$$

And from equations (29) and (33) one obtains the following:

$$\underline{G} \cdot (\underline{T} \cdot \underline{\mathcal{L}}_{sr}) = \begin{bmatrix} 0 & 0 & L_{akqm} \\ -L_{afm} & -L_{akdm} & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (38)$$

Accordingly, the final state model in expanded form is as follows:

$$
\begin{bmatrix} v_d \\ v_q \\ v_0 \\ \hline v_f \\ v_{kd}=0 \\ v_{kq}=0 \end{bmatrix}
=
\left[\begin{array}{ccc|ccc}
r_s & 0 & 0 & 0 & 0 & 0 \\
0 & r_s & 0 & 0 & 0 & 0 \\
0 & 0 & r_s & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & r_f & 0 & 0 \\
0 & 0 & 0 & 0 & r_{kd} & 0 \\
0 & 0 & 0 & 0 & 0 & r_{kq}
\end{array}\right]
\begin{bmatrix} i_d \\ i_q \\ i_0 \\ \hline i_f \\ i_{kd} \\ i_{kq} \end{bmatrix}
$$

$$
+
\left[\begin{array}{ccc|ccc}
0 & -\omega(L_{sa}+L_{ma}-\frac{3}{2}L_v) & 0 & 0 & 0 & -\omega L_{akqm} \\
\omega(L_{sa}+L_{ma}+\frac{3}{2}L_v) & 0 & 0 & \omega L_{afm} & \omega L_{akdm} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
\begin{bmatrix} i_d \\ i_q \\ i_0 \\ \hline i_f \\ i_{kd} \\ i_{kq} \end{bmatrix}
$$

$$
+
\left[\begin{array}{ccc|ccc}
(L_{sa}+L_{ma}+\frac{3}{2}L_v) & 0 & 0 & L_{afm} & L_{akdm} & 0 \\
0 & (L_{sa}+L_{ma}-\frac{3}{2}L_v) & 0 & 0 & 0 & L_{akqm} \\
0 & 0 & (L_{sa}-2L_{ma}) & 0 & 0 & 0 \\
\hline
\frac{3}{2}L_{afm} & 0 & 0 & L_{ff} & L_{fkd} & 0 \\
\frac{3}{2}L_{akdm} & 0 & 0 & L_{fkd} & L_{kdkd} & 0 \\
0 & \frac{3}{2}L_{akqm} & 0 & 0 & 0 & L_{kqkq}
\end{array}\right]
\begin{bmatrix} \dot{i}_d \\ \dot{i}_q \\ \dot{i}_0 \\ \hline \dot{i}_f \\ \dot{i}_{kd} \\ \dot{i}_{kq} \end{bmatrix}
\qquad (39)
$$

$$\begin{bmatrix} v_d \\ v_q \\ v_o \\ v_f \\ 0 \\ 0 \end{bmatrix} = \left[\begin{array}{ccc|ccc} r_s & -\omega L_q & 0 & 0 & 0 & -\omega KL_{akqm} \\ \omega L_d & r_s & 0 & \omega KL_{afm} & \omega KL_{akdm} & 0 \\ 0 & 0 & r_s & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & r_f & 0 & 0 \\ 0 & 0 & 0 & 0 & r_{kd} & 0 \\ 0 & 0 & 0 & 0 & 0 & r_{kq} \end{array}\right] \left[\begin{array}{c} i_d \\ i_q \\ i_o \\ \hline i_f \\ i_{kd} \\ i_{kq} \end{array}\right] +$$

$$\left[\begin{array}{ccc|ccc} L_d & 0 & 0 & KL_{afm} & KL_{akdm} & 0 \\ 0 & L_q & 0 & 0 & 0 & KL_{akqm} \\ 0 & 0 & L_o & 0 & 0 & 0 \\ \hline KL_{afm} & 0 & 0 & L_{ff} & L_{fkd} & 0 \\ KL_{akdm} & 0 & 0 & L_{fkd} & L_{kdkd} & 0 \\ 0 & KL_{akqm} & 0 & 0 & 0 & L_{kqkq} \end{array}\right] \cdot \frac{d}{dt} \left[\begin{array}{c} i_d \\ i_q \\ i_o \\ \hline i_f \\ i_{kd} \\ i_{kq} \end{array}\right] \tag{39}$$

The expanded model of equation (39) can be written in compact form as follows:

$$\underline{V} = \underline{R}\,\underline{I} + \omega\,\underline{\mathcal{L}}_1\,\underline{I} + \underline{\mathcal{L}}_2\,\underline{\dot{I}} \qquad (40)$$

where $\underline{R}$, $\underline{\mathcal{L}}_1$ and $\underline{\mathcal{L}}_2$ are time independent.

Equation (40) can be written in the standard form as follows:

$$\underline{\dot{I}} = -\underline{\mathcal{L}}_2^{-1}\,(\underline{R} + \omega\,\underline{\mathcal{L}}_1)\cdot\underline{I}_1 + \underline{\mathcal{L}}_2^{-1}\cdot\underline{V} \qquad (41)$$

Which is of the form:

$$\underline{\dot{X}} = \underline{A}\,\underline{X} + \underline{B}\,\underline{U} \qquad (42)$$

which can be solved using the routines handed in class.

Electromagnetic (T_{em}) Torque Expression in the dq0 Frame of Reference

$$T_{em} = \frac{3}{2}\,[\{L_{afm}\, i_f\, i_q\} + \{(L_d - L_q)\, i_d\, i_q\} + \{(L_{akdm}\, i_{kd}\, i_q - L_{akqm}\, i_{kd}\, i_q - L_{akqm}\, i_{kq}\, i_d)\}]$$

{Field Synchronous Torque} {Reluctance Torque} {Damping Induction Torque}

EMF Components: e_d and e_q Expressions in the dq0 Frame of Reference

$$e_d = -\,\omega[L_q\, i_q + L_{akqm}\, i_{kq}]$$

$$e_q = \omega[L_d\, i_d + L_{afm}\, i_f + L_{akdm}\, i_{kd}]$$

Case Study Details

Consider a 3-phase, field wound, 2-pole *ac* synchronous generator, rated at 30 kVA, 208 V, and 400 Hz, whose cross-section is shown.

For full details on the application of state space model in the dq0 frame of reference, see the following paper:

Arkadan, A.A., Hijazi, T.M., Demerdash, N.A., Vaidya, J.G., and Maddali, V.K., "Theoretical Development and Experimental Verification of a DC-AC Electronically Rectified Load-Generator System Model Compatible with Common Network Analysis Software Packages," IEEE Trans. on Energy Conversion, Vol. EC-3, No. 1, pp. 123-131, Mar. 1988.

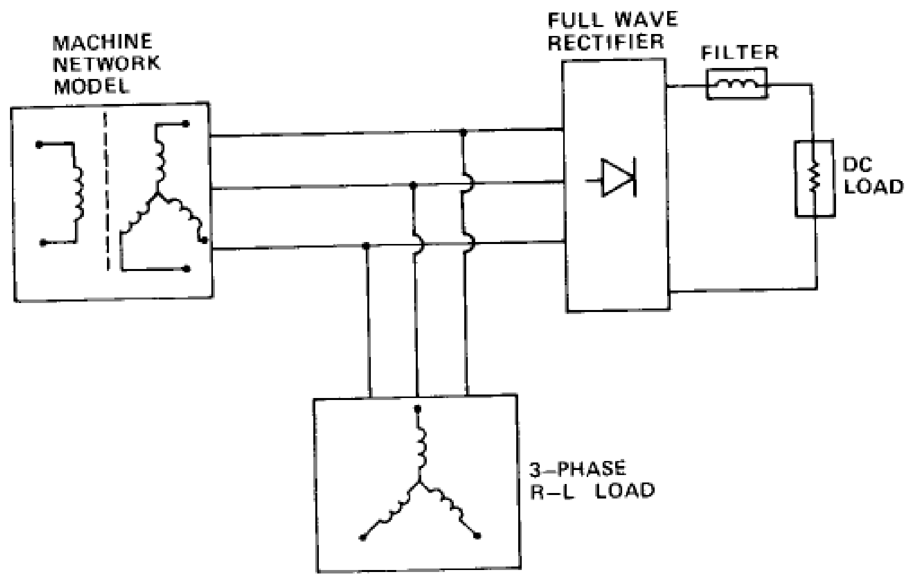


Fig.(1): Stand-Alone Generator – DC Load System

In the generator system of Figure (1), the machine consists of a three phase (stator) armature winding connected to a fullwave rectifier bridge, while its rotating member consists of a salient-pole rotor structure on which the field winding and appropriate damper windings are mounted. Figure (2) depicts a schematic of the three armature phase windings, represented by the stationary coils (a), (b) and (c), and the field as well as damper windings represented by the rotating coils (f), (kd) and (kq), respectively. Here, kd and kq represent damping effects along the direct and quadrature axes of the machine, and are both shorted coils. Notice, throughout the paper, the current is taken positive flowing into the (+) designated terminal of any winding according to standard consumer (or load) notation.

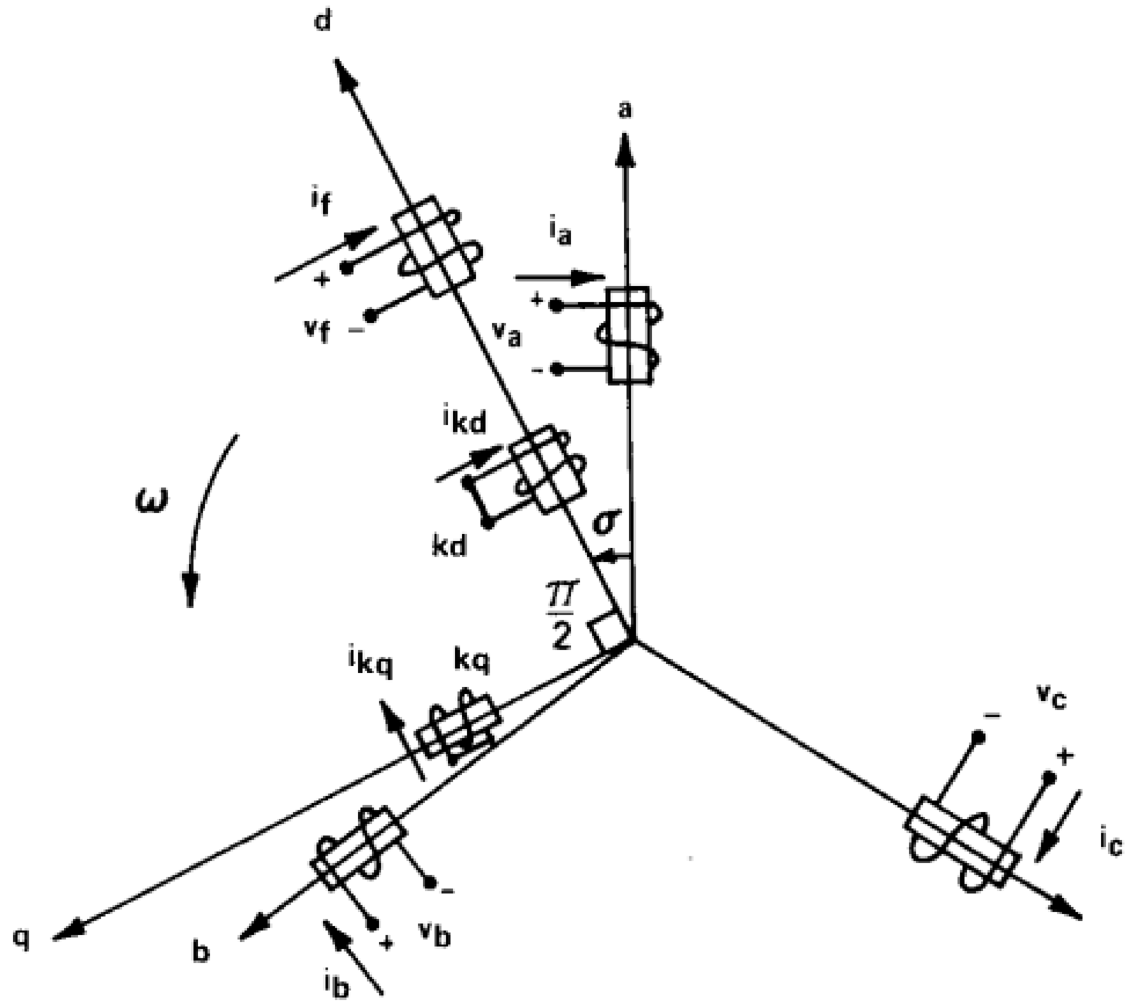


Fig. 2: Schematic of the Generator Windings

Accordingly, in compact matrix notation the state equations governing the dynamics of the generator can be written in the abc frame of reference as follows (all voltages and currents are instantaneous quantities):

$$\begin{bmatrix} \underline{V}_{abc} \\ \underline{V}_{fkdkq} \end{bmatrix} = \begin{bmatrix} \underline{R}_{ss} & \underline{0} \\ \underline{0} & \underline{R}_{rr} \end{bmatrix} \begin{bmatrix} \underline{I}_{abc} \\ \underline{I}_{fkdkq} \end{bmatrix} + \frac{d}{dt}\left\{ \begin{bmatrix} \underline{L}_{ss} & \underline{L}_{sr} \\ \underline{L}_{rs} & \underline{L}_{rr} \end{bmatrix} \begin{bmatrix} \underline{I}_{abc} \\ \underline{I}_{fkdkq} \end{bmatrix} \right\} \quad (1)$$

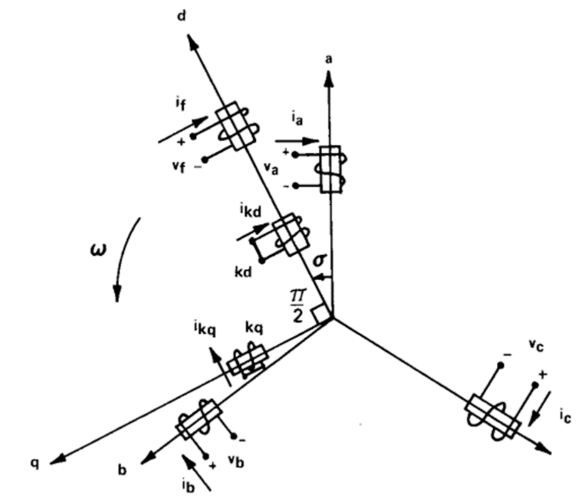
Here, $\underline{V}_{abc}$ is the vector of the three phase line to neutral terminal voltages, v_a, v_b and v_c,

$\underline{V}_{fkdkq}$ is the vector of the field and equivalent damper winding voltages, v_f, $v_{kd} = 0$, and $v_{kq} = 0$,

$\underline{I}_{abc}$ is the vector of the three phase currents, i_a, i_b and i_c,

$\underline{I}_{fkdkq}$ is the vector of the field, direct axis damping, and quadrature axis damping currents, i_f, i_{kd} and i_{kq}, respectively,

$\underline{R}_{ss}$ is a diagonal (3 x 3) matrix representing the a, b and c phase winding resistances, $r_a = r_b = r_c = r_s$,



In this development, the axis of phase (a), namely the a-axis, is chosen as the reference throughout. Also, the angular position of the rotor, σ, is defined as the angle between the d-axis and the a-axis at any instant in time. Throughout, positive rotor rotation is taken counter-clockwise, and again the standard consumer "load" notation is used in conjunction with any current-voltage relationships.

$$\begin{bmatrix} \underline{V}_{abc} \\ \underline{V}_{fkdkq} \end{bmatrix} = \begin{bmatrix} \underline{R}_{ss} & \underline{0} \\ \underline{0} & \underline{R}_{rr} \end{bmatrix} \begin{bmatrix} \underline{I}_{abc} \\ \underline{I}_{fkdkq} \end{bmatrix} + \frac{d}{dt} \left\{ \begin{bmatrix} \underline{L}_{ss} & \underline{L}_{sr} \\ \underline{L}_{rs} & \underline{L}_{rr} \end{bmatrix} \begin{bmatrix} \underline{I}_{abc} \\ \underline{I}_{fkdkq} \end{bmatrix} \right\} \tag{1}$$

$\underline{R}_{rr}$ is a diagonal (3 x 3) matrix representing the field, direct axis damper, and quadrature axis damper winding resistances, r_f, r_{kd}, and r_{kq}, respectively,

$\underline{L}_{ss}$ is a (3 x 3) matrix representing the armature phase windings' self and mutual inductances between the phases, L_{aa}, L_{bb}, L_{cc}, $L_{ab} = L_{ba}$, $L_{bc} = L_{cb}$, and $L_{ca} = L_{ac}$, all of which are well known functions of the rotor position angle, σ, see reference [8] for details,

$\underline{L}_{sr} = \underline{L}_{rs}^{t}$, is a (3 x 3) matrix representing the mutual inductances between the armature (stator) phase windings and the rotor windings, all of which are well known functions of the rotor position angle, σ.

$\underline{L}_{rr}$ is a (3 x 3) matrix representing the rotor windings' self and mutual inductances, in which the mutuals between any windings along the d-axis and any windings along the q-axis are zero, and all other nonzero inductance terms are independent of the rotor position angle, σ.

In order to get rid of the dependence of the various inductance terms in the second right hand side term of equation (1) on the rotor position angle, σ, the practice of transformation to a dqo frame of reference is widely accepted by various machine analysts and designers. This transformation approach has been adopted here using a power invariant form of Park's dqo transformation, see reference [9], such that

$$\underline{V}_{dqo} = \underline{T}\cdot\underline{V}_{abc}, \text{ and } \underline{V}_{abc} = \underline{T}^{-1}\cdot\underline{V}_{dqo} \tag{2}$$

Also, $\underline{I}_{dqo} = \underline{T}\cdot\underline{I}_{abc}$, and $\underline{I}_{abc} = \underline{T}^{-1}\cdot\underline{I}_{dqo}$ (3)

where

$$\underline{T} = \sqrt{\frac{2}{3}}\begin{bmatrix} \cos(\sigma) & \cos(\sigma-2\pi/3) & \cos(\sigma-4\pi/3) \\ -\sin(\sigma) & -\sin(\sigma-2\pi/3) & -\sin(\sigma-4\pi/3) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \tag{4}$$

Here, the instantaneous power, $p = (i_a v_a + i_b v_b + i_c v_c) = (i_d v_d + i_q v_q + i_o v_o)$. Applying this transformation to equation (1) yields the following state equations which govern the dynamics of the generator in the dqo frame of reference:

$$
\begin{bmatrix} v_d \\ v_q \\ v_o \\ v_f \\ 0 \\ 0 \end{bmatrix} =
\left[\begin{array}{ccc|ccc}
r_s & -\omega L_q & 0 & 0 & 0 & -\omega K L_{akqm} \\
\omega L_d & r_s & 0 & \omega K L_{afm} & \omega K L_{akdm} & 0 \\
0 & 0 & r_s & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & r_f & 0 & 0 \\
0 & 0 & 0 & 0 & r_{kd} & 0 \\
0 & 0 & 0 & 0 & 0 & r_{kq}
\end{array}\right]
\left[\begin{array}{c} i_d \\ i_q \\ i_o \\ \hline i_f \\ i_{kd} \\ i_{kq} \end{array}\right] +
$$

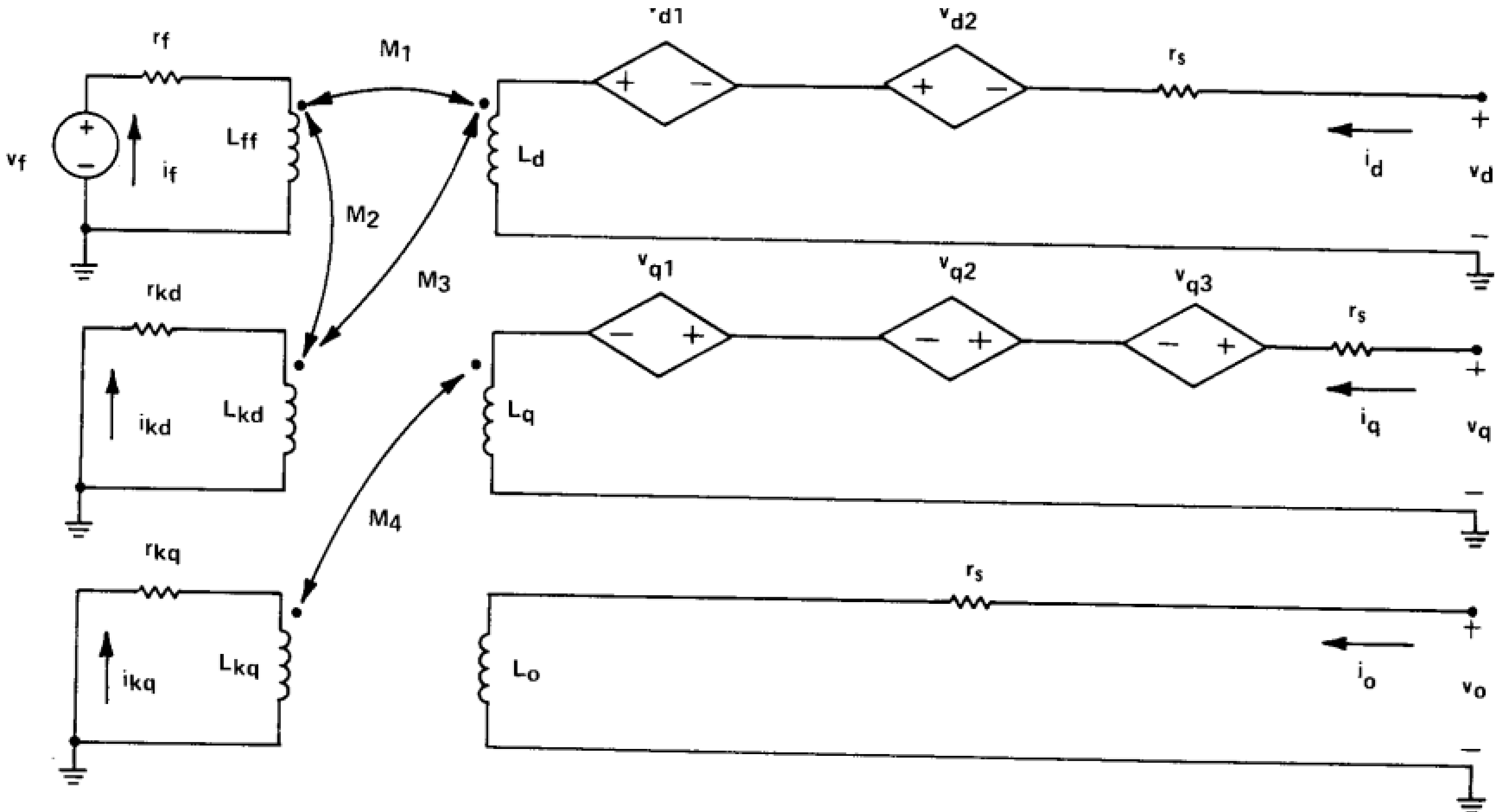
$$
\left[\begin{array}{ccc|ccc}
L_d & 0 & 0 & K L_{afm} & K L_{akdm} & 0 \\
0 & L_q & 0 & 0 & 0 & K L_{akqm} \\
0 & 0 & L_o & 0 & 0 & 0 \\
\hline
K L_{afm} & 0 & 0 & L_{ff} & L_{fkd} & 0 \\
K L_{akdm} & 0 & 0 & L_{fkd} & L_{kdkd} & 0 \\
0 & K L_{akqm} & 0 & 0 & 0 & L_{kqkq}
\end{array}\right] \cdot \frac{d}{dt}
\left[\begin{array}{c} i_d \\ i_q \\ i_o \\ \hline i_f \\ i_{kd} \\ i_{kq} \end{array}\right] \qquad (5)
$$

Here,

* v_d, v_q and v_o are the direct, quadrature and zero sequence components of the armature voltage,
* i_d, i_q and i_o are the direct, quadrature and zero sequence components of the armature current,
* L_d and L_q are the well known direct and quadrature axes armature inductances, respectively, and L_o is the zero sequence armature inductance,
* L_{ff}, L_{kdkd}, L_{kqkq}, and L_{fkd} are the well known self and mutual inductances of the equivalent rotor windings,
* L_{afm}, L_{akdm} and L_{akqm} are the maximum mutual inductances between an armature phase winding and the field, direct axis damper, and quadrature axis damper windings, respectively,
* K is a constant resulting from the transformation where $K = \sqrt{3/2}$,

and * ω is the angular frequency in electrical radians per second, that is $\omega = \dot{\sigma}$, thus throughout this analysis, $\sigma = \omega t$, with ω = constant in this simulation and analysis of the dynamic steady state performance of the generator system

The state equation given above can be represented by an equivalent circuit model as shown.



$M_1 = KL_{afm}$

$M_2 = L_{fkd}$

$M_3 = KL_{akdm}$

$M_4 = KL_{akqm}$

$v_{d1} = (\omega KL_{akqm})\, i_{kq}$

$v_{d2} = (\omega L_q)\, i_q$

$v_{q1} = (\omega KL_{akdm})\, i_{kd}$

$v_{q2} = (\omega KL_{afm})\, i_f$

$v_{q3} = (\omega L_d)\, i_d$

Generator Equivalent Circuit in the dqo Frame of Reference

The state equation given above can be represented by an equivalent circuit model in which the following types of circuit elements are present: resistances, self and mutual inductances, independent voltage sources, and current controlled voltage sources. The resistances, self and mutual inductances, and the single independent voltage source in the network topology are self explanatory. The current controlled voltage sources are the diamond shaped elements and are expressed as follows:

$$v_{d1} = (\omega K L_{akqm}) i_{kq}, \quad v_{d2} = (\omega L_q) i_q$$

and

$$v_{q1} = (\omega K L_{akdm}) i_{kd}, \quad v_{q2} = (\omega K L_{afm}) i_f, \quad v_{q3} = (\omega L_d) i_d$$

From the equivalent network of Figure (3), one can write the following loop equations:

$$v_d = r_s i_d + L_d \frac{di_d}{dt} + KL_{afm} \frac{di_f}{dt} + KL_{akdm} \frac{di_{kd}}{dt}$$

$$- v_{d1} - v_{d2}$$

$$v_q = r_s i_q + L_q \frac{di_q}{dt} + KL_{akqm} \frac{di_{kq}}{dt} + v_{q1} + v_{q2} + v_{q3}$$

$$v_o = r_s i_o + L_o \frac{di_o}{dt}$$

$$v_f = r_f i_f + L_{ff} \frac{di_f}{dt} + KL_{afm} \frac{di_d}{dt} + L_{fkd} \frac{di_{kd}}{dt}$$

$$v_{kd} = 0 = r_{kd} i_{kd} + L_{kd} \frac{di_{kd}}{dt} + L_{fkd} \frac{di_f}{dt} + KL_{akdm} \frac{di_d}{dt}$$

and

$$v_{kq} = 0 = r_{kq} i_{kq} + L_{kq} \frac{di_{kq}}{dt} + KL_{akqm} \frac{di_q}{dt}$$

Upon substituting for v_{d1}, v_{d2}, v_{q1}, v_{q2}, and v_{q3}, one would obtain a set of equations which is identical to the state model of equation . Hence, the network is a full representation of the machine's state model in the dqo frame

The dynamics of this network are identical to the dynamics of the generator, subject to the external constraints, and initial conditions.

Reference: Arkadan, A.A., Hijazi, T.M., Demerdash, N.A., Vaidya, J.G., and Maddali, V.K., “Theoretical Development and Experimental Verification of a DC-AC Electronically Rectified Load-Generator System Model Compatible with Common Network Analysis Software Packages,” IEEE Trans. on Energy Conversion, Vol. EC-3, No. 1, pp. 123-131, Mar. 1988.