COLORADO SCHOOL OF MINES

ELECTRICAL ENGINEERING DEPARTMENT

EENG 577

ADVANCED ELECTRICAL MACHINE DYNAMICS FOR SMART-GRID SYSTEMS

Advanced Topics

Synchronous Machine Initial Conditions

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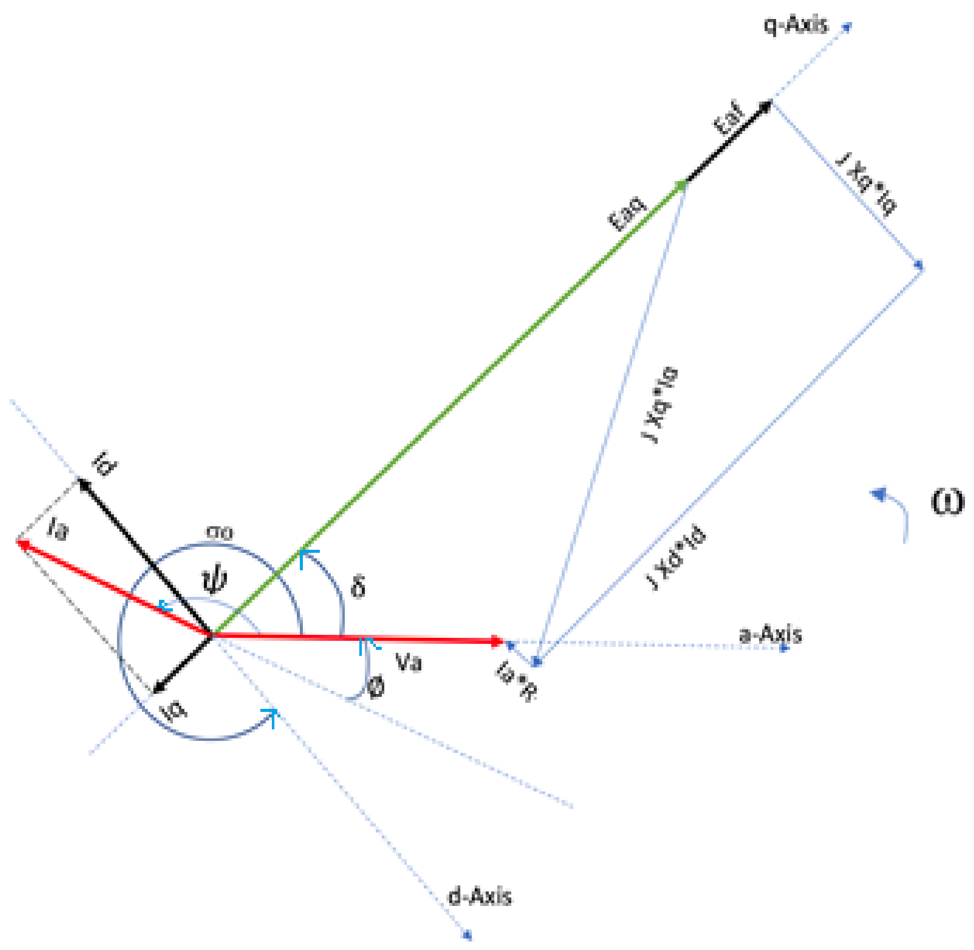
- **<u>Synchronous Machine Initial Conditions</u>**

- To carry out a State Space (SS) model analysis for a given operating load condition, one needs to compute the suitable excitation currents for all the windings.
- Each operating load condition is represented by an output power, S, a power factor, cos(φ), and an output phase voltage, Va. These excitation currents are determined from the phasor diagram of the machine.
- A sample phasor diagram representing an overexcited synchronous machine, i.e. the synchronous generator produces both real power, P, and reactive power, Q, is given as shown.



Phasor Diagram of the Synchronous Generator

In a 3-phase balanced system, the voltage expressions are given as:

$v_{an}(t)=V_m\cos(\omega t)$

$v_{bn}(t)=V_m\cos(\omega t-120°)$

$v_{cn}(t)=V_m\cos(\omega t-240°)$

Also, the current expressions are given as:

$i_a(t)=I_m\cos(\omega t-\phi)$

$i_b(t)=I_m\cos(\omega t-\phi-120°)$

$i_c(t)=I_m\cos(\omega t-\phi-240°)$

In this work, the consumer power notation is being used where generated power is considered negative. Accordingly, if one uses the terminal voltage V_a as a reference, then the current i_a should be in the 3rd quadrant.

The terminal voltage, $\overline{V}_a$ could be expressed as:

$$\overline{V_a} = V_a \angle 0 = V_a \ + \ j0$$

The armature phase current, $\overline{I}_a$, is expressed as:

$$\overline{I}_a \ = \ -I_a \cos\phi \ - \ jI_a \sin\phi$$

where ϕ is the power factor angle.

The complex power, $\overline{S}$, is related to the phase voltage and current as follows:

$$\overline{S} \;=\; 3\overline{I_a^*}\,\overline{V_a} \;=\; -3I_aV_a\cos\phi \;-\; j3I_aV_a\sin\phi$$

The real power is given by:

$$P \;=\; -3I_aV_a\cos\phi$$ (Negative, meaning Watts are generated)

and the reactive power is given by:

$$Q \;=\; -3I_aV_a\sin\phi$$ (Negative, meaning VARs are generated)

From the above phasor diagram, we can see that the equivalent quadrature axis voltage

$\overline{E_q}$, is related to the terminal voltage by the following equation:

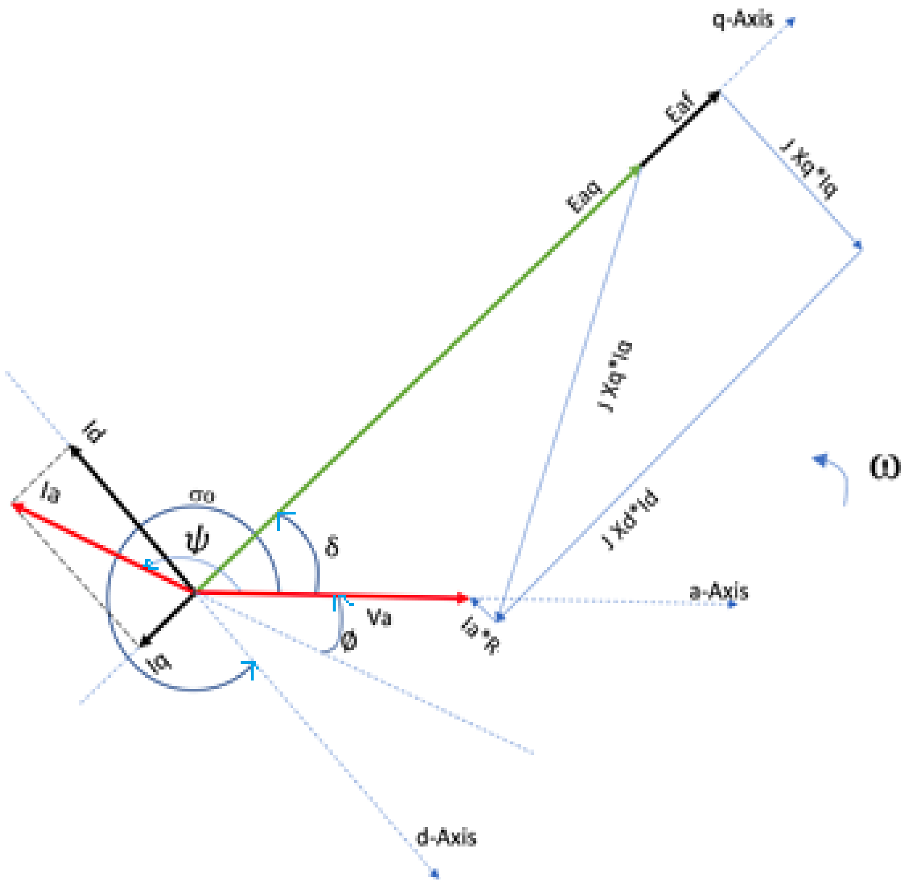
$$\overline{V_a} = \overline{E}_q + jx_q\overline{I_a} + r_a\overline{I_a}$$

We can then write:

$$\overline{E}_q = \overline{V}_a - jx_q\overline{I_a} - r_a\overline{I_a}$$

Neglecting the phase winding resistance r_a, we can write:

$$\begin{aligned}\overline{E}_q &= \overline{V}_a + j\overline{0} - jx_q(-\overline{I}_a\cos\phi - j\overline{I}_a\sin\phi) \\ &= (\overline{V}_a - x_q\overline{I}_a\sin\phi) + j(x_q\overline{I}_a\cos\phi)\end{aligned}$$



Accordingly, we can write the following using phasor notation:

$$\overline{E}_q = \sqrt{(V_a + x_q I_a \sin\phi)^2 + j(x_q I_a \cos\phi)^2} \angle \tan^{-1}\left(\frac{x_q I_a \cos\phi}{V_a - x_q I_a \sin\phi}\right)$$

where the angle δ is the torque angle and is given by:

$$\delta = \angle \tan^{-1}\left(\frac{x_q I_a \cos\phi}{V_a - x_q I_a \sin\phi}\right)$$

we also get:

$$\overline{I}_d = \overline{I}_a \sin(\delta + \phi)$$

$$\overline{I}_q = \overline{I}_a \cos(\delta + \phi)$$

$$\overline{E}_{af} = \overline{E}_q + (x_d - x_q)\overline{I}_d$$

The field voltage $\overline{E}_{af}$ is related to the field winding current and mutual inductance between the field and armature windings as follows:

$$E_{af} = \omega L_{afm} i_f \Big/ \sqrt{2}$$

The field winding current, i_f, and voltage, V_f, initial conditions can be determined by:

$$i_f = \sqrt{2} E_{af} \Big/ \left(\omega L_{afm}\right) \qquad \text{and} \qquad V_f = r_f \; i_f$$

Note also the following from the phasor diagram:

$\sigma_\circ = \delta + 3\pi/2$

$\psi = \pi - \phi$

$\overline{E}_{af} = E_{af} \angle \delta$

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[1] Electric Machinery, by Fitzgerald, Kinsley, and Umans, Recent Edition, McGraw-Hill Publishing Company, Inc., New York, NY.

[2] Arkadan, A.A., VanderHeiden, R.H. and Defenbaugh, J.F., “Effects of Forced Power Transfer on High Speed Generator-Load Systems,” *IEEE Transactions on Energy Conversion*, Vol. 11, No. 2, pp. 344-352, June 1996.