EENG 577 W5-M7

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Switched Reluctance Motor

Part 1)

The Simulink model for the switched reluctance motor has five components that include phase switching, inductance matrices, state space model, the developed torque, and the torque state model. Matlab code is provided in the appendix of this document which contains the function blocks to calculate the inductance with respect to theta and the rate of change of inductance with respect to theta. Also, there is a supplementary section of code for creating Matlab plots from the output variables from the Simulink model. Here we used intermediate scopes throughout the code as diagnostics. Figure (1) shows the overall signal paths between different subsystems in the model. This section will go into more detail on each subsystem below.

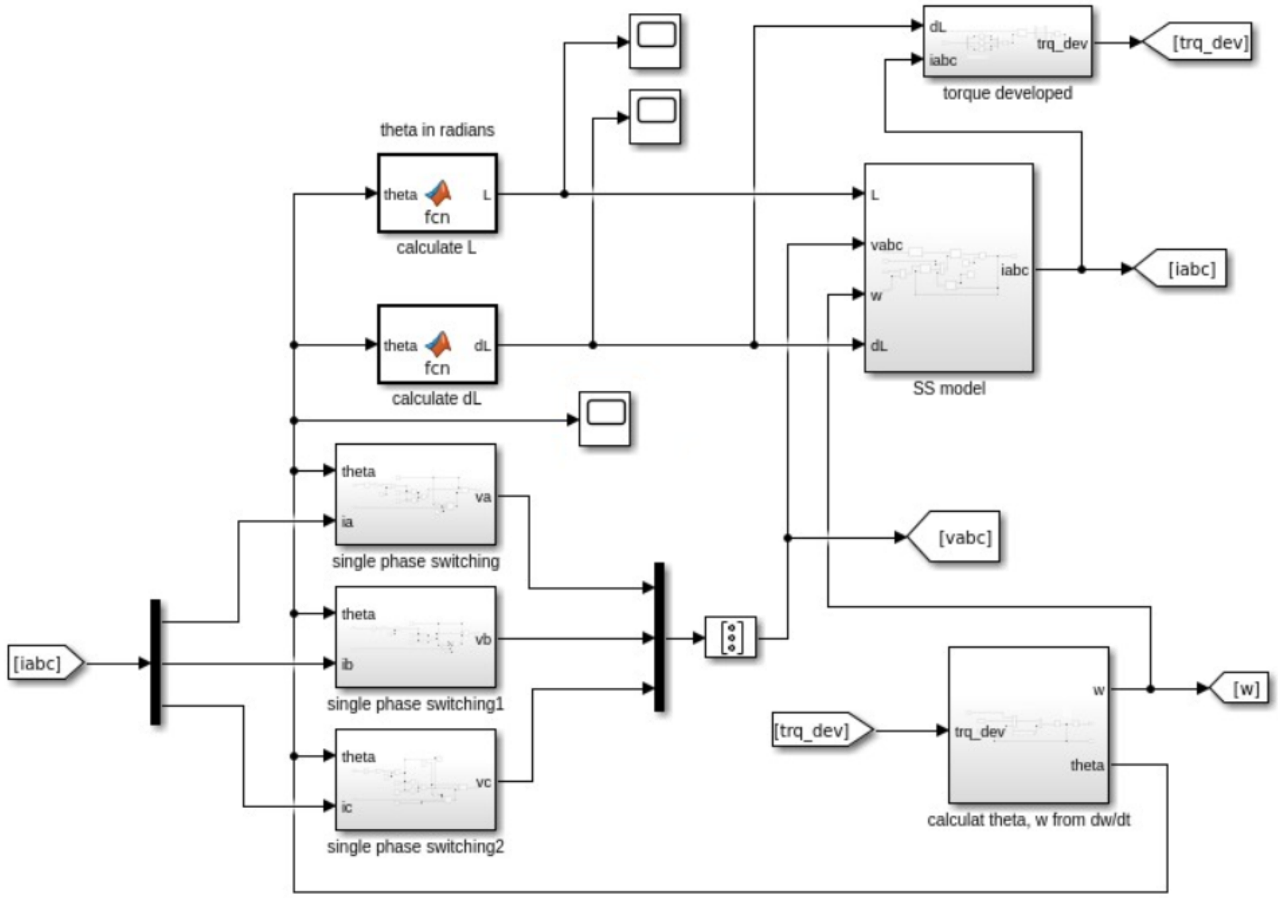

Figure 1: Primary Simulink System

A system of equations can be developed on the switched reluctance motor using Kirchhoff's voltage law across any phase a, b, or c. KVL states the voltage for a given phase is equivalent to the sum of the ohmic voltage and the flux linkage time rate-of-change for a given phase. There are a few considerations. The self and mutual inductance of a switched reluctance motor are functions of the phase current and the rotor position.[1]. We can ignore the change in inductance with respect to current[1] and we can ignore the mutual inductance and get accurate results.[2]. This gives the system of equations

$$V = RI + L\frac{dI}{dt} + \omega_m \frac{dL}{d\theta} I$$

where the voltage is equal to the sum of the ohmic voltage, the transformer voltage, and the reluctance voltage.[1] Figure (2) shows the SS model subsystem.

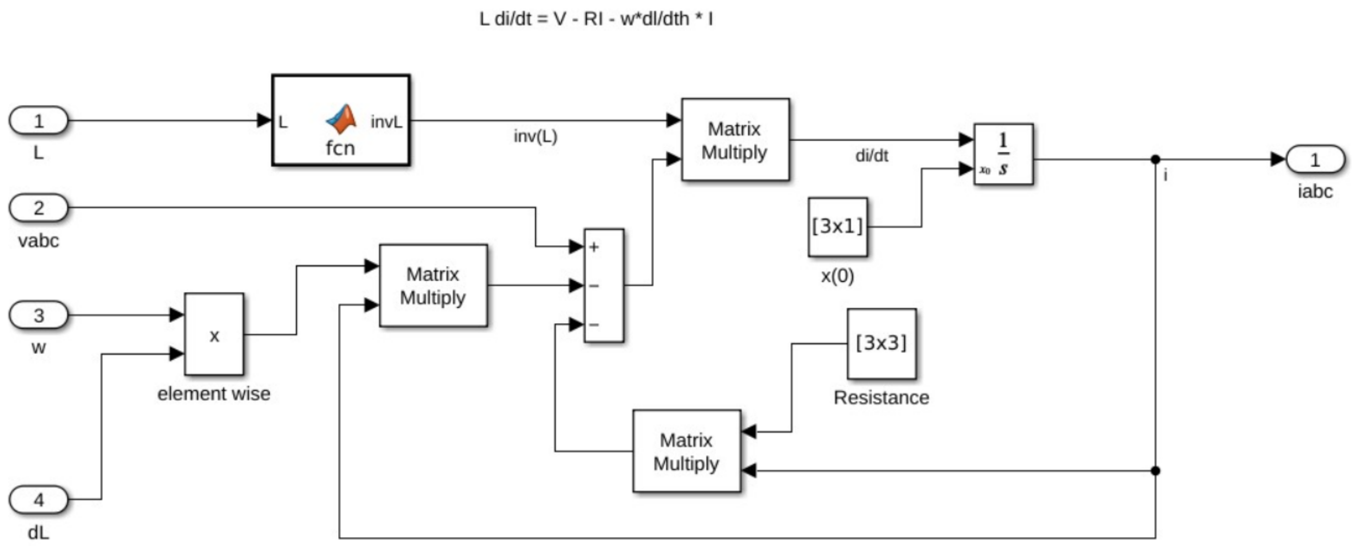

Figure 2: State Space Subsystem

The rotation of the motor is governed by the following expression:

$$\theta = \frac{2\pi}{m\,Nr} = \frac{2\pi}{3 \cdot 4} = \frac{\pi}{6}$$

Where $m = 3$ is the number of phases and $Nr = 4$ is the number of rotor poles. The rotor turns $\frac{\pi}{6}$ radians or 30° before it reaches the next phase. The switching sequence is given as B,C,A such that the A phase of the H-Bridge drive circuit turns on between 60 and 90 degrees. The B phase turns on between 0 and 30 degrees and the C phase turns on between 30 and 60 degrees. Figure (3) shows the subsystem of the phase switching where the output is either $V_{bus} = 43.5[V]$, $-V_{bus}$, or 0 depending on θ and i_a.
In the state space model the inductance for the phases are each shifted by $2\pi/3$ radians because the three stator windings are equally spaced around the full rotation of the motor.

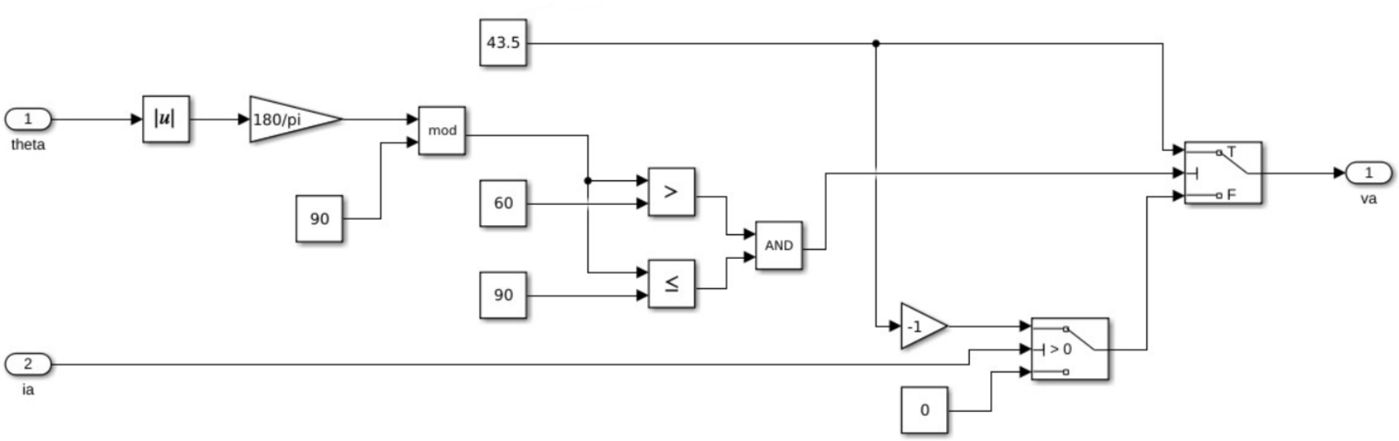

Figure 3: Phase A Switching Sequence Subsystem

The torque state model subsystem is governed by the equation

$$\frac{d}{dt}\omega = \left(\frac{1}{J}\right)(T_{em} - B\omega - T_L)$$

where J, B, and T_L are the Inertia of the Rotor and Load, coefficient of viscous friction, and load torque respectively. In the subsystem block, we integrated $\frac{d\omega}{dt}$ twice to get ω and θ.

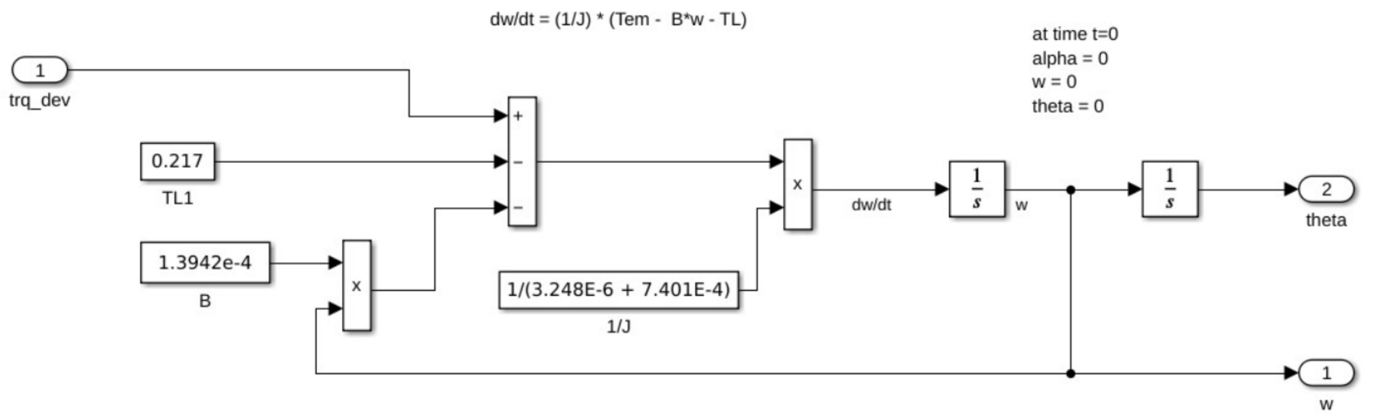

Figure 4: Torque State Model Subsystem

The electromechanical torque can be calculated from

$$T_{EM} = \frac{P}{\omega_m} = \frac{ei}{\omega_m}$$

where e is the rotational voltage term in the state space equation[2]

$$V = RI + L\frac{dI}{dt} + \omega_m \frac{dL}{d\theta} I$$

and if we consider the currents to be constant in the partial differential equation[1], we can use the form

$$T = \frac{1}{2}\frac{dL}{d\theta}\sum_{j=a,b,c} i_j^2$$

The developed torque subsystem is shown in Figure (5).

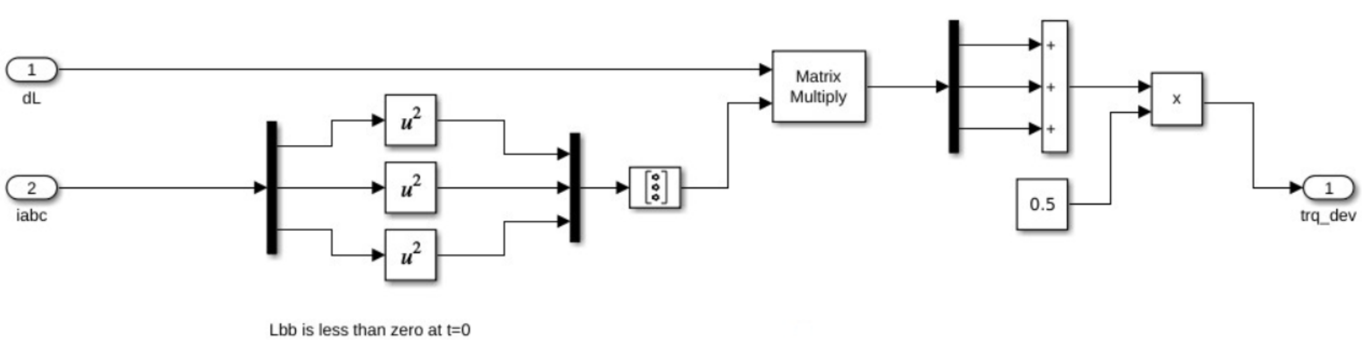

Figure 5: Developed Torque Subsystem

Part of the model takes the outputs from vary subsystems in the model and outputs the results as a function of time shown in Figure (6).

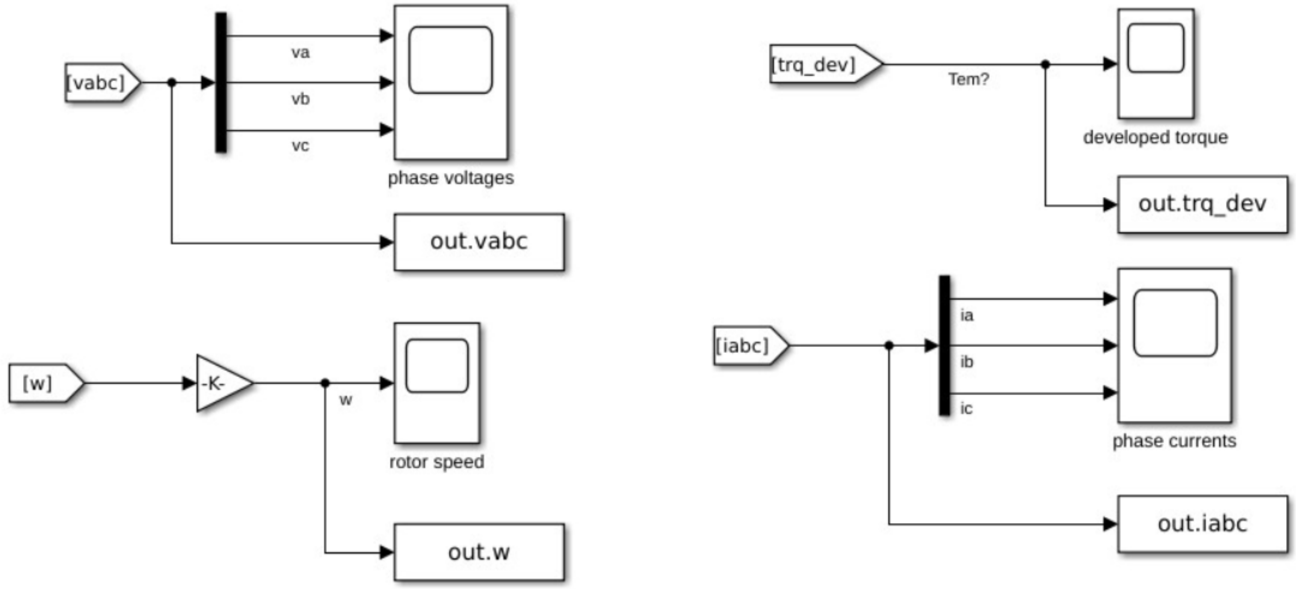

Figure 6: Variable Outputs to Scopes

Part 2)

The resulting time domain waveforms for the phase voltages, phase currents, developed torque, and the rotor speed is given below. Here we show both the startup transient from 0 to 0.25 seconds and slow data from 0 to 2 seconds.

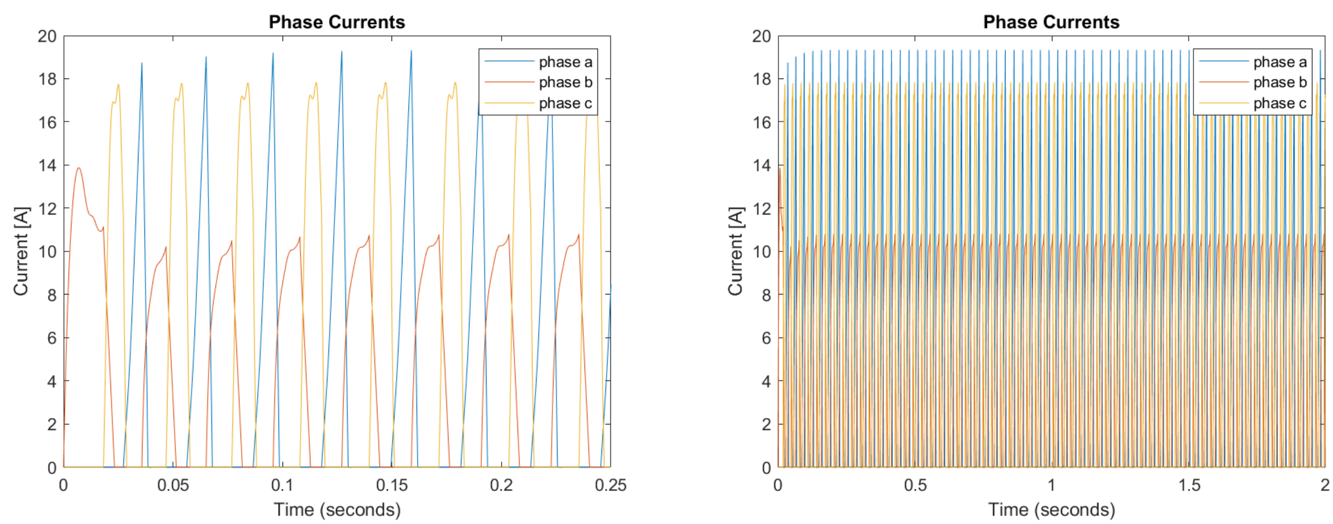

Figure 7: Phase Currents

RMS phase currents

$$i_a = 3.452[A]$$
$$i_b = 6.174[A]$$
$$i_c = 8.281[A]$$

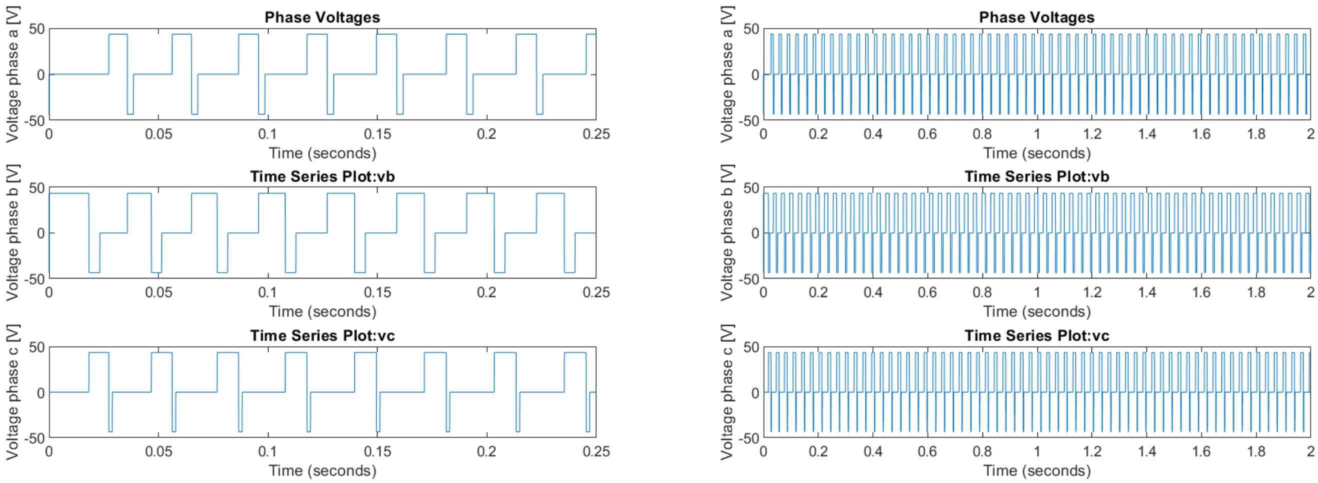

Figure 8: Phase Voltage

RMS phase voltages

$$v_a = 26.75[V]$$
$$v_b = 32.38[V]$$
$$v_c = 26.16[V]$$

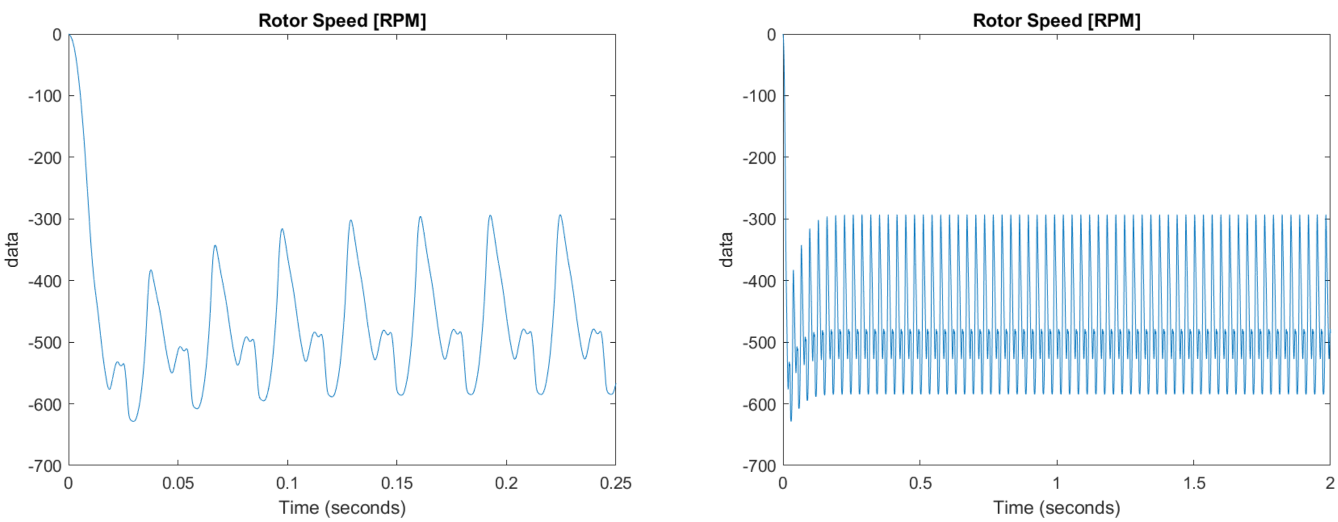

Figure 9: Rotor Speed

RMS rotor speed

$$Rotor\ Speed = 477.1[RPM]$$

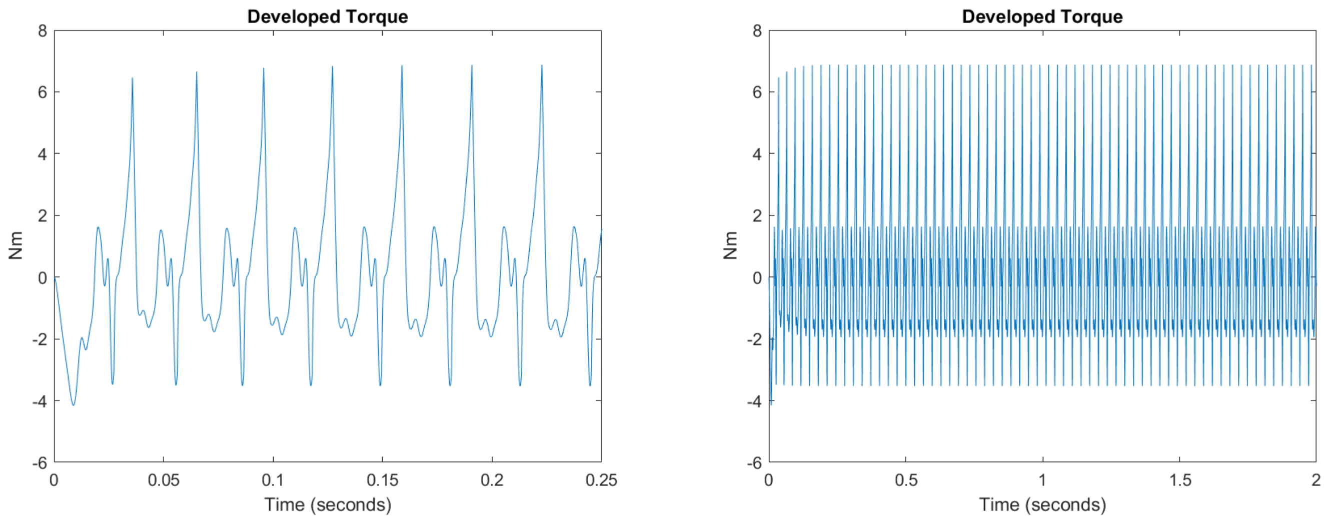

Figure 10: Developed Torque

RMS developed torque

$$\text{Developed Torque} = 2.15 \text{ Nm}$$

Contributions

Tasks	Eric Hildenbrandt	Devon Davis	Joseph Brownlee
Formulations & Calculations	15 %	40 %	45 %
MATLAB/Simulink Coding	55 %	20 %	25 %
Report Writing	30 %	40 %	30 %
Overall % Contribution/Member	100 %	100 %	100 %

References

- [1] A.A. Arkadan EENG577 Class Notes, Colorado School of Mines.
- [2] Stephen J. Chapman. (2005). Electric Machinery Fundamentals. McGraw-Hill.
- [3] A.A. Arkadan, and B.W. Kielgas, "Switched Reluctance Drive System and Dynamic Performance Prediction and Experimental Verification," IEEE Trans. On Energy Conversion, Vol. 9, No. 1, pp. 36-43, March 1994.
- [4] A.A. Arkadan, and B.W. Kielgas, "Switched Reluctance Drive System and Dynamic Performance Prediction Under Internal and External Fault Conditions," IEEE Trans. On Energy Conversion, Vol. 9, No. 1, pp. 45-52, March 1994.
- [5] A.A. Arkadan, and B.W. Kielgas, "Effects of Force Fitting on the Inductance Profile of a Switched Reluctance Motor," IEEE Trans. On Magnetics, Vol. 29, No. 2, pp. 2006-2009, March 1993.

Appendix: Matlab

Inductance of Motor

```
function L = fcn(theta)

aa = 0.1431e-1;
a4 = 0.1068e-1*cos(4*theta + 0.4272e1);
a8 = 0.3517e-3*cos(8*theta + 0.2084e2);
a12 = 0.5599e-3*cos(12*theta + 0.5148e2);
a16 = 0.4487e-3*cos(16*theta + 0.1744e2);
a20 = 0.1866e-3*cos(20*theta - 0.2523e2);
a24 = 0.1429e-3*cos(24*theta - 0.6382e2);
a28 = 0.3222e-4*cos(28*theta + 0.2141e2);
a32 = 0.1237e-3*cos(32*theta - 0.1178e2);

La = aa + a4 + a8 + a12 + a16 + a20 + a24 + a28 + a32;

%-----------------------------------------------------------
bb = 0.1431e-1;
b4 = 0.1068e-1*cos(4*theta + 0.4272e1 - 2*pi/3);
b8 = 0.3517e-3*cos(8*theta + 0.2084e2 - 2*pi/3);
b12 = 0.5599e-3*cos(12*theta + 0.5148e2 - 2*pi/3);
b16 = 0.4487e-3*cos(16*theta + 0.1744e2 - 2*pi/3);
b20 = 0.1866e-3*cos(20*theta - 0.2523e2 - 2*pi/3);
b24 = 0.1429e-3*cos(24*theta - 0.6382e2 - 2*pi/3);
b28 = 0.3222e-4*cos(28*theta + 0.2141e2 - 2*pi/3);
b32 = 0.1237e-3*cos(32*theta - 0.1178e2 - 2*pi/3);

Lb = bb + b4 + b8 + b12 + b16 + b20 + b24 + b28 + b32;

%-----------------------------------------------------------

cc = 0.1431e-1;
c4 = 0.1068e-1*cos(4*theta + 0.4272e1 - 4*pi/3);
c8 = 0.3517e-3*cos(8*theta + 0.2084e2 - 4*pi/3);
c12 = 0.5599e-3*cos(12*theta + 0.5148e2 - 4*pi/3);
c16 = 0.4487e-3*cos(16*theta + 0.1744e2 - 4*pi/3);
c20 = 0.1866e-3*cos(20*theta - 0.2523e2 - 4*pi/3);
c24 = 0.1429e-3*cos(24*theta - 0.6382e2 - 4*pi/3);
c28 = 0.3222e-4*cos(28*theta + 0.2141e2 - 4*pi/3);
c32 = 0.1237e-3*cos(32*theta - 0.1178e2 - 4*pi/3);

Lc = cc + c4 + c8 + c12 + c16 + c20 + c24 + c28 + c32;

%-----------------------------------------------------------

L = diag([La, Lb, Lc]);
```

Rate of Change Inductance with Respect to Theta of Motor

```
function dL = fcn(theta)

da4 =  -4  * 0.1068e-1 * sin(4*theta + 0.4272e1);
da8 =  -8  * 0.3517e-3 * sin(8*theta + 0.2084e2);
da12 = -12 * 0.5599e-3 * sin(12*theta + 0.5148e2);
da16 = -16 * 0.4487e-3 * sin(16*theta + 0.1744e2);
da20 = -20 * 0.1866e-3 * sin(20*theta - 0.2523e2);
da24 = -24 * 0.1429e-3 * sin(24*theta - 0.6382e2);
da28 = -28 * 0.3222e-4 * sin(28*theta + 0.2141e2);
da32 = -32 * 0.1237e-3 * sin(32*theta - 0.1178e2);

dLa = da4 + da8 + da12 + da16 + da20 + da24 + da28 + da32;

db4 =  -4  * 0.1068e-1 * sin(4*theta + 0.4272e1 - 2*pi/3);
db8 =  -8  * 0.3517e-3 * sin(8*theta + 0.2084e2 - 2*pi/3);
db12 = -12 * 0.5599e-3 * sin(12*theta + 0.5148e2 - 2*pi/3);
db16 = -16 * 0.4487e-3 * sin(16*theta + 0.1744e2 - 2*pi/3);
db20 = -20 * 0.1866e-3 * sin(20*theta - 0.2523e2 - 2*pi/3);
db24 = -24 * 0.1429e-3 * sin(24*theta - 0.6382e2 - 2*pi/3);
db28 = -28 * 0.3222e-4 * sin(28*theta + 0.2141e2 - 2*pi/3);
db32 = -32 * 0.1237e-3 * sin(32*theta - 0.1178e2 - 2*pi/3);

dLb = db4 + db8 + db12 + db16 + db20 + db24 + db28 + db32;

dc4 =  -4  * 0.1068e-1 * sin(4*theta + 0.4272e1 - 4*pi/3);
dc8 =  -8  * 0.3517e-3 * sin(8*theta + 0.2084e2 - 4*pi/3);
dc12 = -12 * 0.5599e-3 * sin(12*theta + 0.5148e2 - 4*pi/3);
dc16 = -16 * 0.4487e-3 * sin(16*theta + 0.1744e2 - 4*pi/3);
dc20 = -20 * 0.1866e-3 * sin(20*theta - 0.2523e2 - 4*pi/3);
dc24 = -24 * 0.1429e-3 * sin(24*theta - 0.6382e2 - 4*pi/3);
dc28 = -28 * 0.3222e-4 * sin(28*theta + 0.2141e2 - 4*pi/3);
dc32 = -32 * 0.1237e-3 * sin(32*theta - 0.1178e2 - 4*pi/3);

dLc = dc4 + dc8 + dc12 + dc16 + dc20 + dc24 + dc28 + dc32;

dL = diag([dLa, dLb, dLc]);
```

Figure Generator

```
% Output plots from Simulink

out=sim("M5_group_v10.slx")

%%

mkdir figs

xlims_zoom=[1.75 2]

figure(1)
plot(out.iabc)
title("Phase Currents")
ylabel("Current [A]")
legend(["phase a", "phase b", "phase c"])
saveas(gcf,'figs/currents.png')
xlim(xlims_zoom)
saveas(gcf,'figs/currents_zoom.png')

figure(2)
subplot(3,1,1)
plot(out.va)
title("Voltage phase a")
ylabel("Voltage phase a [V]")

subplot(3,1,2)
plot(out.vb)
title("Voltage phase b")
ylabel("Voltage phase b [V]")

subplot(3,1,3)
plot(out.vc)
title("Voltage phase c")
ylabel("Voltage phase c [V]")

saveas(gcf,'figs/voltages.png')
subplot(3,1,1)
xlim(xlims_zoom)
subplot(3,1,2)
xlim(xlims_zoom)
subplot(3,1,3)
xlim(xlims_zoom)
saveas(gcf,'figs/voltages_zoom.png')

figure(3)
plot(out.w)
title("Rotor Speed [rev/min]")
saveas(gcf,'figs/speed.png')
xlim(xlims_zoom)
saveas(gcf,'figs/speed_zoom.png')

figure(4)
plot(out.trq_dev)
title("Developed Torque")
saveas(gcf,'figs/torque.png')
xlim(xlims_zoom)
saveas(gcf,'figs/torque_zoom.png')
```