COLORADO SCHOOL OF MINES
ELECTRICAL ENGINEERING DEPARTMENT

EENG 577

ADVANCED ELECTRICAL MACHINE DYNAMICS FOR SMART-GRID SYSTEMS

M1-P2 Single & Three-Phase Power

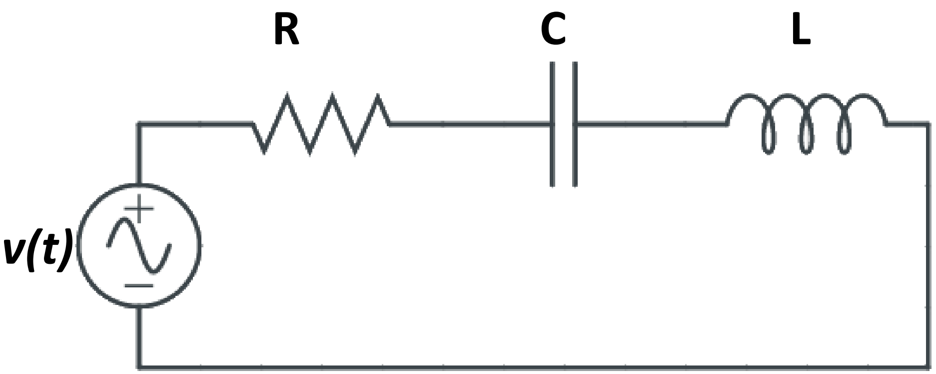
Objectives

I. Single Phase Power

- **Power concepts:**
  - ✓ instantaneous power,
  - ✓ average power, (real power); P (W)
  - ✓ reactive power, (imaginary power); Q (VAR)
  - ✓ complex power, **S**=P+jQ= ISI∟θ (VA)
  - ✓ apparent power = ISI (VA)
  - ✓ power factor angle=θ
  - ✓ Power factor= cos (θ)
- **Power Triangle:**
  - ✓ Relationships among power concepts.

II. Three-Phase Power

Given



In a constant frequency ac system

$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$i(t) = I_m \cos(\omega t + \theta_I)$$

Root Mean Square (RMS) voltage of sinusoid (*V*):

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2\, dt} = \frac{V_m}{\sqrt{2}}$$

As such, v(t) is expressed in terms of the Root RMS value as follows:

$$v(t) = \sqrt{2}\, V_{rms} \cos(\omega t + \theta_v)$$

Instantaneous power:

Given

$$p(t) = v(t)\, i(t)$$
$$v(t) = V_m \cos(\omega t + \theta_V)$$
$$i(t) = I_m \cos(\omega t + \theta_I)$$
$$Note: \cos\alpha \cos\beta = \frac{1}{2}[\cos(\alpha-\beta)+\cos(\alpha+\beta)]$$

$$p(t) = v(t) i(t)$$
$$= [V_m \cos(\omega t + \theta_v)][I_m \cos(\omega t + \theta_i)]$$

... *(lots of trigonometry)* ...

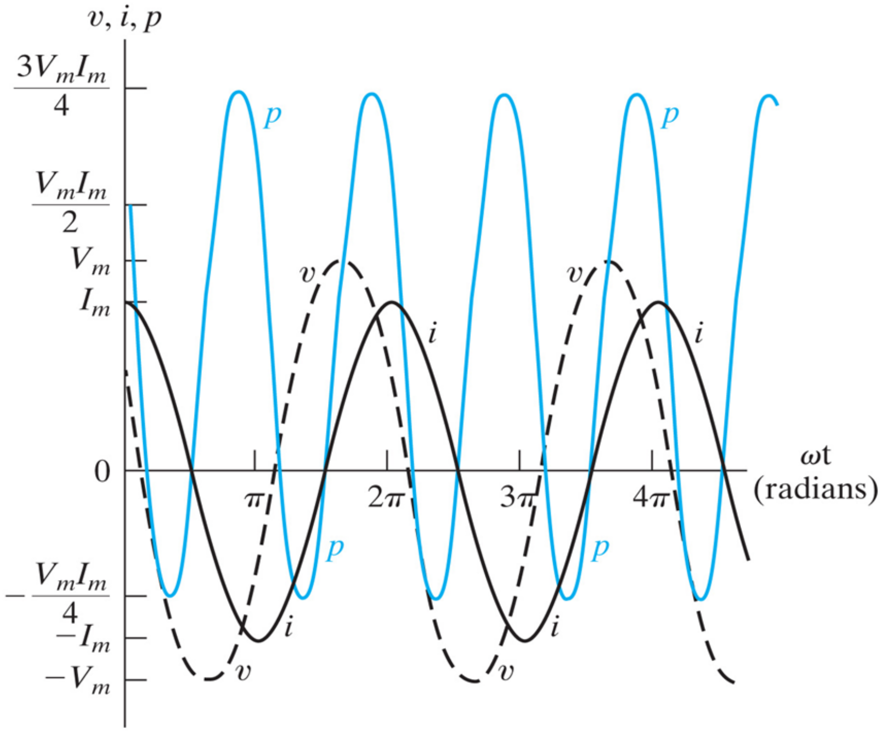
$$= \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\cos 2\omega t$$
$$- \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)\sin 2\omega t$$

Rewrite

$$p(t) = P + P\cos 2\omega t - Q \sin 2\omega t$$

where

$$P = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)$$
$$Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)$$



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Instantaneous power, rewritten:

$$p(t) = P + P\cos 2\,\omega t - Q\sin 2\,\omega t$$

where $\quad P = \frac{V_m I_m}{2}\cos(\,\theta_v - \theta_i)$

and $\quad Q = \frac{V_m I_m}{2}\sin(\,\theta_v - \theta_i)$

Definitions:

Power Factor Angle = $\theta = \theta_V - \theta_I$

Power Factor, PF= cosθ

P is the average (or real) power and has the unit Watt [W]

$$P = \frac{1}{T}\int_{t_0}^{t_0+T} p(t)dt$$

Q is the reactive power and has the unit **V**olt-**A**mpere-**R**eactive [VAR]

Complex Power (Definition)

$$\bar{S} = P + jQ = S\angle\theta$$

Note

$$p(t) = v(t)\, i(t)$$
$$v(t) = V_m \cos(\omega t + \theta_V)$$
$$i(t) = I_m \cos(\omega t + \theta_I)$$

- For a **resistor** in the presence of an AC source, **voltage and current are in-phase**!

$$\cos(\theta_v - \theta_i) = \cos(0°) = 1; \qquad \sin(\theta_v - \theta_i) = 0$$

$$P = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) = \frac{V_m I_m}{2}; \qquad Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i) = 0$$

- For an **inductor** the **current lags the voltage by 90°**:

$$\cos(\theta_v - \theta_i) = \cos(90°) = 0; \qquad \sin(\theta_v - \theta_i) = \sin(90°) = 1$$

$$P = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) = 0; \qquad Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i) = \frac{V_m I_m}{2}$$

- For a **capacitor**, the **current leads voltage by 90°**:

$$\cos(\theta_v - \theta_i) = \cos(-90°) = 0; \qquad \sin(\theta_v - \theta_i) = \sin(-90°) = -1$$

$$P = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) = 0; \qquad Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i) = -\frac{V_m I_m}{2}$$

Complex Power

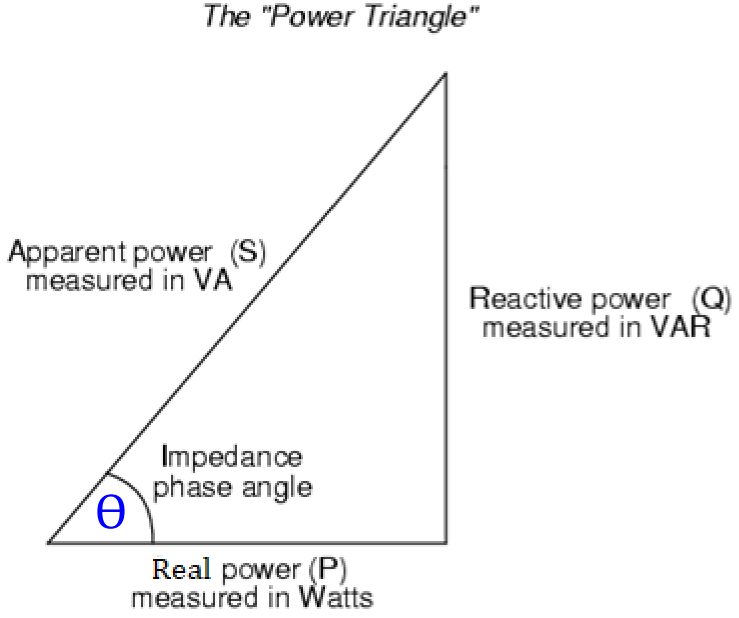
$$\bar{S} = P + jQ = S\angle\theta$$
$$\bar{S} = \bar{V}\bar{I}*$$

Note the apparent power S is the magnitude of the complex power where S=VI in VA
As such

$$P = S\ cos\theta$$

$$Q = S\ sin\theta = S\sqrt{1 - pf^2}$$

Some utilities charge for the Apparent Power:
|S|=VI in VA



Power Consumption in Devices

Resistors only consume real power

$$P_{\text{Resistor}} = \left| I_{\text{Resistor}} \right|^2 R$$

$P_{Resistor}$ >0 Consume

Inductors only consume reactive power

$$Q_{\text{Inductor}} = \left| I_{\text{Inductor}} \right|^2 X_{\text{L}} \quad ;X_L=\omega L$$

$Q_{Inductor}$ >0 Consume

Capacitors only generate reactive power

$$Q_{\text{Capacitor}} = -\left| I_{\text{Capacitor}} \right|^2 X_{\text{C}} \qquad X_C = \frac{1}{\omega C}$$

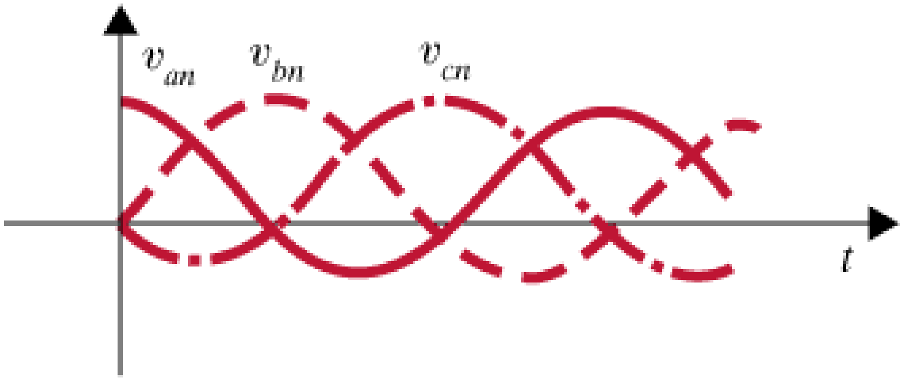
$$Q_{\text{Capacitor}} = -\frac{\left| V_{\text{Capacitor}} \right|^2}{X_{\text{C}}}$$

$Q_{Capacitor}$ <0 Generate

THREE-PHASE CIRCUITS

- **Three Phase Circuits**
- **Three Phase Connections**
- **Source/Load   Delta-Y connections**
- **Power Relationships**

THREE PHASE CIRCUITS

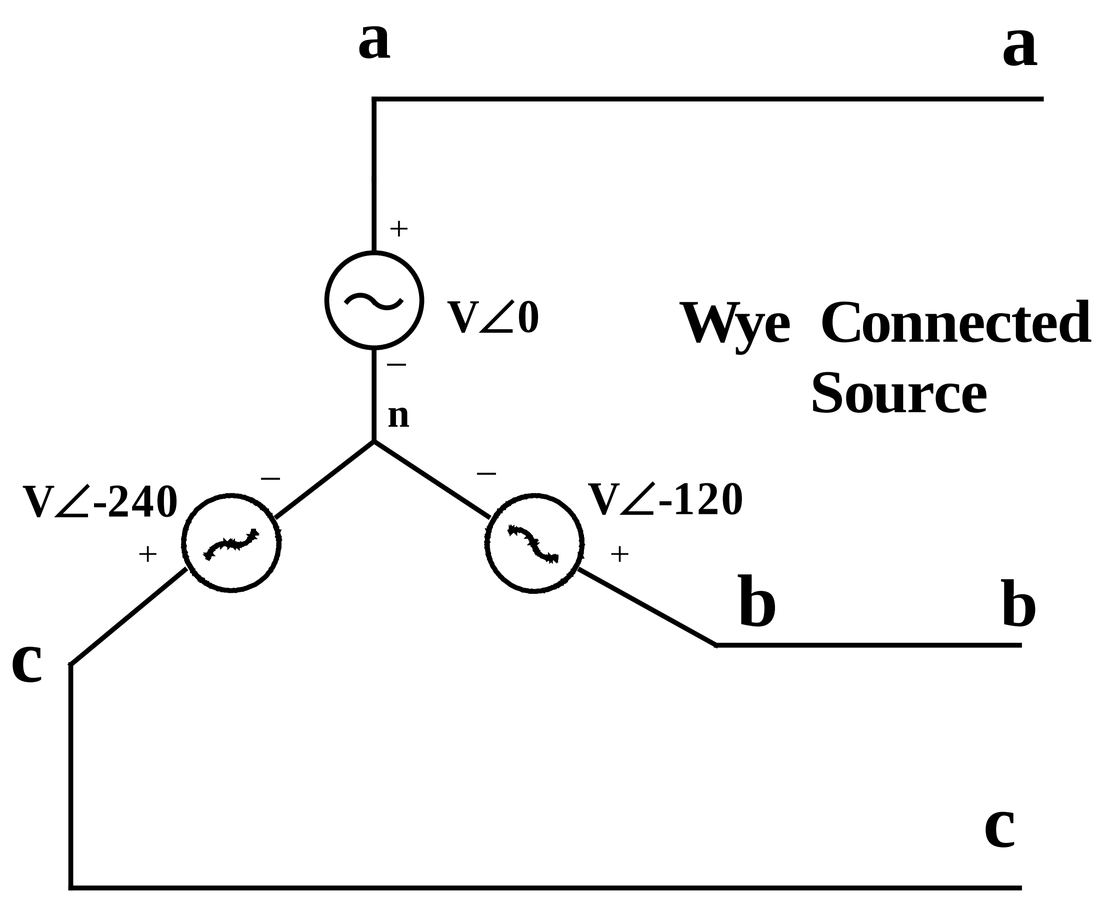


Instantaneous Phase Voltages

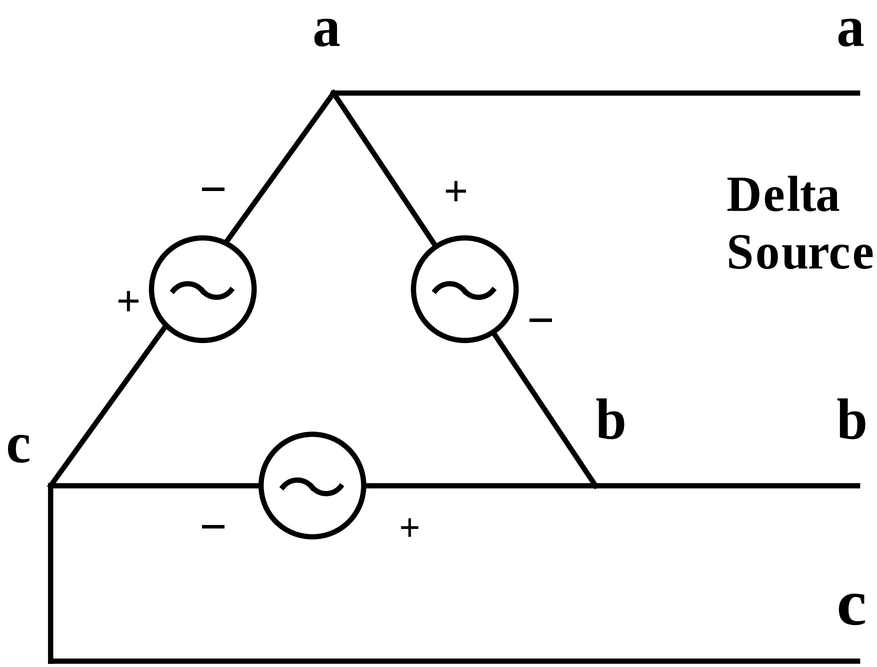
$$v_{an}(t) = V_m \cos(\omega t)(V)$$

$$v_{bn}(t) = V_m \cos(\omega t - 120°)(V)$$

$$v_c(t) = V_m \cos(\omega t - 240°)(V)$$


a
a
+
V∠0
–
n
Wye Connected
Source
V∠-240
–
+
–
V∠-120
+
b
b
c
c

Delta Source

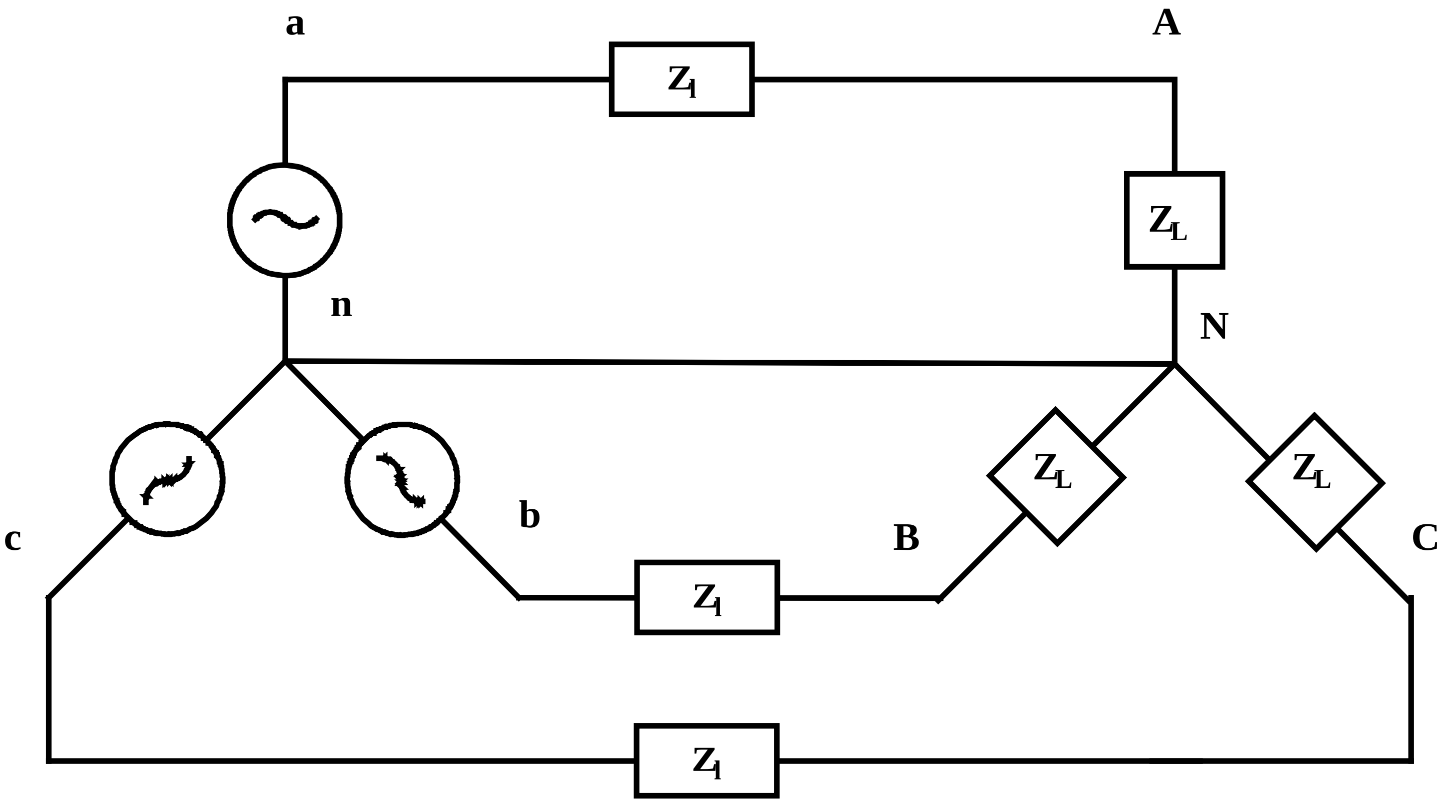


$$\mathbf{V_{ab}} = |V_{ab}| \angle 0$$

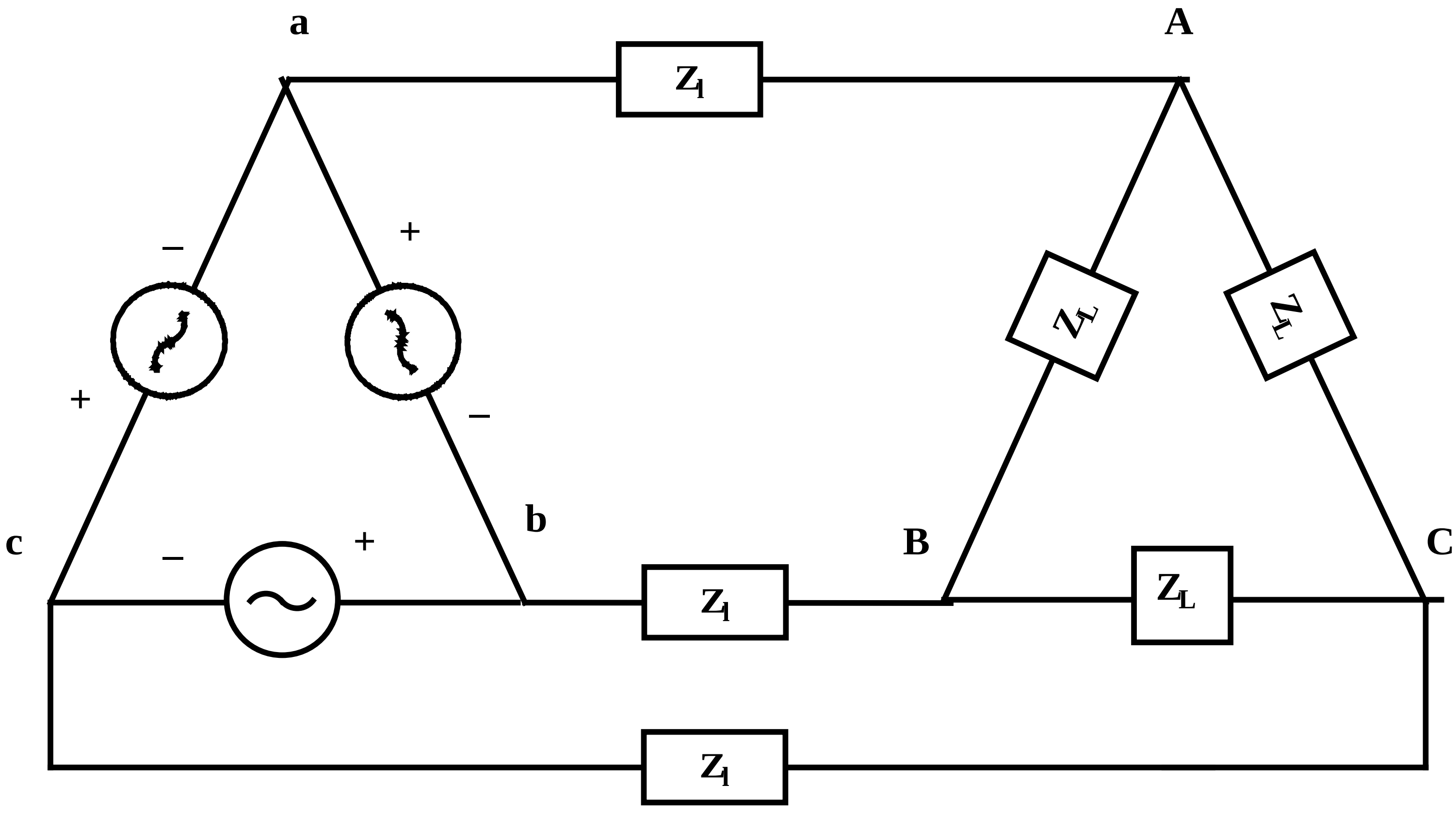
$$V_{bc} = V_{ab} \angle -120$$

$$V_{ca} = V_{ab} \angle -240$$

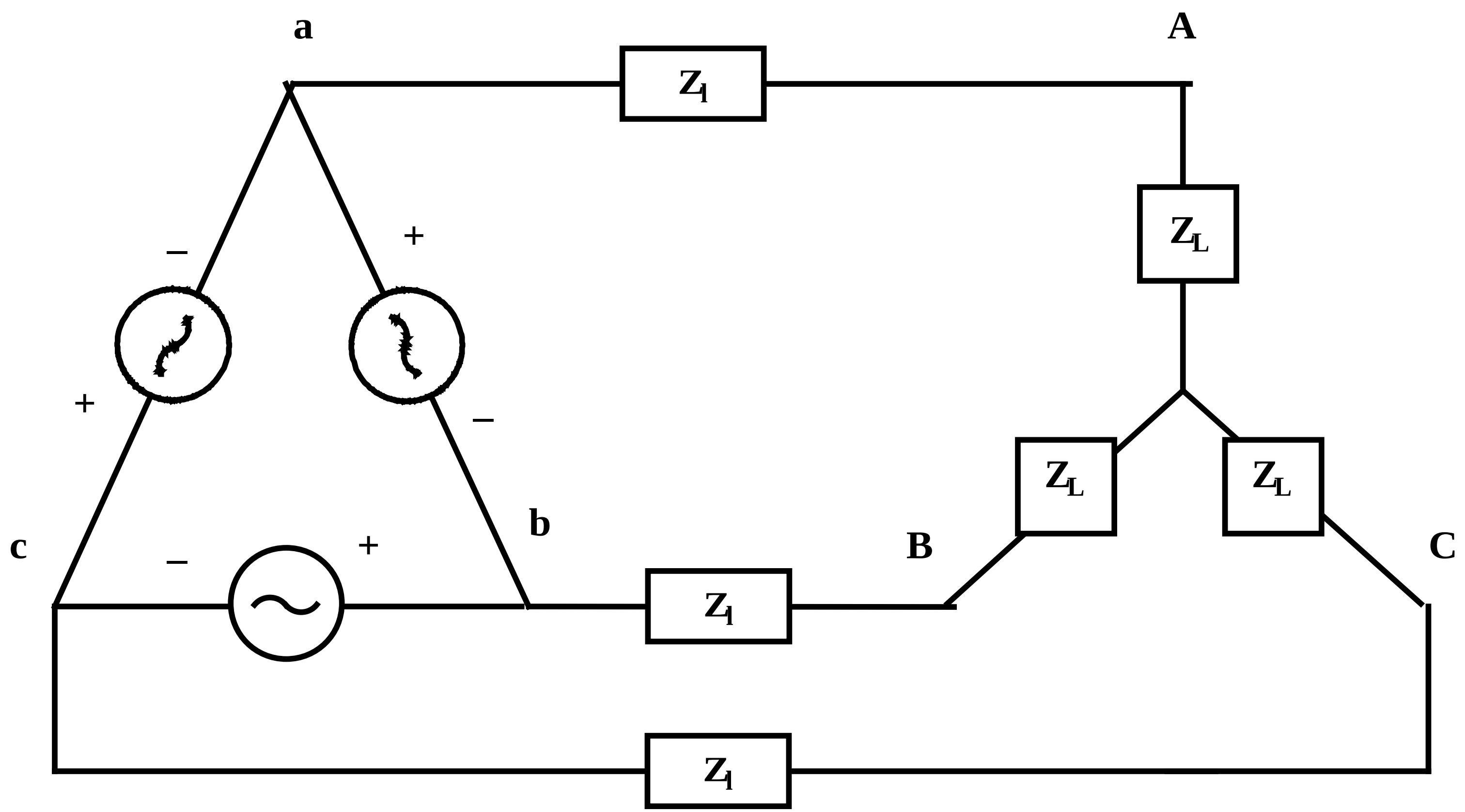
Wye – Wye System

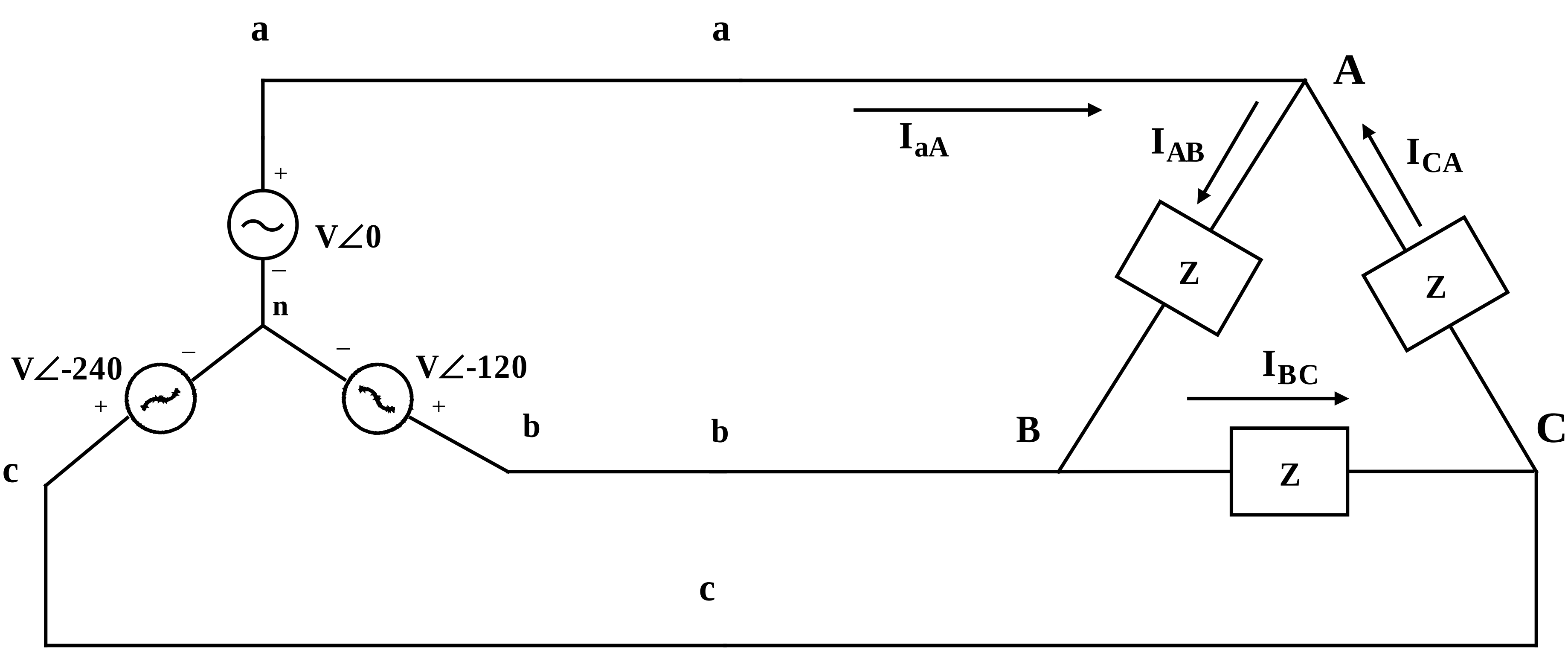


Delta – Delta System

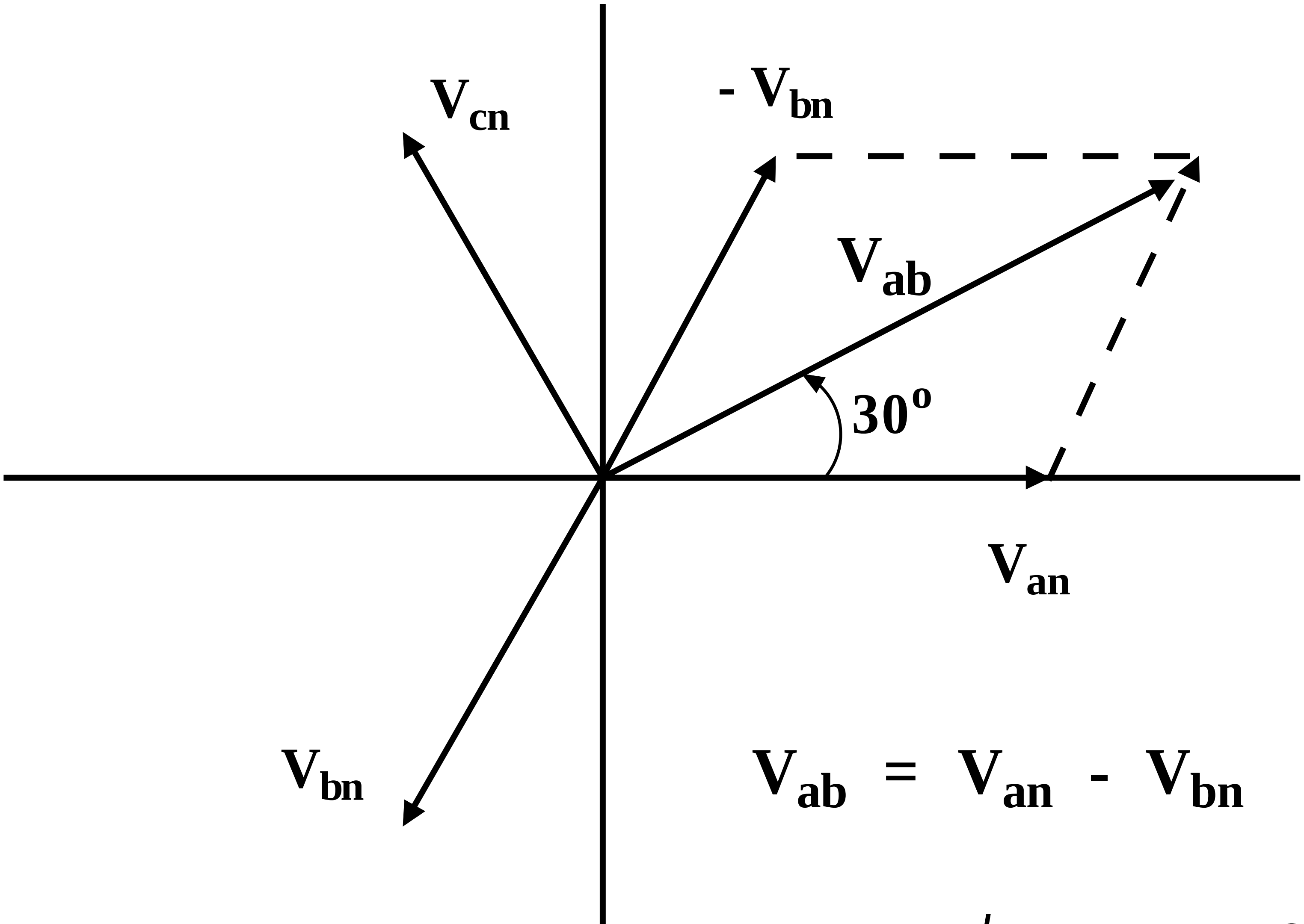


Delta – Wye System



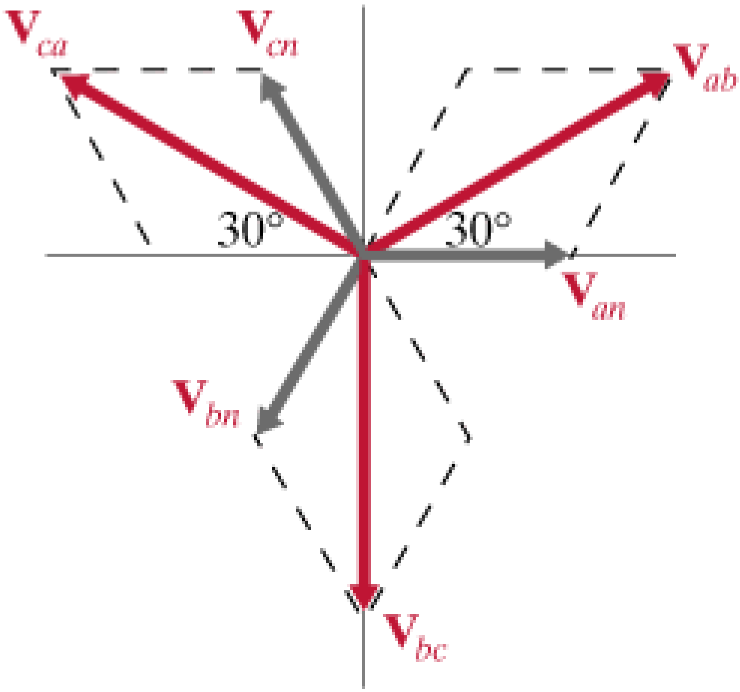


a
a
A
+
V∠0
–
n
IaA
IAB
ICA
Z
Z
V∠-240
–
–
V∠-120
+
+
IBC
b
b
B
C
c
Z
c



$$V_{ab} = V_{an} - V_{bn}$$

$$V_{ab} = \sqrt{3}\, V_{an} \angle 30^o$$



(b)

$$V_{ab} = V_{an} - V_{bn}$$
$$= |V_p| \angle 0° - |V_p| \angle -120°$$
$$= |V_p| \left(1 - (\cos 120 - j \sin 120)\right)$$
$$= |V|_p - |V_p| \left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)$$
$$= \sqrt{3} |V_p| \angle 30°$$

$$V_{bc} = \sqrt{3} |V_p| \angle -90°$$
$$V_{ca} = \sqrt{3} |V_p| \angle -210°$$

$V_L = \sqrt{3} |V_p|$ = Line Voltage

INSTANTANEOUS POWER

Balanced 3−Phase Voltages

$v_{an}(t) = V_m \cos(\omega t)(V)$

$v_{bn}(t) = V_m \cos(\omega t - 120°)(V)$

$v_{cn}(t) = V_m \cos(\omega t - 240°)(V)$

Balanced 3−Phase Currents

$i_a(t) = I_m \cos(\omega t - \theta)$

$i_b(t) = I_m \cos(\omega t - \theta - 120°)$

$i_c(t) = I_m \cos(\omega t - \theta - 240°)$

Instantaneous power

$p(t) = v_{an}(t)i_a(t) + v_{bn}(t)i_b(t) + v_{cn}(t)i_c(t)$

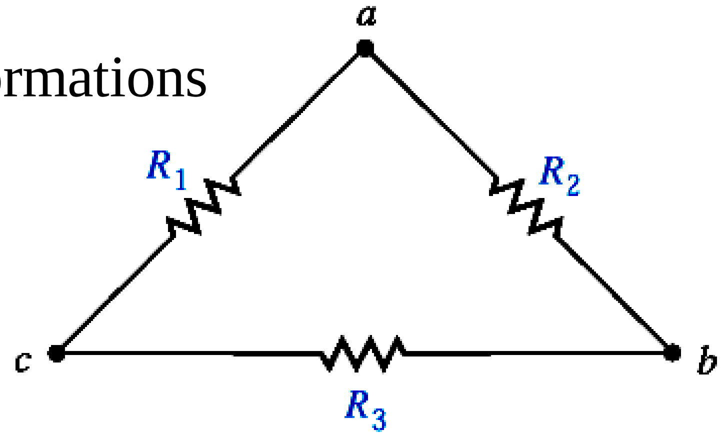
Three-Phase Balanced Instantaneous Power

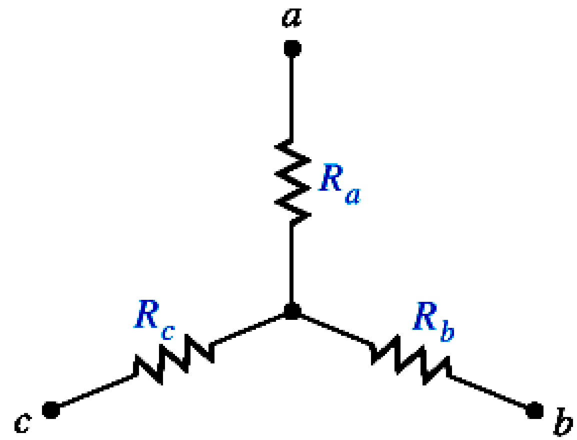
$$p(t) = 3\frac{V_m I_m}{2}\cos\theta\ (W)$$

$$p(t) = \frac{\sqrt{3}}{2}V_L I_L \cos\theta\ (W)$$

$\Delta \leftrightarrow$ Y

Transformations





$$R_a = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

$$R_b = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

$$R_c = \frac{R_3 R_1}{R_1 + R_2 + R_3}$$

$\Delta \rightarrow Y$

$$R_1 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_b}$$

$$R_2 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_c}$$

$$R_3 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_a}$$

$Y - \Delta$

Note: In the **3-Phase Balanced case,** the above expressions are simplified as $R_1 = R_2 = R_3$ **&** $R_a = R_b = R_c$