COLORADO SCHOOL OF MINES
ELECTRICAL ENGINEERING DEPARTMENT

EENG 577

ADVANCED ELECTRICAL MACHINE DYNAMICS FOR SMART-GRID SYSTEMS

M1-P3 Energy Conversion and Magnetic Circuits

Objectives

1. Describe and explain magnetic circuits.
2. Describe and explain the basics of rotational mechanics: angular velocity, angular acceleration, torque, and Newton’s law for rotation.
3. Describe and explain how to produce a magnetic field.
4. Describe and explain Faraday’s law.
5. Describe and explain how to produce an induced force on a wire.
6. Describe and explain how to produce an induced voltage across a wire.
7. Describe and explain self and mutual inductances of a magnetic circuit.

Rotational Motion

- ω_m, **Angular Velocity**, radians/second

$$\omega_m = \frac{d\theta_m}{dt}$$

- f_m, Angular Velocity, revolution/second

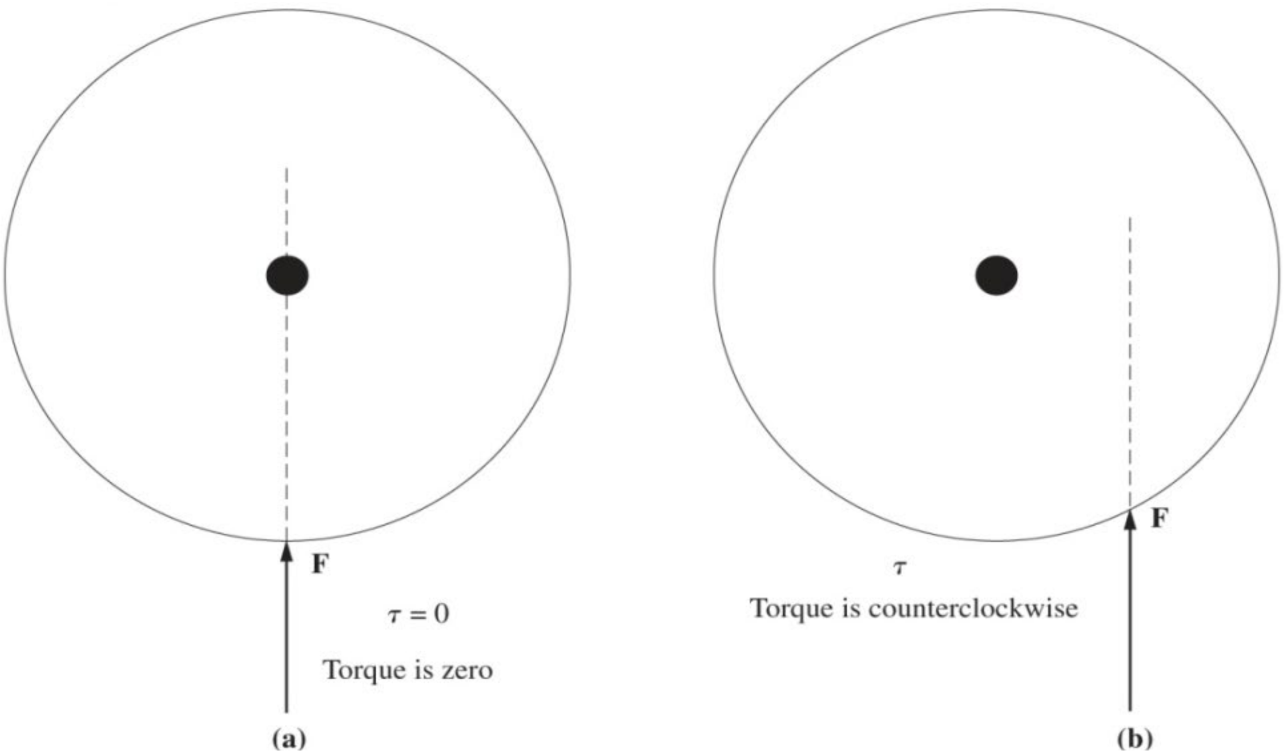
- n_m, Angular Velocity, revolution/minute

- α, **Angular Acceleration**, radians/second^2

$$\alpha = \frac{d\omega_m}{dt}$$

Torque

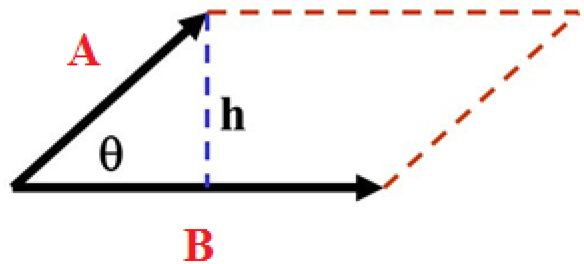
- The torque on an object is the product of force applied and the *smallest distance* between the *line of action of the force and the object's axis of rotation*.



(a)A force applied to a cylinder so that it passes through the axis of rotation.

(b)A force applied to a cylinder so that the line of action misses the axis of rotation.

Cross Product

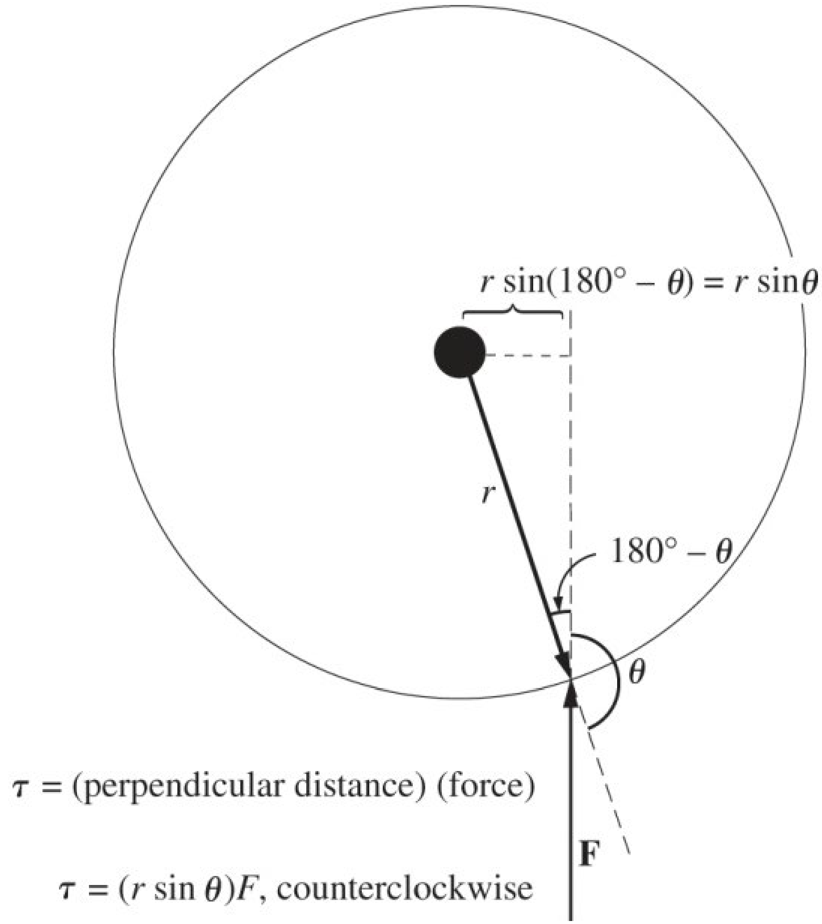


$$|\mathbf{A} \times \mathbf{B}| = |\mathbf{A}| \sin(\theta)\, |\mathbf{B}|$$

$$= \mathbf{h}\, |\mathbf{B}|$$

= area of Parallelogram

- More formally, $\tau = \boldsymbol{r} \times \boldsymbol{F}$



Derivation of the equation for the torque on an object.

<u>Power and Torque</u>

- Work is defined as the application of force, F, through a distance, r.
- **<u>For linear motion</u>** and constant force:

  $$W = Fr$$

- **<u>For rotational motion</u>**, work is the application of torque, τ, through an angle, θ :

  $$W = \tau\theta$$

- Power is the rate of doing work, or the increase of work per unit time

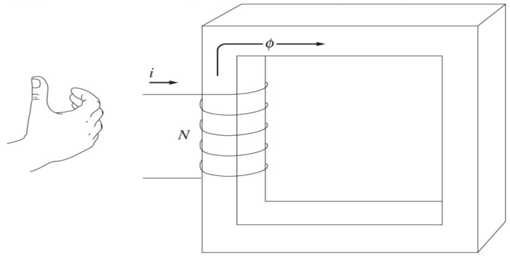
  $$P = dW/dt = d(\tau\theta)/dt = \tau(d\theta/dt) = \tau\omega$$

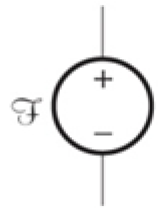
Or $\mathbf{P = \omega\tau}$

Very important relationship in the study of electric machinery.

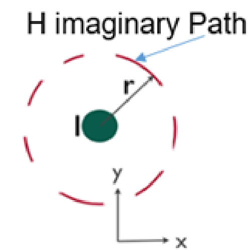
The Magnetic Field

- Magnetic Field Intensity, H, (AT/m) per Ampere's Law $\oint \mathbf{H}\cdot d\mathbf{L} = I_{enc}$





MMF=F=NI



- Magnetic Flux Density, B, accounts for medium property

$$B = \mu H = \mu_{\circ} \mu_r H$$

where, μ is magnetic permeability of medium

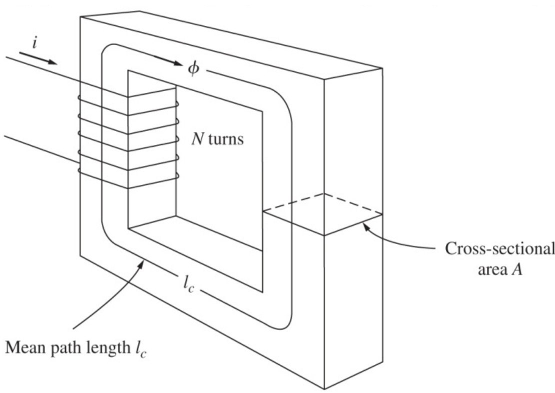
$\mu_{\circ}$ is permeability of the free space = $4\pi x10^{-7}$ H/m

μ_r is the relative permeability of the material

- Flux, ϕ is related to flux density B as:

$$\phi = \int_A B.dA$$

where A is the cross-sectional area shown

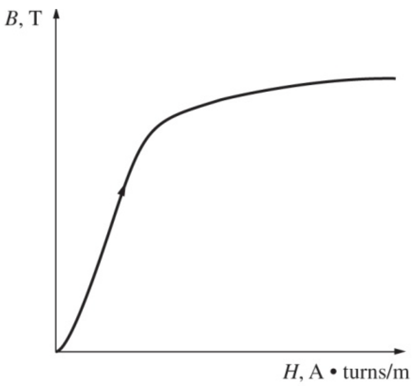


- Flux Linkage, is defined as $\lambda = N\phi$
- Inductance, L, is defined as $L=\lambda/i$ in Henries.

The **B-H Curve**

- Magnetic Flux density B (Tesla or T) accounts for the magnetic properties of the medium

- B vs H relationship is frequently expressed by a non-linear curve called B-H curve

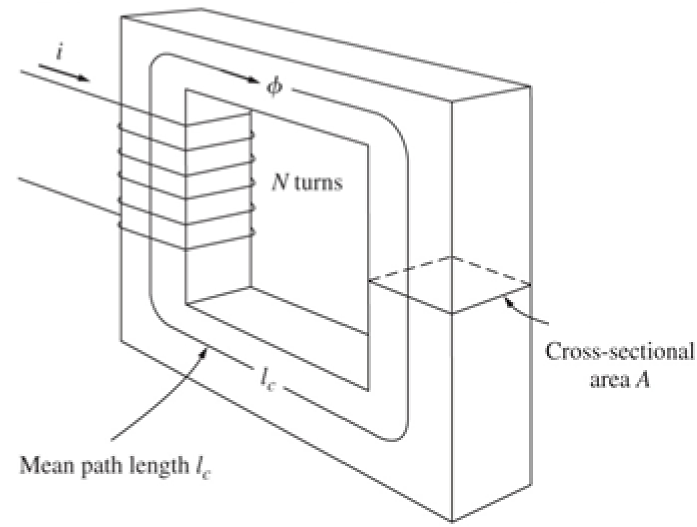


- In most *ferromagnetic* materials, the curve starts at a very high slope which tends to be constant. This is the *linear portion* of the B-H curve.

- At higher values of H, the flux density levels off and the material is said to be in *saturation region*.

Magnetic Circuit

- Ampere's law applied over the mean length l_c as:



$$\oint \mathbf{H} \cdot d\mathbf{L} = I_{enc}$$

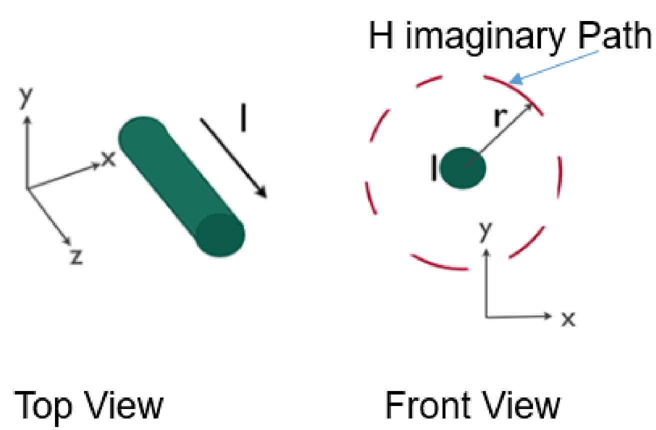
$$H = \frac{Ni}{l_c}$$

where H is the average magnitude of **H** in the core

- Total Flux, ϕ, crossing surface A is obtained as:

$$B = \mu H = \mu_o \mu_r H$$

$$\phi = \int_A \mathbf{B.dA}$$



- Defining Reluctance

$$\Re = \frac{\ell}{\mu A}$$

- Magnetic circuit is an analogous DC circuit for a magnetic device so that the *magnetomotive* force MMF or F is:

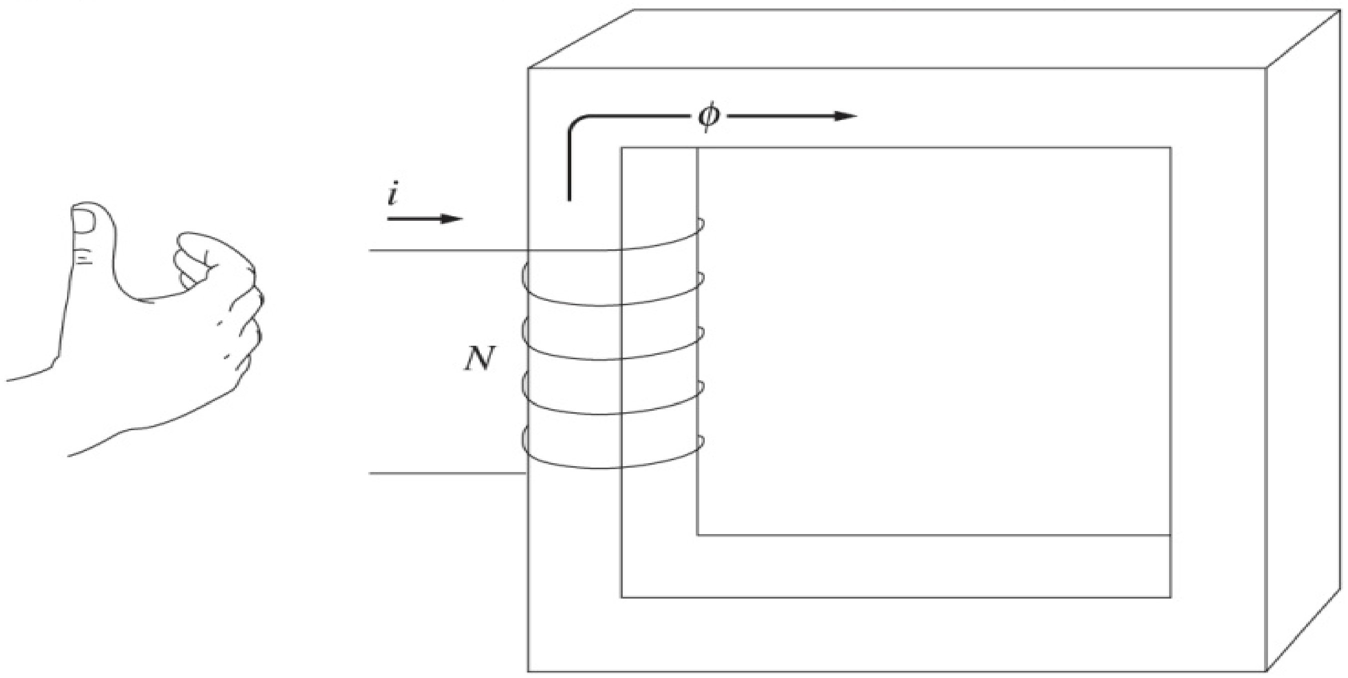
$$F = \Re\phi$$

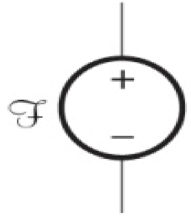
which is analogous to Ohm's law

$$V=Ri$$

Where *magnetomotive force* F and flux ϕ are analogous to *electromotive force* V and current I, respectively.

Polarity of magnetomotive force (mmf)

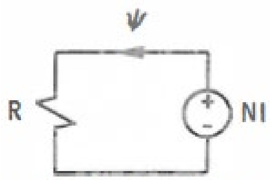




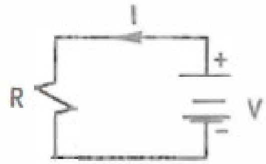
Determining the polarity of a mmf in a magnetic circuit

MAGNETIC & ELECTRIC CIRCUITS RELATIONSHIPS

QUANTITIES \ CIRCUITS	MAGNETIC CIRCUIT	ELECTRIC CIRCUIT
FLOW QUANTITY	FLUX Ψ [Wb]	CURRENT I [A]
FLOW DENSITY	FLUX DENSITY B [Wb / m^2]	CURRENT DENSITY J [A/m]
FLOW RESISTANCE	RELUCTANCE $R = \frac{l}{\mu A}$ $\left[\frac{AT}{Wb}\right]$	RESISTANCE $R = \frac{l}{\sigma A}$ $\left[\frac{V}{A}\right]$
MOTIVE FORCE	MAGNETOMOTIVE FORCE MMF [AT]	VOLTAGE V [V]
MOTIVE FORCE INTENSITY	MAGNETIC FIELD INTENSITY H [A / m]	ELECTRIC FIELD INTENSITY E [V / m]
FLOW RELATIONSHIP	MAGNETIC OHM'S LAW $MMF = \Psi R$	OHM'S LAW $V = IR$
MATERIAL PROPERTY	PERMEABILITY $\mu = \mu_r \mu_o$ [Wb / A·m]	CONDUCTIVITY σ [A / V·m]
CONSTITUTIVE RELATIONSHIP	$\bar{B} = \mu \bar{H}$	$\bar{J} = \sigma \bar{E}$



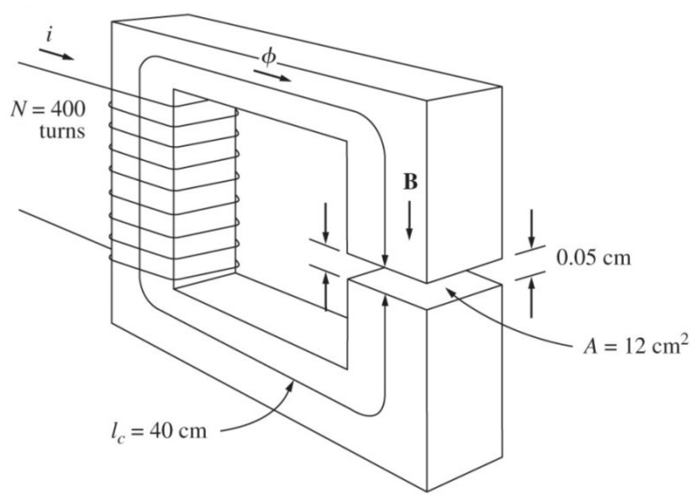
MAGNETIC CIRCUIT



ELECTRIC CIRCUIT

Problem

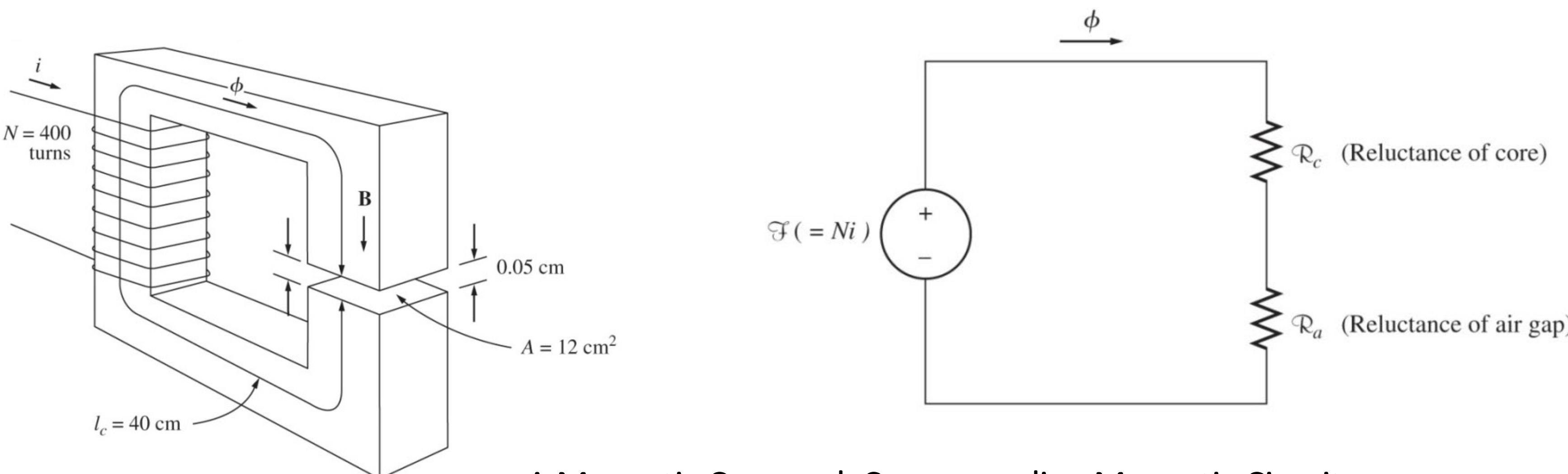
A magnetic core with a relative permeability of 4000 is shown. Assuming a fringing coefficient of 1.05 for the air gap, find the flux density in the air gap if i=0.60A.



A Magnetic Core

Problem-1

A magnetic core with a relative permeability of 4000 is shown in Assuming a fringing coefficient of 1.05 for the air gap, find the flux density in the air gap if i=0.60A.



A Magnetic Core and Corresponding Magnetic Circuit

Note:

If there is an air gap in the flux path in a magnetic core, the flux tends to spread and hence the cross section area of the air gap, A_g, will be larger than A_c, the cross section area on the core surfaces on either side of the air gap. This phenomena is called fringing is accounted for by increasing the cross section of the air gap, A_a, by a "fringing coefficient", i.e $A_g = K_f A_c$

The magnetic circuit shown can be analyzed as follows:

$$R_c = \frac{l_c}{\mu A_c} = \frac{0.4m}{(4000)(4\pi x10^{-7})(0.002m^2)} = 66{,}300\text{A.T/Wb}$$

$$R_g = \frac{l_g}{\mu_0 A_g} = \frac{0.0005m}{(4\pi x10\_7)(1.05)(0.002m^2)} = 316{,}000\text{ A.T/Wb}$$

$$\varphi = \frac{Ni}{(R_c + R_g)} = \frac{400X0.60}{382{,}300} = 0.628\text{mWb}$$

$$B_g = \frac{\varphi}{A_g} = \frac{0.628x10^{-3}}{(1.05)(0.002)} = 0.50\text{ T}$$

Production of Induced Force on a Wire

- Force on a current-carrying conductor of length placed in a magnetic field *B* is given by

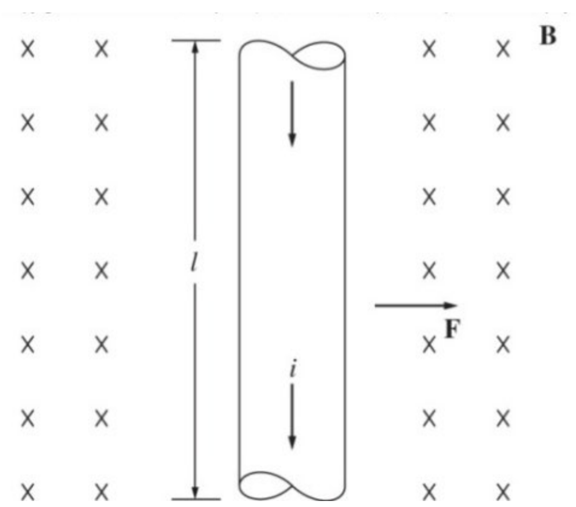
$$F = i(\ell \mathbf{x} \boldsymbol{B})$$

where,

i = magnitude of the current in the wire

ℓ = vector length of wire its direction defined to be in the direction of the current flow.

B = magnetic flux density vector



Force on a current-carrying wire in a magnetic field

Induced Voltage on a Conductor Moving in a Magnetic Field

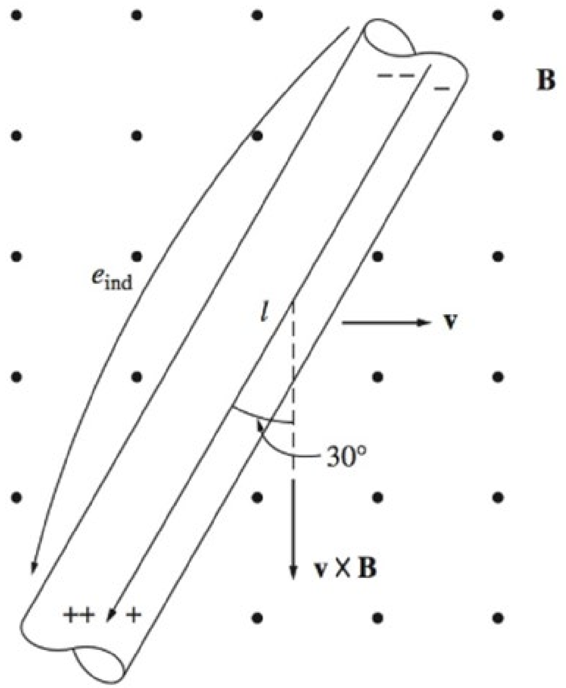
$$e_{ind} = (\mathbf{v} \mathbf{x} \boldsymbol{B}) . l$$

where

v = velocity of the wire

B = magnetic flux density vector

l = vector length of the conductor in the field with ***l*** pointing along the direction of the wire toward the end making the smallest angle with respect to the vector $(\mathbf{vxB})$

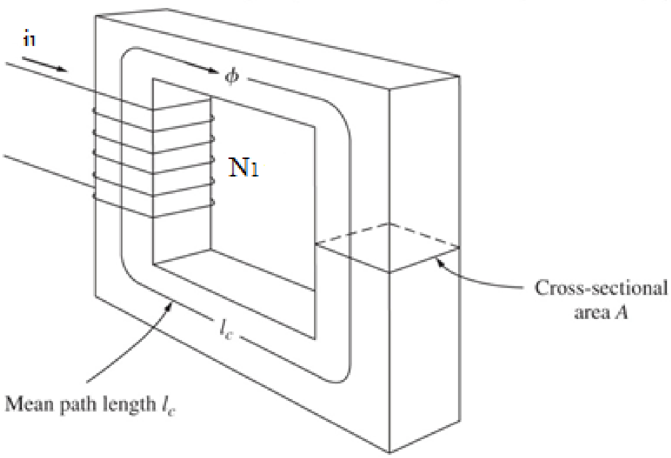


Induced voltage in a wire

Self and Mutual Inductances

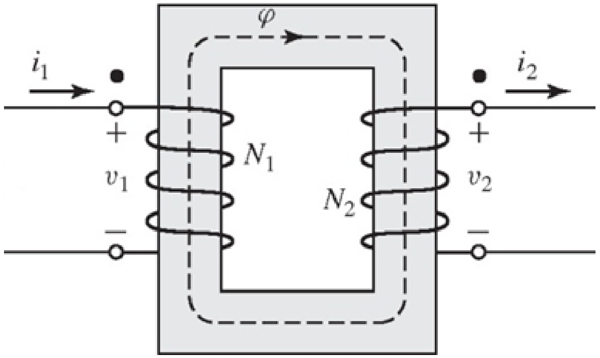
Self and Mutual Inductances

- The flux linkage in coil is $\lambda = N\,\varphi$
- Also, in terms of the coil inductance L, define $\lambda = Li$
- Note $e = d\lambda/dt$ (Faraday's Law) & Energy stored in an inductor is $W = 1/2(Li^2)$
- If we excite Coil #1, where $MMF1 = N_1 I_1$ will produce flux φ_1
- The flux linkage in coil 1 due to coil 1 current excitation is λ_{11}
- $\lambda_{11} = N_1\,\varphi_1 = N_1\,(N_1 I_1/R) = N_1^2\,i_1/\mathfrak{R} = L_{11} i_1$
- where the Reluctance $R = l/\mu A$
- $\rightarrow = \mathbf{L_{11} = N_1^2/\mathfrak{R}}$

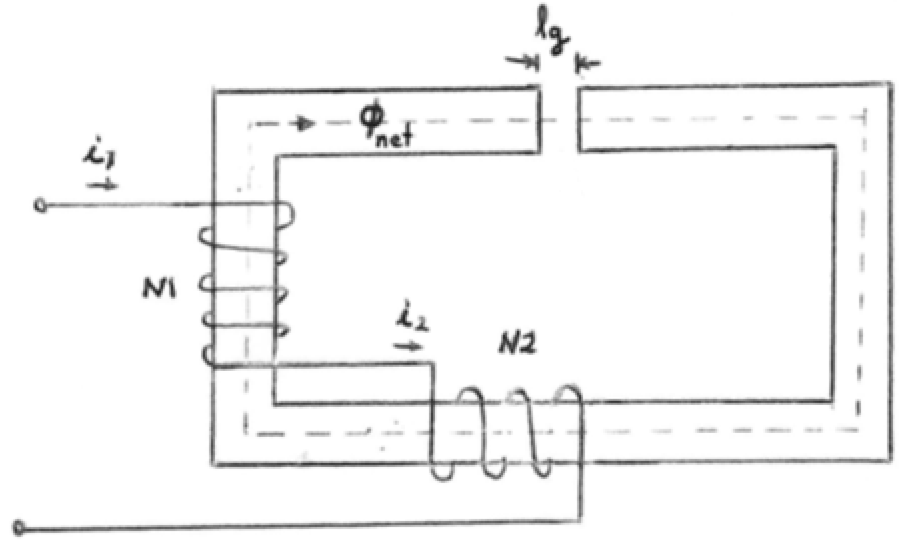


In the case of two coils with turns N_1 & N_2

- It can be shown similarly that $\mathbf{L_{22} = N_2^2/\mathfrak{R}}$
- Also, the mutual inductances $\mathbf{L_{12} = L_{21} = (N_1 N_2)/\mathfrak{R}}$



Problem



For the magnetic circuit shown, the air gap cross-sectional area is 0.004 m^2, the air gap length l_g=0.002 m, winding #1 has N_1=600 turns, winding #2 has N_2=200 turns, the core permeability is infinite, and leakage and fringing of the gap are neglected.

a) Find the reluctance R of the magnetic circuit

$$R = R_g = \frac{l_g}{\mu_0 A} = \frac{0.002}{(4\pi \times 10^{-7})(0.004)} = \frac{1}{8\pi(10^{-7})} = 397.887 \text{ K AT/Wb}$$

b) Find the current required to produce a net flux Φ_{net}=0.004 Wb in the airgap

$$\phi_{net} = \phi_1 - \phi_2 = \frac{N_1 i_1}{R} - \frac{N_2 i_2}{R} = \frac{i(N_1 - N_2)}{R}$$

$$\Rightarrow i = \frac{R\,\phi_{net}}{(N_1 - N_2)} = \frac{397,887 \times (0.004)}{(600 - 200)} = 3.98 \text{ A}$$

c) Find the self inductances L_{11} and L_{22} of windings #1 and #2

$L_{11}=N_1^2/\Re$=$(600)^2$/397,887=0.9 H & $L_{22}=N_2^2/R$ =$(200)^2$/397,887=0.1 H

d) Find the mutual inductances L_{12} and L_{21} between windings #1 and #2.

$L_{12}=L_{21}=(N_1N_2)/\Re$=(200)(600)/397,887=0.3 H