EENG 577 M2 Assignment 2 (Group)

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1) If we assume a 42MVA, 118 - 14.3 kV three-legged magnetic core with primary and secondary coil winding around each phase connected Δ - Y. The resistance of the primary and secondary windings are given. Also, the self inductance for the primary, self inductance on the secondary, and mutual inductances are given. The following state-space model can be used to describe the instantaneous voltages and currents assuming a linear unsaturated magnetic core where the inductance is constant for the time rate-of-change of current on the primary and secondary coils.

$$V = [v_{p1}, v_{p2}, v_{p3}, v_{s1}, v_{s2}, v_{s3}]^T \qquad I = [i_{p1}, i_{p2}, i_{p3}, i_{s1}, i_{s2}, i_{s3}]^T$$

$$\dot{I} = \frac{d}{dt}[i_{p1}, i_{p2}, i_{p3}, i_{s1}, i_{s2}, i_{s3}]^T$$

$$\begin{bmatrix} v_{p1} \\ v_{p2} \\ v_{p3} \\ v_{s1} \\ v_{s1} \\ v_{s1} \end{bmatrix} = \begin{bmatrix} r_p & 0 & 0 & 0 & 0 & 0 \\ 0 & r_p & 0 & 0 & 0 & 0 \\ 0 & 0 & r_p & 0 & 0 & 0 \\ 0 & 0 & 0 & r_s & 0 & 0 \\ 0 & 0 & 0 & 0 & r_s & 0 \\ 0 & 0 & 0 & 0 & 0 & r_s \end{bmatrix} \begin{bmatrix} i_{p1} \\ i_{p2} \\ i_{p3} \\ i_{s1} \\ i_{s1} \\ i_{s1} \end{bmatrix} + \begin{bmatrix} L_{pp} & M_{pp} & M_{pp} & L_{ps} & M_{ps} & M_{ps} \\ M_{pp} & L_{pp} & M_{pp} & M_{ps} & L_{ps} & M_{ps} \\ M_{pp} & M_{pp} & L_{pp} & M_{ps} & M_{ps} & L_{ps} \\ L_{ps} & M_{ps} & M_{ps} & L_{ss} & M_{ss} & M_{ss} \\ M_{ps} & L_{ps} & M_{ps} & M_{ss} & L_{ss} & M_{ss} \\ M_{ps} & M_{ps} & L_{ps} & M_{ss} & M_{ss} & L_{ss} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_{p1} \\ i_{p2} \\ i_{p3} \\ i_{s1} \\ i_{s2} \\ i_{s3} \end{bmatrix} \tag{1}$$

2) We can use compact matrix notation to get the state-space model into the general form $\dot{X} = AX + BU$ where A, B are given below and X, and U are the current and voltage vectors respectively.

$$V = RI + \frac{d}{dt}(LI) \qquad \dot{I} = -L^{-1}RI + L^{-1}V$$

$$V = RI + L\frac{d}{dt}I \qquad \dot{X} = AX + BU$$

$$L^{-1}V = L^{-1}RI + \dot{I} \qquad A = -L^{-1}R \qquad B = L^{-1}$$

3 - 5) The power relationships and efficiency in terms of instantaneous voltage, current are given below. The power in is calculated using the primary currents and voltages and accounts for the copper losses in the primary and secondary windings.

$$P_{in} = v_{p1}i_{p1} + v_{p2}i_{p2} + v_{p3}i_{p3} \tag{2}$$

The output power is calculated using the secondary currents and voltages and does not account for the copper or core losses.

$$P_{out} = v_{s1}i_{s1} + v_{s2}i_{s2} + v_{s3}i_{s3} \tag{3}$$

Transformer efficiency is a function of output power and losses, where output power is divided by output power + losses multiplied by 100%.

$$\eta = \frac{P_{out}}{P_{out} + P_{cu}} \cdot 100\% = \frac{v_{s1}i_{s1} + v_{s2}i_{s2} + v_{s3}i_{s3}}{v_{s1}i_{s1} + v_{s2}i_{s2} + v_{s3}i_{s3} + (i_{p1}^2 + i_{p2}^2 + i_{p3}^2)r_p + (i_{s1}^2 + i_{s2}^2 + i_{s3}^2)r_s} \cdot 100\% \tag{4}$$

6) Solving Equation (1) for the instantaneous voltage on the primary coils gives equations (5) through (7).

$$v_{p1}(t) = r_p i_{p1} + L_{pp}\frac{d}{dt}i_{p1} + M_{pp}\frac{d}{dt}(i_{p2} + i_{p3}) + L_{ps}\frac{d}{dt}i_{s1} + M_{ps}\frac{d}{dt}(i_{s2} + i_{s3}) \tag{5}$$

$$v_{p2}(t) = r_p i_{p2} + L_{pp}\frac{d}{dt}i_{p2} + M_{pp}\frac{d}{dt}(i_{p1} + i_{p3}) + L_{ps}\frac{d}{dt}i_{s2} + M_{ps}\frac{d}{dt}(i_{s1} + i_{s3}) \tag{6}$$

$$v_{p3}(t) = r_p i_{p3} + L_{pp}\frac{d}{dt}i_{p3} + M_{pp}\frac{d}{dt}(i_{p1} + i_{p2}) + L_{ps}\frac{d}{dt}i_{s3} + M_{ps}\frac{d}{dt}(i_{s1} + i_{s2}) \tag{7}$$

7) Given the apparent power, $S_{3\phi}$, of transformer and the Line-Line voltage on the secondary, we can find the full load impedance on the secondary.

$$V_\phi = \frac{V_L}{\sqrt{3}} = \frac{14.3\ [\text{kV}]}{\sqrt{3}} = 8.27\ [\text{kV}] \tag{8}$$

$$\cos^{-1}(0.8) = 36.87\ [\text{deg}] \tag{9}$$

$$Z_\phi = \frac{3(V_\phi)^2}{S_3\phi} = \frac{3(8.27)^2}{42} = 4.885\angle 36.87^o\ [\Omega] \tag{10}$$