COLORADO SCHOOL OF MINES
ELECTRICAL ENGINEERING
DEPARTMENT

ENG 577

M2 Project KEY

1. **For the system shown, find the equivalent electric circuit and draw it.**

Solution:

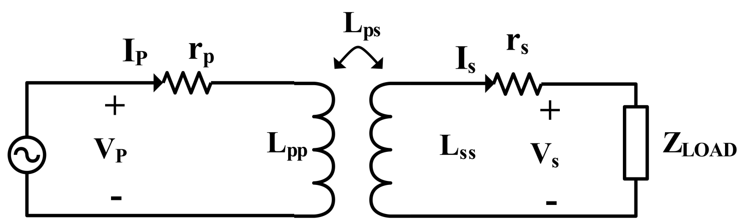

Figure 1 – Equivalent Circuit.

2. **Write the governing voltage equations for $v_1(t)$ and $v_2(t)$ in terms of currents $i_1(t)$ and $i_2(t)$ given $Z_{Load}=R_L+jX_L$ and coils 1 and 2 resistances R_1 and R_2 (use symbols only).**

Solution:

The transformer state-space model is expressed as [1],

$$\overline{V} = \overline{R} \cdot \overline{I} + \frac{d\overline{\lambda}}{dt}$$

Where λ_j is the flux linkage (obtained as $\lambda_j = \sum_{k=1}^{n} L_{jk} \cdot i_k$). for winding jth, and n is the number of coils. Therefore, the equation can be rewritten as,

$$\overline{V} = R \cdot \overline{I} + L \cdot \frac{d\overline{I}}{dt}$$

Using the equivalent electric circuit, the following equation can be obtained:

$$v_1 = r_1 \cdot i_1 + \frac{d}{dt}(L_{11} \cdot i_1 + \mathrm{M} \cdot i_2)$$

$$v_2 = r_2 \cdot i_2 + \frac{d}{dt}(L_{22} \cdot i_2 + \mathrm{M} \cdot i_1)$$

Using the voltage equations, the state space model can be expressed as matrices:

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + \begin{bmatrix} L_{pp} & M \\ M & L_{ss} \end{bmatrix} \cdot \frac{d}{dt} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

The secondary voltage is related to the load, Z_{load}, as $v_s = Z_{load} \cdot i_s$ where Z_{load} is represented by r_l and L_l. As such, the equation above can be rearranged and can be used to solve for the current vector $\bar{I}$ as follows:

$$\frac{d}{dt} \begin{bmatrix} i_p \\ i_s \end{bmatrix} = -\begin{bmatrix} L_{pp} & M \\ M & (L_{ss} + L_l) \end{bmatrix}^{-1} \cdot \begin{bmatrix} r_p & 0 \\ 0 & (r_s + r_l) \end{bmatrix} \cdot \begin{bmatrix} i_p \\ i_s \end{bmatrix} + \begin{bmatrix} L_{pp} & M \\ M & (L_{ss} + L_l) \end{bmatrix}^{-1} \cdot \begin{bmatrix} v_p \\ 0 \end{bmatrix}$$

3. Use MATLAB to plot the current $i_1(t)$ and $i_2(t)$, assuming the following: f=60 Hz, v1(t)=120 cos(ωt) & Z_{Load}=20 ∠+30° Ω, R_1=0.2 Ω and R_2=0.4 Ω.

Solution:

First, the inductances (self and mutual) are calculated using the transformer parameters available.

```
N1 = 800.
N2=400.
Gaplength = .0005;
area = 0.0012;
gapCoef = 1.05;
mu0 = 4*pi*1e-7;
Relc = (len)/(area*gapCoef*mu0);
L11 =N1^2/Relc;
L22 = N2^2/Relc;
L12 = (N2*N1)/Relc;
```

Second, find the load parameters. Where the load is $Z_{Load} = 20\angle 30$

```
RL = 20*cosd(30);
Ll = (20*sind(30))/((2*pi*60)); % Ll = XL/(2*pi*f)
```

Finally, put everything together and build the SS script to solve for the currents based in the equations above. First, using MATLAB ODE45 function.

```
%% State Space Model (4th-Order Runge Kutta Method for ODEs)
tspan = [0 1]; % integrates the system of differential equations from 0 to 0.0667
                              % time of running. Note: the time is longer to get accurate results for the current values.
iniCon = [0;0]; %initial conditions or initial current value
[t,x] = ode45(@myode, tspan, iniCon); %
Is = rms(x(:,2));
Ip = rms(x(:,1));
plot(t,x(:,1),t,x(:,2));
legend('i1','i2')
xlabel('Time')
ylabel('Current [Amps]')
title('Kutta Method for ODEs')
grid
function dx = myode(t,x)
   %inductances calculations
   N1 = 800;
   N2=400;
   len = .0005;
   area = 0.0012;
   gapCoef = 1.05;
   mu0 = 4*pi*1e-7;
   Relc = (len)/(area*gapCoef*mu0);
   L11 =N1^2/Relc;
   L22 = N2^2/Relc;
   L12 = (N2*N1)/Relc;
   %Load parameters calculations
   RL = 20*cosd(-30);
   Ll = (20*sind(30))/((2*pi*60));
   %windings' resistance values
   R1 =0.2;
   R2 = 0.4;
   R = [R1 0;0 R2+RL];
   L = [L11 L12;L12  L22+Ll];
   Vp = 120;
   A = -inv(L)*R;
   B = inv(L);
   K = [Vp;0];
   u = K*sin(377.*t+(pi/2));

   dx = A*x+ B*u;
end
```

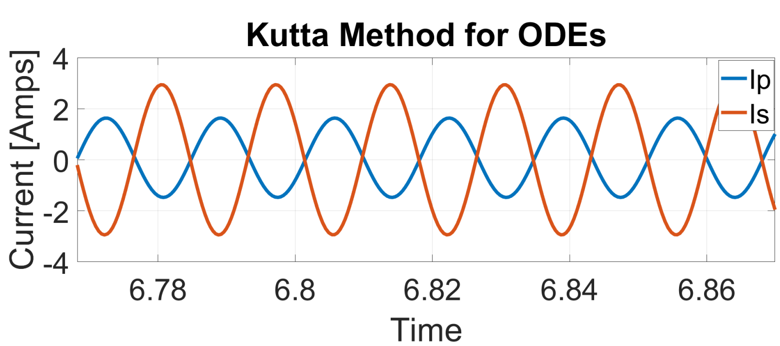

Figure 2 – Transformer Primary and Secondary Current

Or using Euler's Method:

```
n =60;
h=0.000167;                % step size
t = 0:h:1;                 % time of running. Note: the time is longer to get
                             accurate results for the current values.
x = zeros(2,length(t));     % initialize the state vector size
K = [1;0];
A = -inv(L)*R;
B = inv(L);
u = K*Vp;
SS = A*x+ B*u*sin(377*t+pi/2);
dx =@(t,x)(A*x+ B*u*sin(377.*t));
%% State Space Model (Euler's Method)
%
for i = 1:(length(t)-1)
   t(i+1) = t(i)+h;
   x(:,i+1) = x(:,i) +  h*dx(t(i),x(:,i));
end
plot(t,x(1,:),t,x(2,:))
legend({'I1','I2'})
xlabel('time')
ylabel('Current Values')
title('Euler Method')
Is = rms(x(2,:));
Ip = rms(x(1,:));
```

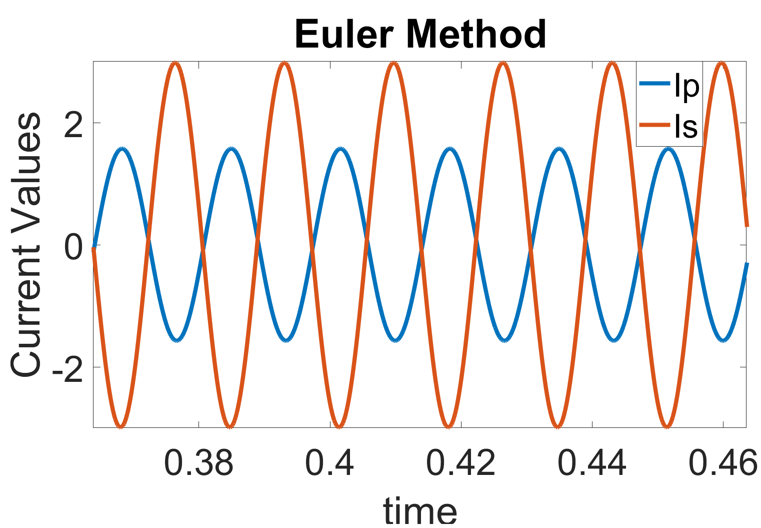

Figure 3 – Transformer Primary and Secondary Current

Or using MATLAB/Simulink

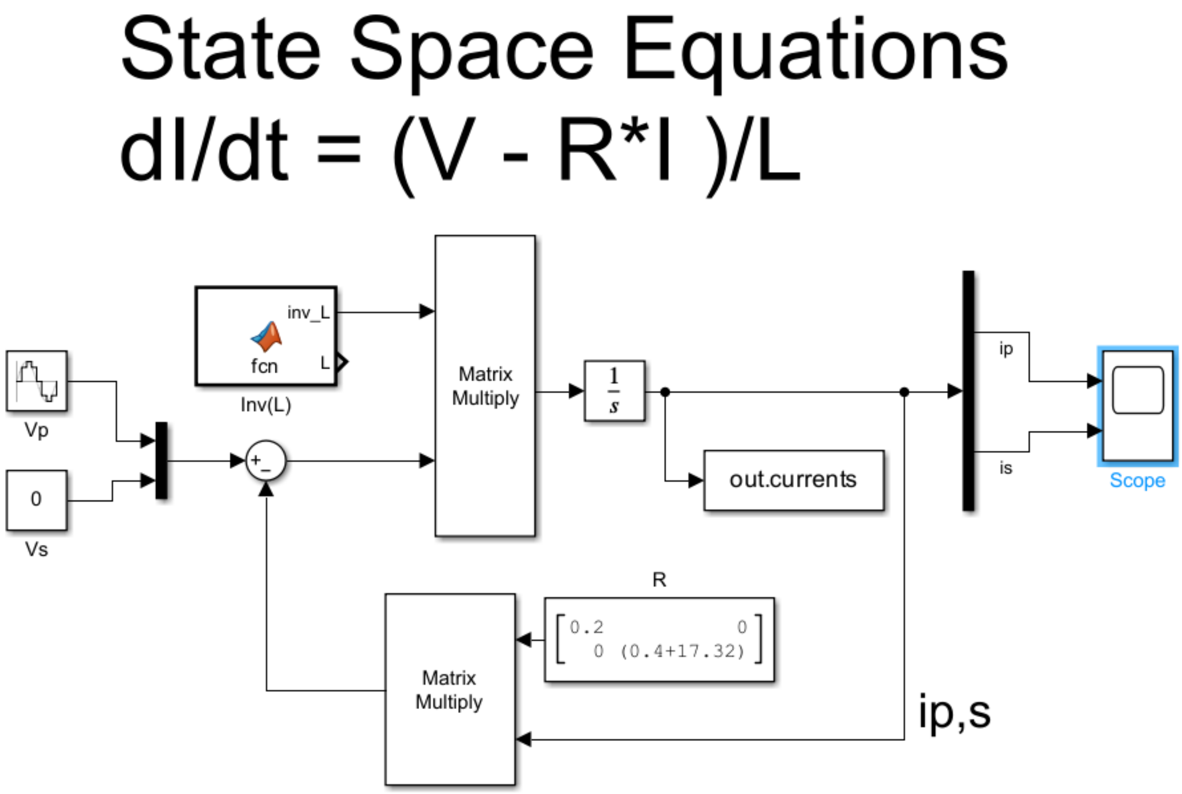

Figure 4 – Transformer Primary and Secondary Current

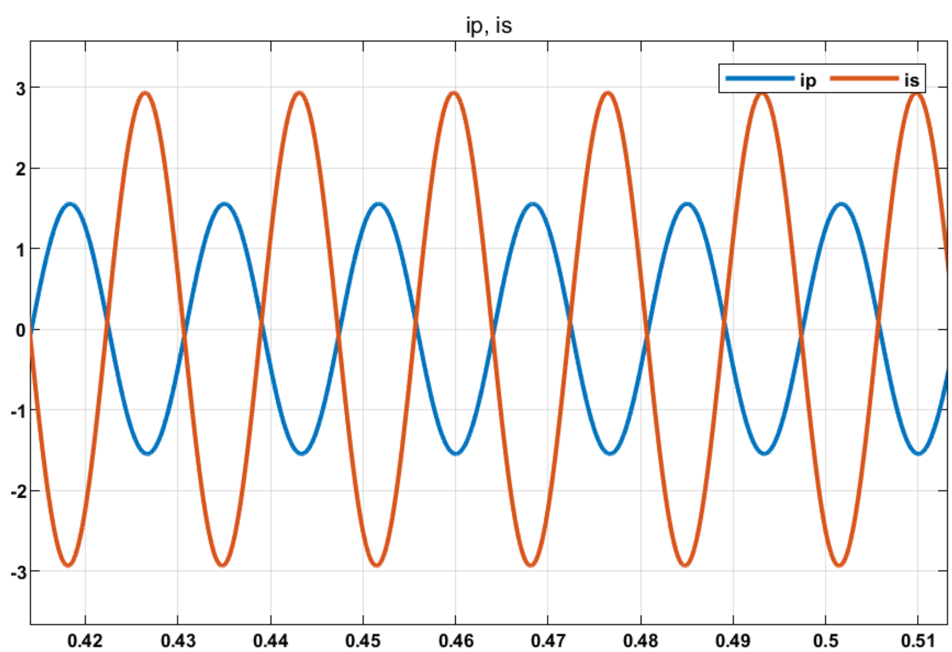

Figure 5 – MATLAB/Simulink Transformer Primary and Secondary Current

4. **Calculate the load real, reactive, and complex powers and write down the values.**

The load real power can be found by multiplying the current squared by the resistance of the load.

$$P = I_{2,rms}^2 \cdot r_l = 77.1\ W$$

With real power calculated, apparent power can be determined by using the power triangle.

$$S = \frac{P}{COS(30)} = \frac{P}{PF} = 88.9\ VA$$

Finally, the reactive power can be found using current and impedance.

$$Q = \sqrt{S_L^2 - P_L^2} = I_{2,rms}^2 \cdot X_{Ll} = S \cdot sin(30) = 44.5\ VAR$$

[1] Colorado School of Mines, EENG577 Class Notes, January, 2023.

[2] R. A. Almazmomi, M. A. Gutierrez-McCoy and A. A. Arkadan, "Transformer EM-FL-PSO Design Optimization," *2021 IEEE Green Technologies Conference (GreenTech)*, Denver, CO, USA, 2021, pp. 430-434, doi: 10.1109/GreenTech48523.2021.00073.