EENG577 M2-A2 Assignment KEY

Consider the schematic of a three-phase 42MVA, 118-14.3 kV, Δ-Y distribution transformer:

The transformer primary and secondary winding parameter values (neglecting saturation) are summarized as follows:

Please answer the following questions:

1) For the transformer voltage equations given below, using compact matrix notation, write the expanded form of **V, I, L,** and **R**, using the notation given above (no numerical values).

V=**R**·**I**+**L** d(**I**)/dt

For example, **V**=[$v_{p1}, v_{p2}, v_{p3}, v_{s1}, v_{s2}, v_{s3}$]^T, where T denotes the vector transpose.

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$$
\begin{bmatrix} v_{p1} \\ v_{p2} \\ v_{p3} \\ v_{s1} \\ v_{s2} \\ v_{s3} \end{bmatrix} =
\left[\begin{array}{ccc|ccc} r_p & 0 & 0 & 0 & 0 & 0 \\ 0 & r_p & 0 & 0 & 0 & 0 \\ 0 & 0 & r_p & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & r_s & 0 & 0 \\ 0 & 0 & 0 & 0 & r_s & 0 \\ 0 & 0 & 0 & 0 & 0 & r_s \end{array}\right]
\begin{bmatrix} i_{p1} \\ i_{p2} \\ i_{p3} \\ i_{s1} \\ i_{s2} \\ i_{s3} \end{bmatrix} +
\left[\begin{array}{ccc|ccc} L_{pp} & M_{pp} & M_{pp} & L_{ps} & M_{ps} & M_{ps} \\ M_{pp} & L_{pp} & M_{pp} & M_{ps} & L_{ps} & M_{ps} \\ M_{pp} & M_{pp} & L_{pp} & M_{ps} & M_{ps} & L_{ps} \\ \hline L_{ps} & M_{ps} & M_{ps} & L_{ss} & M_{ss} & M_{ss} \\ M_{ps} & L_{ps} & M_{ps} & M_{ss} & L_{ss} & M_{ss} \\ M_{ps} & M_{ps} & L_{ps} & M_{ss} & M_{ss} & L_{ss} \end{array}\right]
\frac{d}{dt}
\begin{bmatrix} i_{p1} \\ i_{p2} \\ i_{p3} \\ i_{s1} \\ i_{s2} \\ i_{s3} \end{bmatrix}
$$

2) If the transformer state space equation is written in the general compact form as

d**X**/dt=**A**·**X**+**B.U**

Following the development given in the handout, write down the equations for **A**, **B**, **X**, and **U**, relating it to the quantities given in part 1) above. Use compact matrix notation (do not expand).

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$A = -RL^{-1}$

$B = L^{-1}$

$X = \bar{I}$

$U = \bar{V}$

$$\frac{d}{dt}\begin{bmatrix} i_{p1} \\ i_{p2} \\ i_{p3} \\ i_{s1} \\ i_{s2} \\ i_{s3} \end{bmatrix} = -\begin{bmatrix} r_p & 0 & 0 & 0 & 0 & 0 \\ 0 & r_p & 0 & 0 & 0 & 0 \\ 0 & 0 & r_p & 0 & 0 & 0 \\ 0 & 0 & 0 & r_s & 0 & 0 \\ 0 & 0 & 0 & 0 & r_s & 0 \\ 0 & 0 & 0 & 0 & 0 & r_s \end{bmatrix} \begin{bmatrix} L_{pp} & M_{pp} & M_{pp} & L_{ps} & M_{ps} & M_{ps} \\ M_{pp} & L_{pp} & M_{pp} & M_{ps} & L_{ps} & M_{ps} \\ M_{pp} & M_{pp} & L_{pp} & M_{ps} & M_{ps} & L_{ps} \\ L_{ps} & M_{ps} & M_{ps} & L_{ss} & M_{ss} & M_{ss} \\ M_{ps} & L_{ps} & M_{ps} & M_{ss} & L_{ss} & M_{ss} \\ M_{ps} & M_{ps} & L_{ps} & M_{ss} & M_{ss} & L_{ss} \end{bmatrix}^{-1} \begin{bmatrix} i_{p1} \\ i_{p2} \\ i_{p3} \\ i_{s1} \\ i_{s2} \\ i_{s3} \end{bmatrix}$$

$$+ \begin{bmatrix} L_{pp} & M_{pp} & M_{pp} & L_{ps} & M_{ps} & M_{ps} \\ M_{pp} & L_{pp} & M_{pp} & M_{ps} & L_{ps} & M_{ps} \\ M_{pp} & M_{pp} & L_{pp} & M_{ps} & M_{ps} & L_{ps} \\ L_{ps} & M_{ps} & M_{ps} & L_{ss} & M_{ss} & M_{ss} \\ M_{ps} & L_{ps} & M_{ps} & M_{ss} & L_{ss} & M_{ss} \\ M_{ps} & M_{ps} & L_{ps} & M_{ss} & M_{ss} & L_{ss} \end{bmatrix}^{-1} \begin{bmatrix} v_{p1} \\ v_{p2} \\ v_{p3} \\ v_{s1} \\ v_{s2} \\ v_{s3} \end{bmatrix}$$

3) Write the expression for the input power of the transformer, function of the windings' voltage and current.

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$$P_{in} = P_{out} + P_{losses} = \sqrt{3} V_{s,L} I_s \cos(\theta_s) + P_{losses}$$

$P_{in}=[V_{p1}I_{p1}+ V_{p2}I_{p2}+ V_{p3}I_{p3}] \cos(\theta_p)$

4) Write the expression for the output power of the transformer, function of the windings' voltage and current.

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$$P_{out} = \sqrt{3} V_{s,L} I_s \cos(\theta_s)$$

$P_{out}=[V_{s1}I_{s1}+ V_{s2}I_{s2}+ V_{s3}I_{s3}] \cos(\theta_s)$

5) Write the expression for % transformer efficiency.

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$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{oout}}{P_{out} + P_{losses}}$$ x100% **-2**

6) The transformer is supplied by a 3-phase, 60Hz, 118kV voltage source. Write the equations of the of the voltages $v_{p1}(t)$, $v_{p2}(t)$, and $v_{p2}(t)$, and show all values.

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$$v_{p1}(t) = 118 \times 10^3 \sqrt{2} \cos(wt - \theta)$$

$$v_{p2}(t) = 118 \times 10^3 \sqrt{2} \cos(wt - \theta - 120)$$

$$v_{p3}(t) = 118 \times 10^3 \sqrt{2} \cos(wt - \theta - 240)$$

How to get the value of θ?

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7) Calculate the value of the per phase load impedance, Z_{Load}, for a full load, 0.8 PF load conditions.

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$$I_s = \frac{S}{\sqrt{3}V_{s,L}} = \frac{42 \times 10^6}{\sqrt{3} \cdot 14.3 \times 10^3} = 1695.7\lfloor -36.87^o A$$

$$Z_{Load} = \frac{V_{s,\varphi}}{I_s} = \frac{14.3 \times 10^3 \div \sqrt{3}}{1695.7\lfloor -36.87^o} = 4.87\lfloor 36.87^o = 3.89 + j2.92\Omega$$

The submission is **due on CANVAS at 1:59PM on the designated day** and must be submitted as a **group submission** in **PDF file** format and should be **typed**, **clear, organized, and showing your work.**