COLORADO SCHOOL OF MINES
ELECTRICAL ENGINEERING DEPARTMENT

EENG 577

ADVANCED ELECTRICAL MACHINE DYNAMICS FOR SMART-GRID SYSTEMS

M2-P1 Transformers

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Objectives

- Understand and explain the purpose of a transformer in a power system.
- Know the voltage, current, and impedance relationships across the windings of an ideal transformer.
- Be able to explain how copper losses, leakage flux, hysteresis, and eddy currents are modeled in transformer equivalent circuits.
- Understand how real transformers approximate the operation of an ideal transformer.
- Use a transformer equivalent circuit to find the voltage and current transformations across a transformer.
- Be able to calculate the losses and efficiency of a transformer.
- Be able to calculate the voltage regulation of a transformer.
- Understand, explain and explain operation of three-phase transformers.

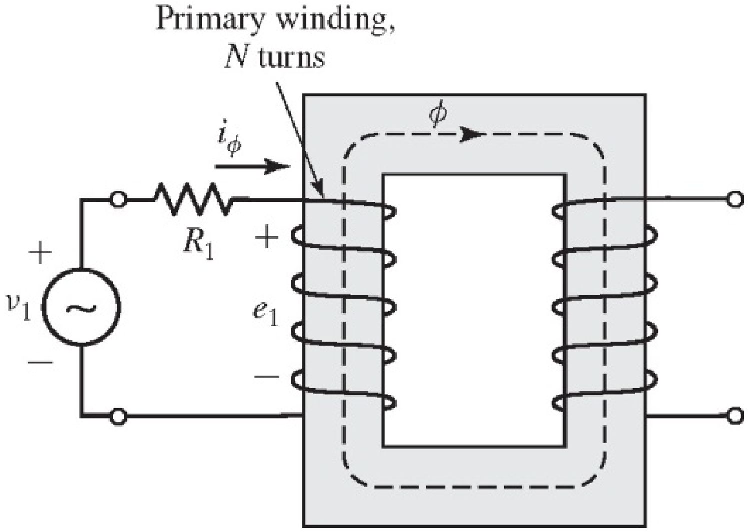
How Transformers work

How Transformers work

Introduction

- The transformer consists of two or more coils (or windings) coupled by mutual magnetic flux
- If the primary winding is connected to an alternating voltage source, an alternating flux will link the secondary winding, inducing an alternating secondary voltage
- Most transformers considered here have high-permeability iron cores to increase the degree of coupling between the windings

No-load conditions



Transformer with open secondary

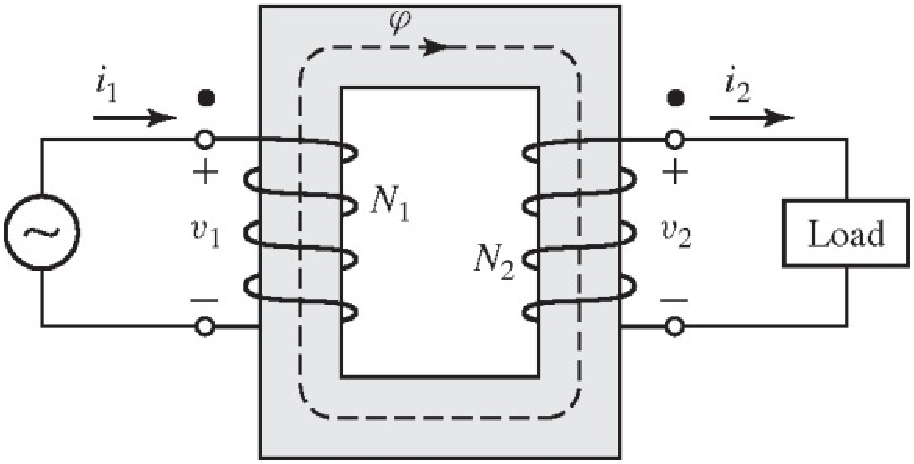
$$e_1 = \frac{d\lambda_1}{dt} = N_1 \frac{d\phi}{dt}$$

- With secondary open, there is no load on the transformer
- All of the primary current is exciting current, that is, it goes to magnetize the core and to overcome core losses
- Currents and voltages are equivalent sinusoidal values

Ideal Transformer

- Ideal transformer model:
  - Neglect losses and magnetizing current, and assume all the flux in the magnetic core links both windings
  - Faraday's Law states that induced voltage equals time rate of change of flux linking a coil
  - Ampere's Law states that primary coil MMF is equal to the secondary coil MMF

Ideal transformer and Load



Faraday's law: $v_1 = e_1 = N_1\, d\phi/dt$ $v_2 = e_2 = N_2\, d\phi/dt$

$v_1/v_2 = N_1/N_2 =$ a, the turns ratio

Ampere's law: $N_1\, i_1 = N_2\, i_2$ $i_1/i_2 = N_2/N_1$

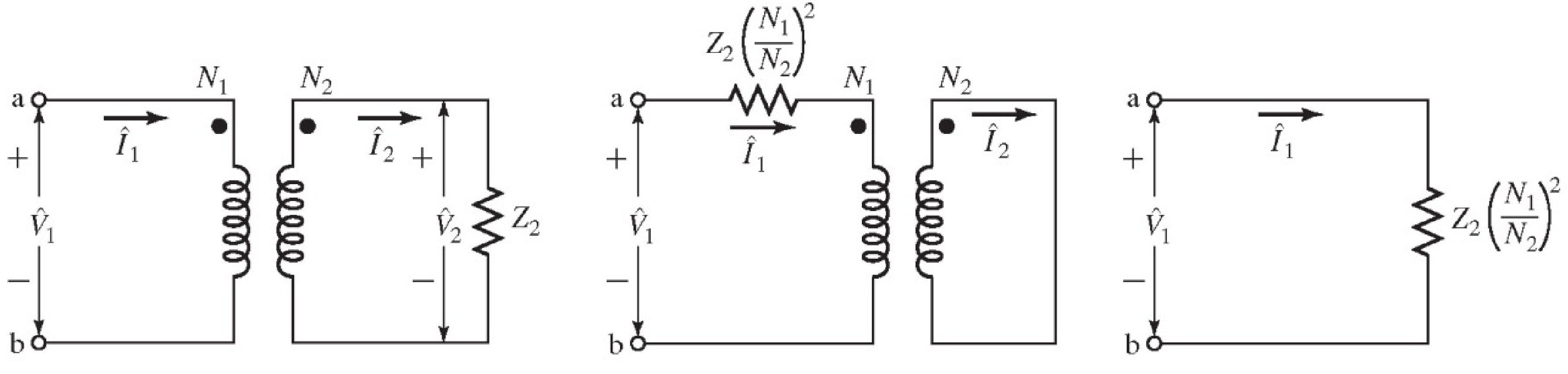
Ideal transformer: $v_1/v_2 = N_1/N_2$ and $i_1/i_2 = N_2/N_1 = 1/a$

Lossless since power in = power out $v_1\, i_1 = v_2\, i_2$

Note: $N_1/N_2 = a$ is the turns ratio

Reflected impedance

- An impedance on the secondary of an ideal transformer may be reflected or referred to the primary by the square of the turns ratio, a.



(a) (b) (c)

Note that the load Z2= V2/I2.

Similarly V1/I1= Z1 (the referred quantity of the load Z2 to the primary side)

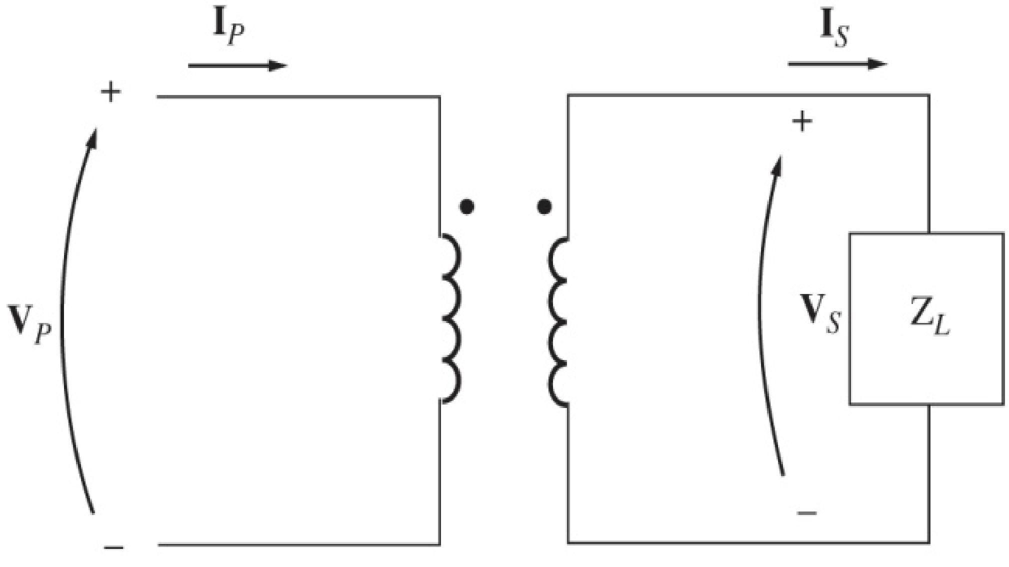
We know V1/V2=a or V1=aV2 Also, I2/I1=a or I1=I2/a

Now Consider Z1=V1/I1 and substitute for V1 and I1 in terms of V2 and I2 as shown above.

It would result in Z1=Z2(a^2) or $Z_1=Z_2(N_1/N_2)^2$

Dot Convention

- The “dots” help to determine the polarity of the voltage and direction of the current in the secondary winding.
- If **the primary *voltage* is positive at the dotted end** of the winding with respect to the undotted end, then the **secondary voltage will be positive at the dotted end** also.
- If the ***primary current*** of the transformer **flows into** the dotted end of the primary winding, the ***secondary current*** will **flow out** of the dotted end of the secondary winding.

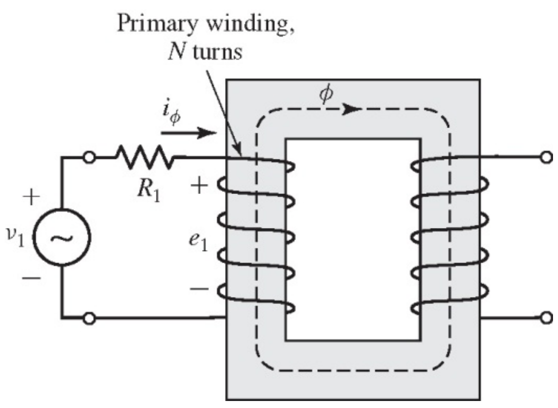


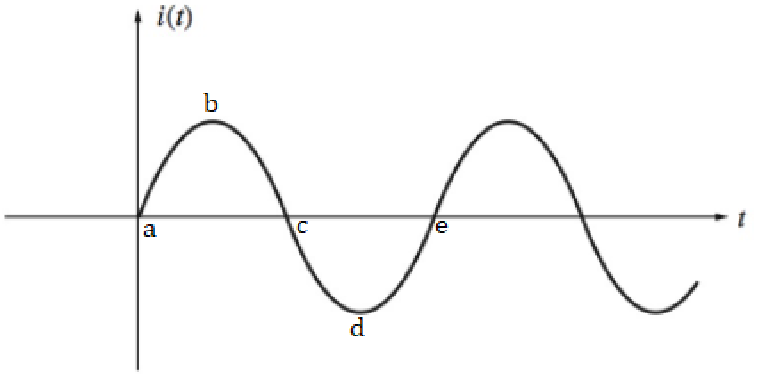
MAGNETIC (CORE) LOSSES

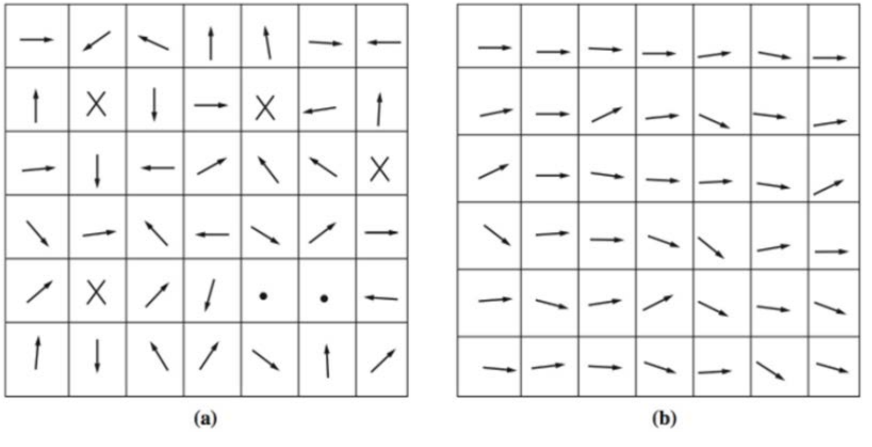
(Eddy Current & Hysteresis Losses)

- Magnetic losses are also known as core or iron losses
- The presence of magnetic material in time varying flux will result in some energy loss
- Magnetic or core losses consist of eddy current and hysteresis losses

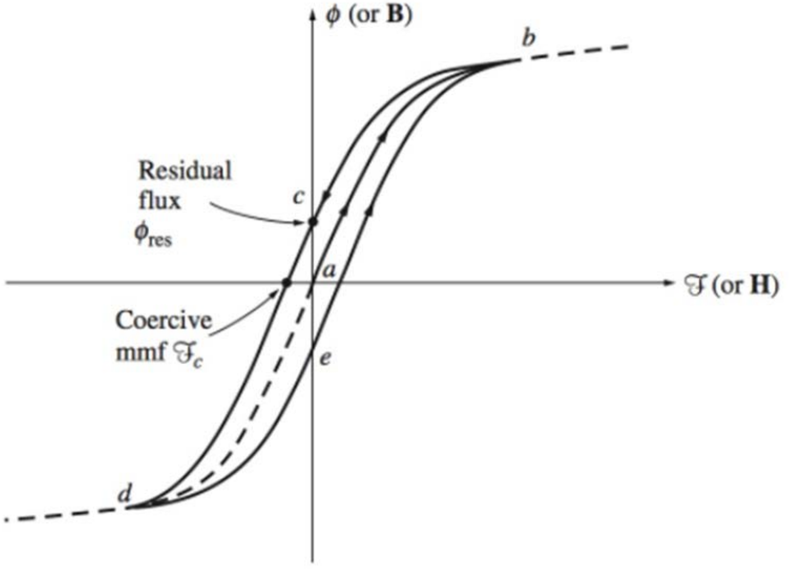
Energy Losses in a Ferromagnetic Core





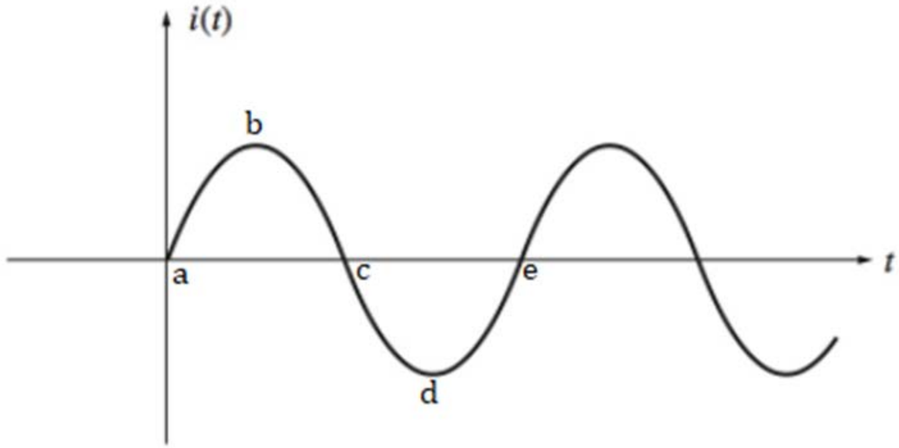


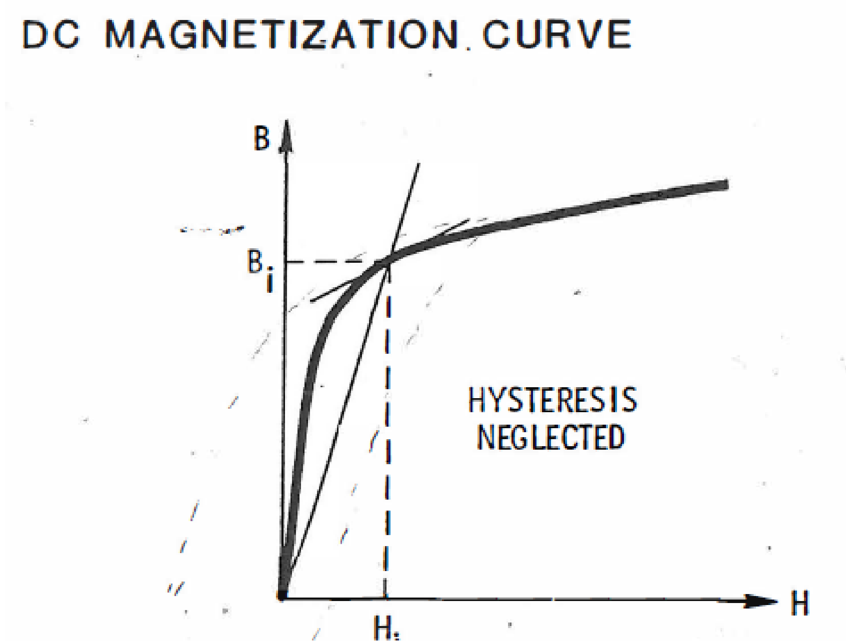
(a) Magnetic domains oriented randomly.
(b) Magnetic domains lined up in the presence of an external magnetic field.

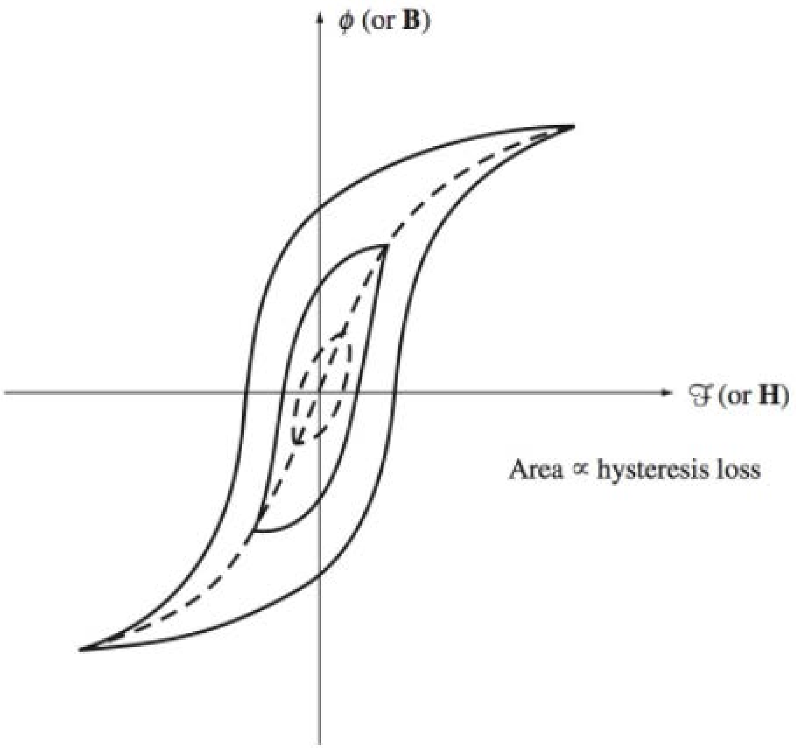


The hysteresis loop traced out by the flux in a core when the current i(t) is applied to it.

Nonlinear Magnetic Material and Core Losses







The effect of the size of magnetomotive force excursions on the magnitude of the hysteresis loss

$$P_h = K_h f B_m^n V$$

where

k_h is an hysteresis constant which depends on the material

f is the frequency

n is the Steinmetz exponent, which was found to vary from 1.5-2.5

B_m is the maximum flux density

V is the volume of the magnetic material

EDDY CURRENT LOSS

- Eddy current loss is caused by currents known as eddy currents which are the result of induced voltages caused by time varying flux (magnetic fields)
- The eddy currents also exert a demagnetization effect on the core
- This effect is higher at the center of the core cross-section perpendicular to the field
- As a result the magnetic field is nonuniform and is more crowded at the surface of the material
- Accordingly the eddy current loss can be reduced by laminating the core

- **Lamination Planes must be parallel to the flux lines and never perpendicular to them**

- **Eddy current loss is dependent on the frequency, flux level, and lamination thickness**

$$P_e \propto \left(f^2 \delta^2 B_m^2 / \rho\right) V$$

$$P_e = k_e f^2 \delta^2 B_m^2 V$$

where

k_e **is an eddy current constant which depends on the material**

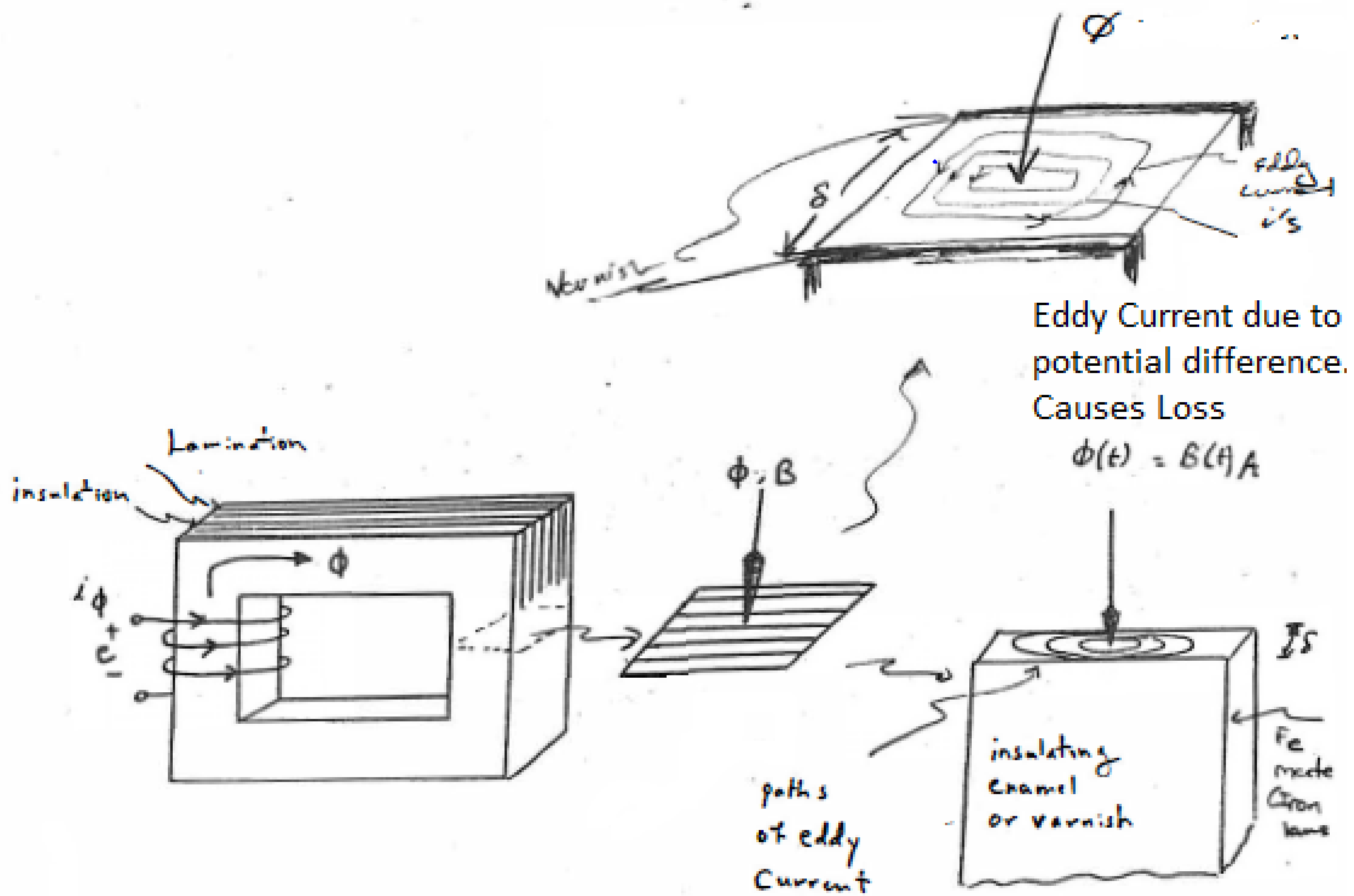
ρ **is the electrical resistivity of material**

f **is the frequency**

δ is the lamination thickness

B_m is the maximum flux density

V is the volume of the magnetic material



MAGNETIC (CORE) LOSSES

The total core losses is the sum of the Eddy Current and Hysteresis Losses

frequency

Volume of Core

$$P_c = P_e + P_h = V(k_e f^2 \delta^2 B_m^2 + K_h f B_m^n)$$

Theory of Operation of Non-Ideal Single-Phase Transformers

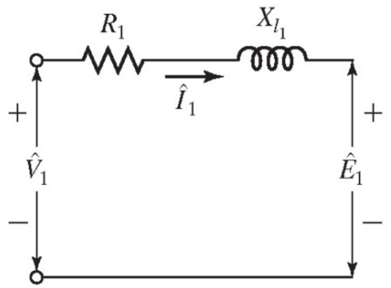
In a real transformer, the following non-ideal facts must be considered:

- Copper Losses in primary and secondary windings: these are modeled as series resistors R_p and R_s for the primary and secondary windings, respectively.
- Small portion of the core flux leaks outside the core and passes through one winding only. This flux will be presented by a leakage inductance. Both primary and secondary coils generate leakage flux which are modeled as series reactances X_p and X_s in the primary and secondary windings, respectively.
- Core Losses which are due to heating losses produced by Eddy Current and Hysteresis Losses in the core. These losses are presented by a shunt resistor R_C in the primary winding.
- Magnetizing Current which flows in the primary current to establish the flux in the core. This is modeled as a shunt reactance X_m in the primary winding.

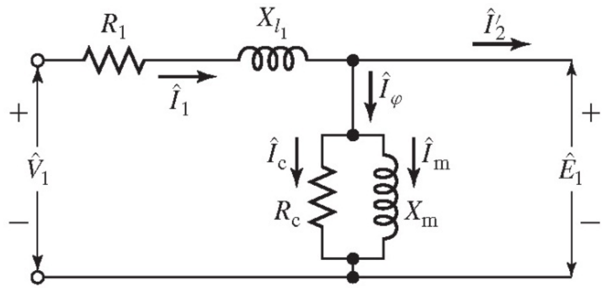
Note: in the literature the following notation is used interchangeably

$N_p=N_1$ and $N_s=N_2$

Steps in development of transformer equivalent circuit

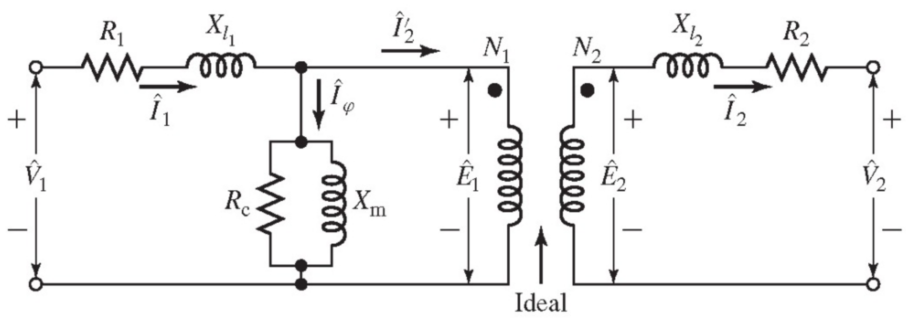


(a)



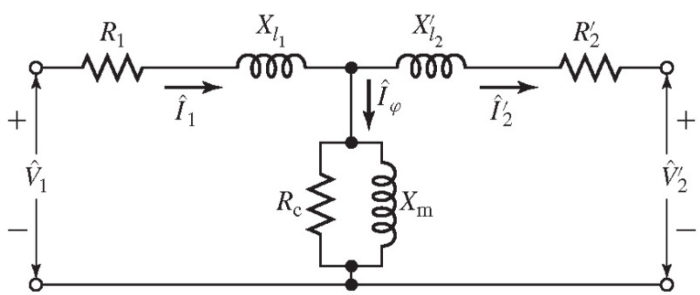
(b)

(a) Primary resistance and leakage reactance.
(b) Adding Magnetizing reactance and core loss resistance.



(c)

(c) Adding ideal transformer and secondary impedance.

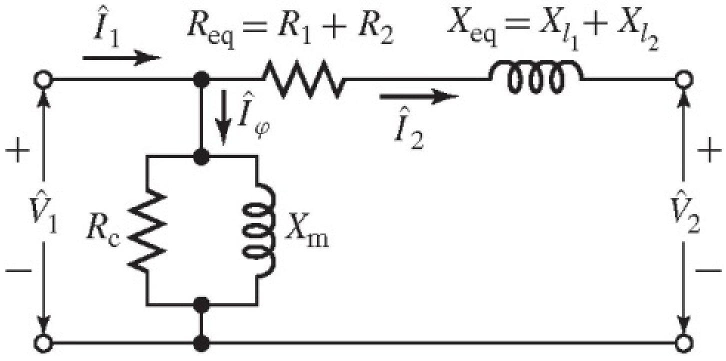


(d)

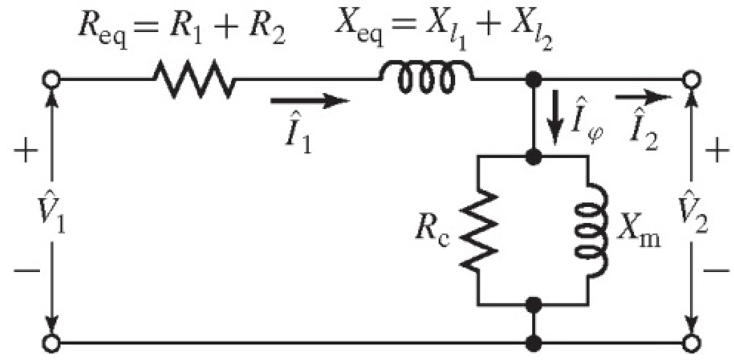
(d) Reflecting secondary impedance to primary of ideal transformer,
The **T-Circuit**.

Approximate equivalent circuits

- Excitation branch impedances are large compared to other branches
  - Approximate equivalent circuits called **cantilever circuits**
  - Use the form that is most convenient



(a)

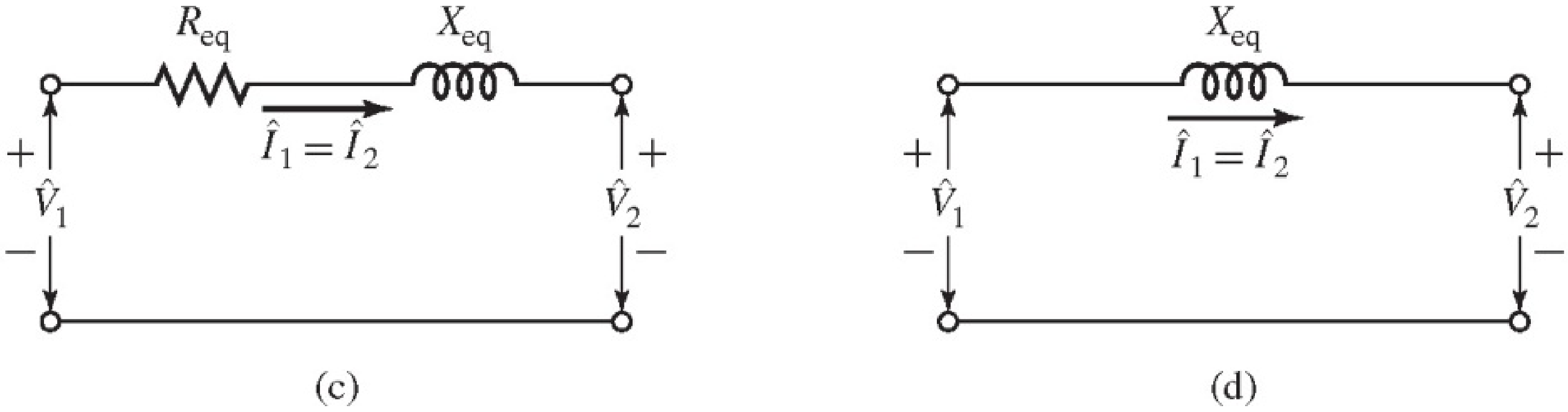


(b)

Approximate equivalent circuits

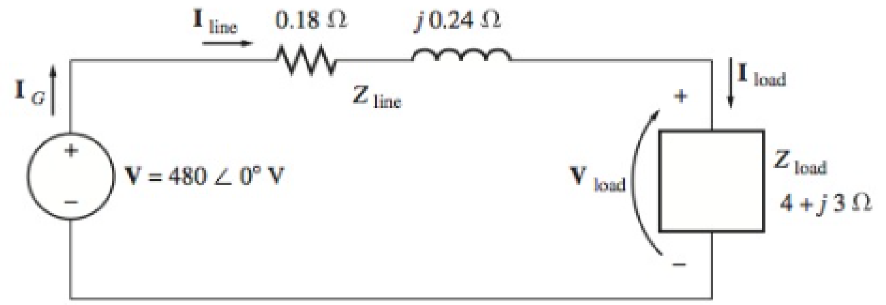
– At normal load, the exciting current may be neglected

– For large power transformers, the winding resistances are small, so it is used as a simplified-circuits



Example 2-1 Chapman): The single-phase power system shown consists of a 480-V 60-Hz generator supplying a load Z_{load}=4+j3 Ω through a transmission line of impedance Z_{line}=0.18 +j0.24Ω.

(a) Find the load voltage and the transmission line losses.



(a)

Solution

(*a*) Figure 2–6a shows the power system without transformers. Here $\mathbf{I}_G = \mathbf{I}_{line} = \mathbf{I}_{load}$. The line current in this system is given by

$$\mathbf{I}_{line} = \frac{\mathbf{V}}{\mathbf{Z}_{line} + \mathbf{Z}_{load}}$$

$$= \frac{480\angle 0^\circ \text{ V}}{(0.18\ \Omega + j0.24\ \Omega) + (4\ \Omega + j3\ \Omega)}$$

$$= \frac{480\angle 0^\circ}{4.18 + j3.24} = \frac{480\angle 0^\circ}{5.29\angle 37.8^\circ}$$

$$= 90.8\angle -37.8^\circ \text{ A}$$

Therefore the load voltage is

$$\mathbf{V}_{load} = \mathbf{I}_{line}\mathbf{Z}_{load}$$

$$= (90.8\angle -37.8^\circ \text{ A})(4\ \Omega + j3\ \Omega)$$

$$= (90.8\angle -37.8^\circ \text{ A})(5\angle 36.9^\circ\ \Omega)$$

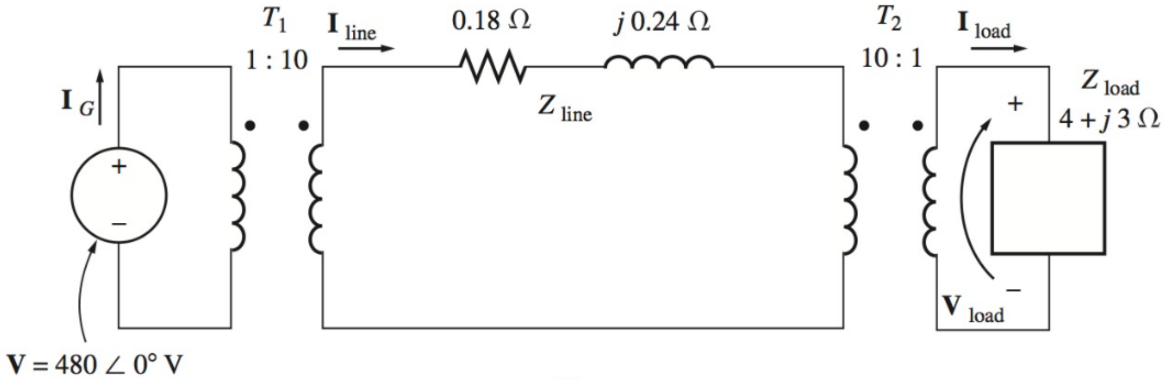
$$= 454\angle -0.9^\circ \text{ V}$$

and the line losses are

$$P_{loss} = (I_{line})^2 R_{line}$$

$$= (90.8 \text{ A})^2 (0.18\ \Omega) = 1484 \text{ W}$$

b) Now the circuit includes two transformers as shown. Find the load voltage and th transmission line losses.



(b)

(*b*) Figure 2–6b shows the power system with the transformers. To analyze this system, it is necessary to convert it to a common voltage level. This is done in two steps:

1. Eliminate transformer T_2 by referring the load over to the transmission line's voltage level.
2. Eliminate transformer T_1 by referring the transmission line's elements and the equivalent load at the transmission line's voltage over to the source side.

The value of the load's impedance when reflected to the transmission system's voltage is

$$\begin{aligned} \mathbf{Z}'_{\text{load}} &= a^2 \mathbf{Z}_{\text{load}} \\ &= \left(\frac{10}{1}\right)^2 (4\ \Omega + j3\ \Omega) \\ &= 400\ \Omega + j300\ \Omega \end{aligned}$$

The total impedance at the transmission line level is now

$$\begin{aligned} Z_{\text{eq}} &= Z_{\text{line}} + Z'_{\text{load}} \\ &= 400.18\ + j300.24\ \Omega = 500.3\angle 36.88^\circ\ \Omega \end{aligned}$$

This equivalent circuit is shown in Figure 2–7a. The total impedance at the transmission line level ($Z_{line} + Z'_{load}$) is now reflected across T_1 to the source's voltage level:

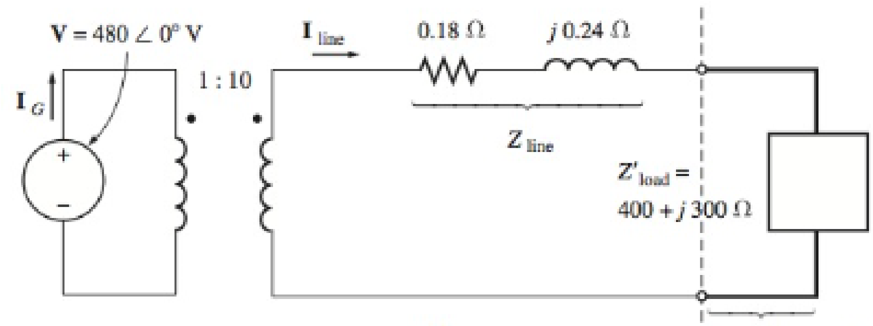
$$\begin{aligned} \mathbf{Z}'_{eq} &= a^2\, \mathbf{Z}_{eq} \\ &= a^2(\mathbf{Z}_{line} + \mathbf{Z}'_{load}) \\ &= \left(\frac{1}{10}\right)^2 (0.18\ \Omega + j0.24\ \Omega + 400\ \Omega + j300\ \Omega) \\ &= (0.0018\ \Omega + j0.0024\ \Omega + 4\ \Omega + j3\ \Omega) \\ &= 5.003\angle 36.88°\ \Omega \end{aligned}$$

Notice that $Z''_{load} = 4 + j3\ \Omega$ and $Z'_{line} = 0.0018 + j0.0024\ \Omega$. The resulting equivalent circuit is shown in Figure 2–7b. The generator's current is

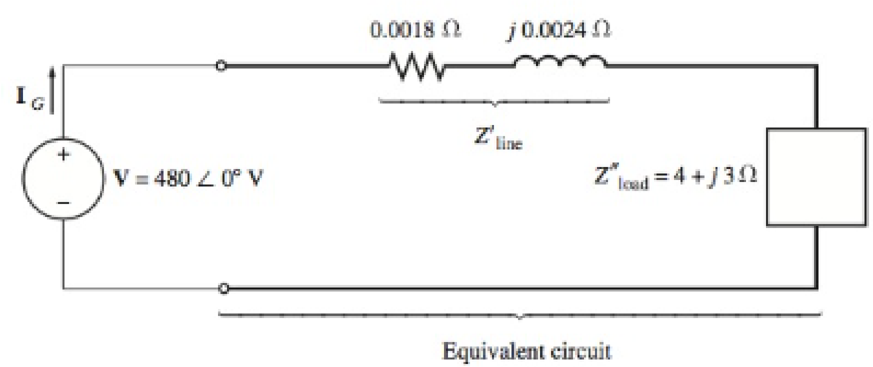
$$\mathbf{I}_G = \frac{480\angle 0°\ \text{V}}{5.003\angle 36.88°\ \Omega} = 95.94\angle -36.88°\ \text{A}$$

Knowing the current $\mathbf{I}_G$, we can now work back and find $\mathbf{I}_{line}$ and $\mathbf{I}_{load}$. Working back through T_1, we get

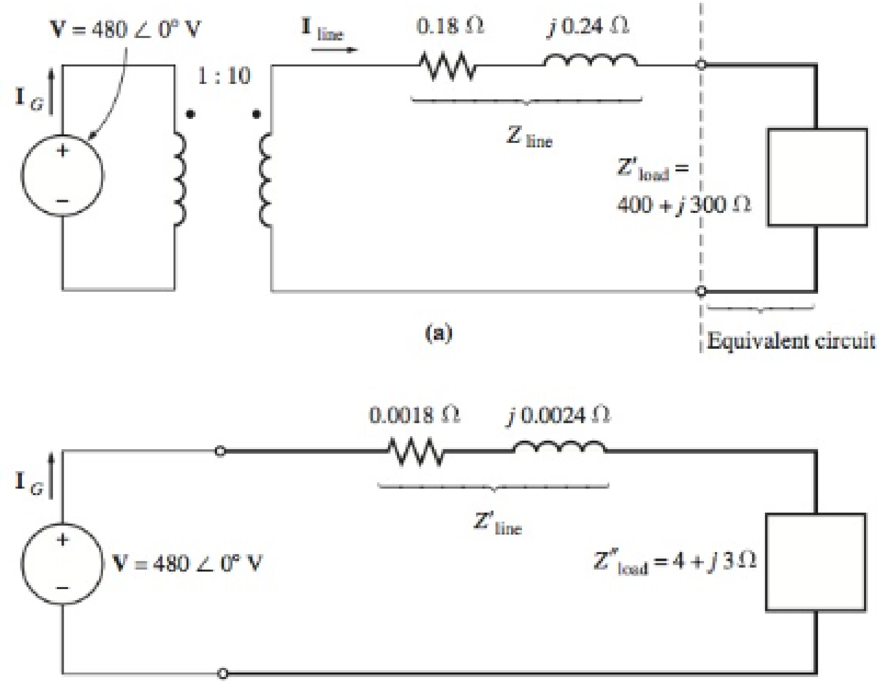
$$\begin{aligned} N_{P1}\mathbf{I}_G &= N_{S1}\mathbf{I}_{line} \\ \mathbf{I}_{line} &= \frac{N_{P1}}{N_{S1}}\mathbf{I}_G \\ &= \frac{1}{10}(95.94\angle -36.88°\ \text{A}) = 9.594\angle -36.88°\ \text{A} \end{aligned}$$



(a)



(b)



(a)

(b)

Working back through T_2 gives

$$N_{P2}\mathbf{I}_{\text{line}} = N_{S2}\mathbf{I}_{\text{load}}$$

$$\mathbf{I}_{\text{load}} = \frac{N_{P2}}{N_{S2}}\mathbf{I}_{\text{line}}$$

$$= \frac{10}{1}(9.594\angle -36.88^\circ \text{ A}) = 95.94\angle -36.88^\circ \text{ A}$$

It is now possible to answer the questions originally asked. The load voltage is given by

$$\mathbf{V}_{\text{load}} = \mathbf{I}_{\text{load}}Z_{\text{load}}$$

$$= (95.94\angle -36.88^\circ \text{ A})(5\angle 36.87^\circ \ \Omega)$$

$$= 479.7\angle -0.01^\circ \text{ V}$$

and the line losses are given by

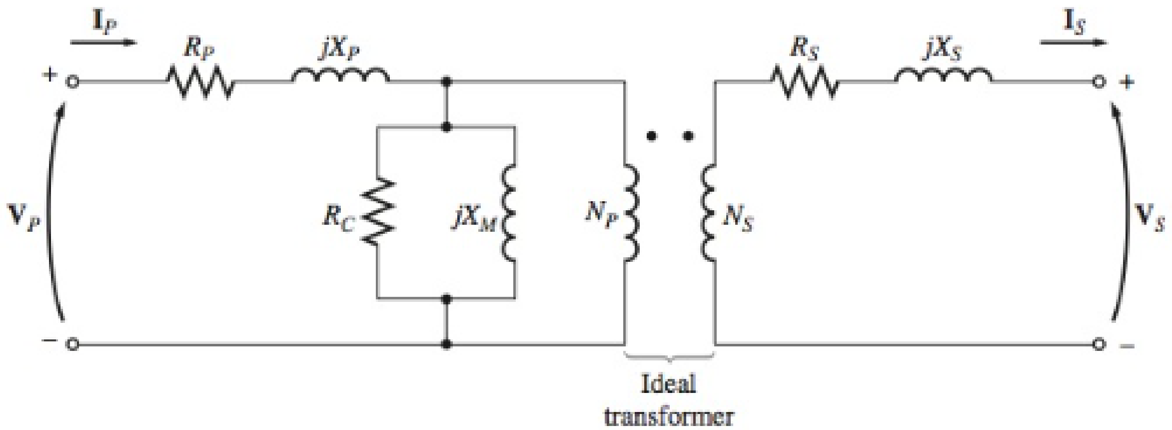
$$P_{\text{loss}} = (I_{\text{line}})^2 R_{\text{line}}$$

$$= (9.594 \text{ A})^2 (0.18 \ \Omega) = 16.7 \text{ W}$$

Compared to 1,484 W

Transformer Voltage Regulation

- Due to the series impedances, the output voltage (secondary) will change as the load current changes.



- Voltage Regulation, VR, compares the output voltage of the transformer at no load with the output voltage at full load while the input voltage is kept constant at a value corresponding to the full load condition.

$$\mathrm{VR} = \frac{V_{S,\mathrm{nl}} - V_{S,\mathrm{fl}}}{V_{S,\mathrm{fl}}} \times 100\%$$

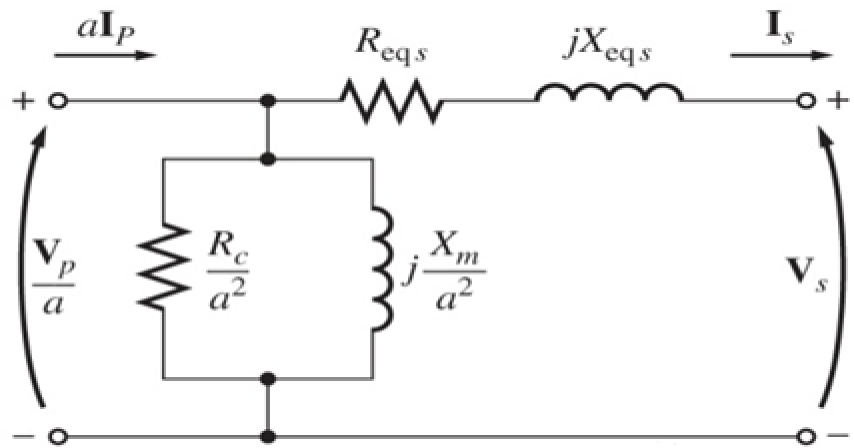
Since at No Load $V_s = V_p/a$

$$\mathrm{VR} = \frac{V_P/a - V_{S,\mathrm{fl}}}{V_{S,\mathrm{fl}}} \times 100\%$$

Transformer Efficiency

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\%$$

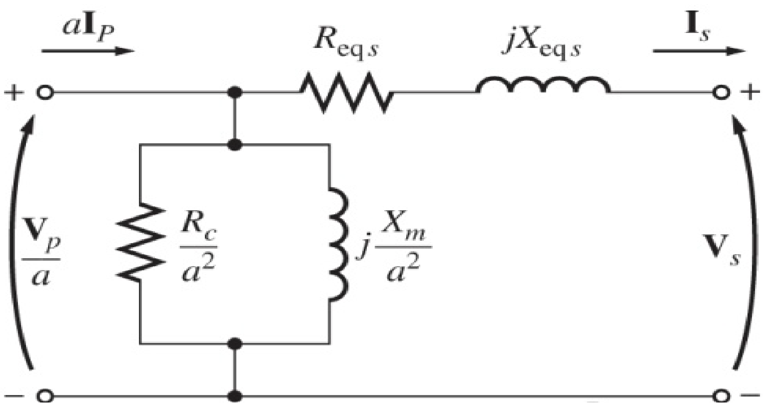
$$\eta = \frac{P_{\text{out}}}{P_{\text{out}} + P_{\text{loss}}} \times 100\%$$



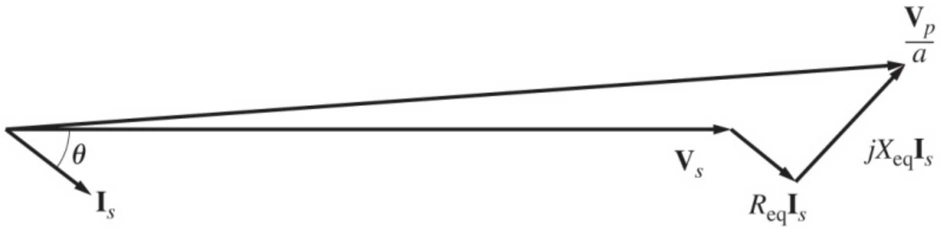
- $P_{out} = P_s = V_s I_s Cos(\theta_s)$
- $P_{in} = P_s + P_{Losses} = V_s I_s Cos(\theta_s) + P_{core} + P_{cu}$

- $$\eta = \frac{P_{out}}{P_{in}} x100\% = \frac{V_s I_s Cos(\theta_s)}{V_s I_s Cos(\theta_s) + P_{core} + P_{cu}} x100\%$$

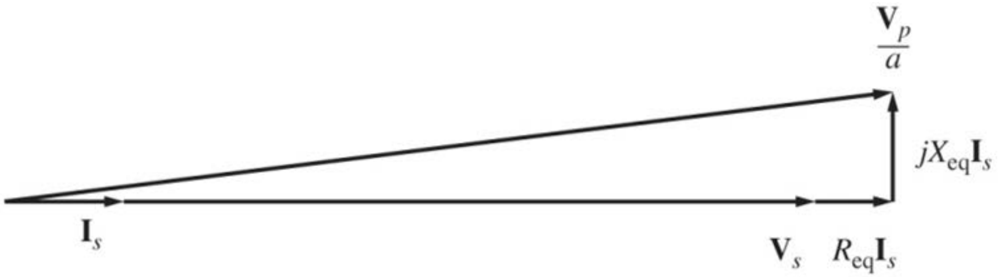
Transformer Phasor Diagram



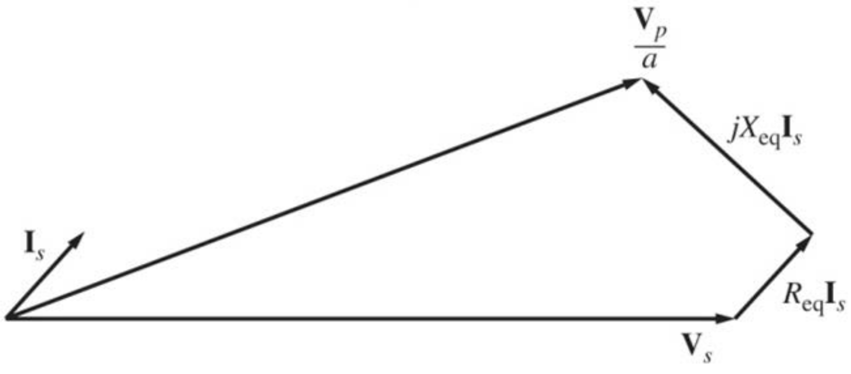
$\mathbf{V}_p/a=\mathbf{V}_s+R_{eq}\mathbf{I}_s+jX_{eq}\mathbf{I}_s$



1-Lagging PF



2-Unity PF



3-Leading PF

Three-phase transformers

- Three single-phase transformers can be connected as a *three-phase transformer bank* in four ways
  - windings at the left are the primaries
  - those at the right are the secondaries
  - primary winding in one transformer corresponds to the secondary winding drawn parallel to it

Three-Phase Transformers

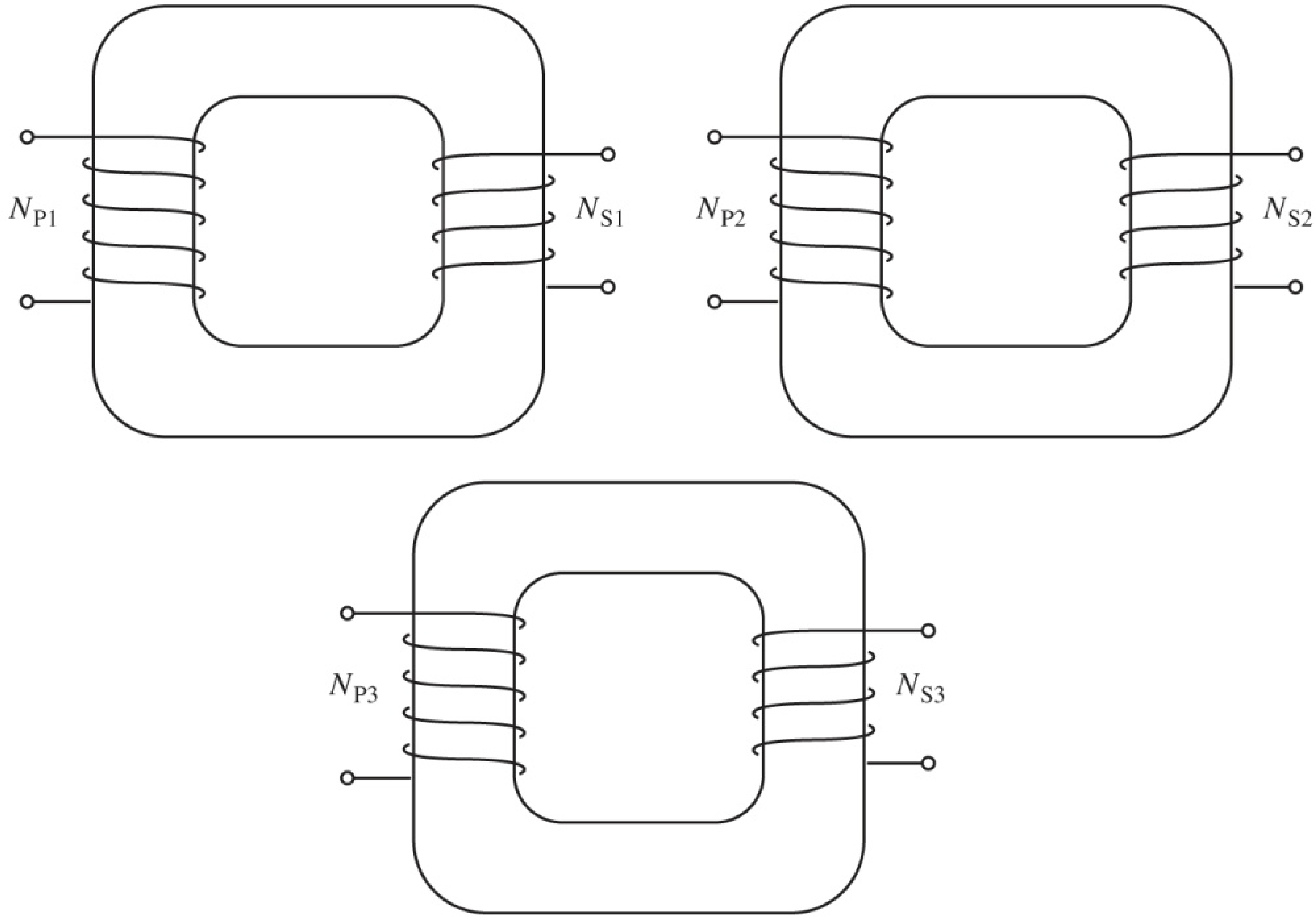


Figure 2-35 A three-phase transformer bank composed of independent transformers

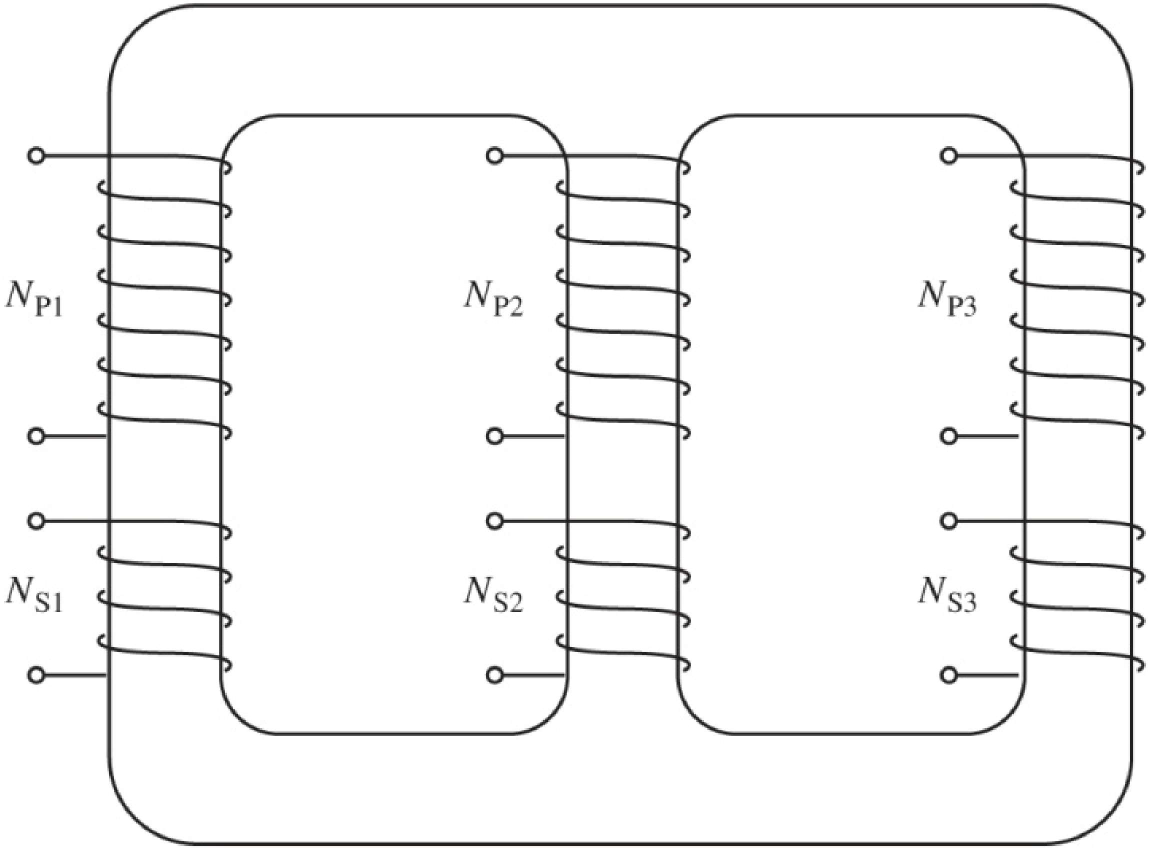


Figure 2-36 A three-phase transformer wound on a single three-legged core.

Transformer Connections

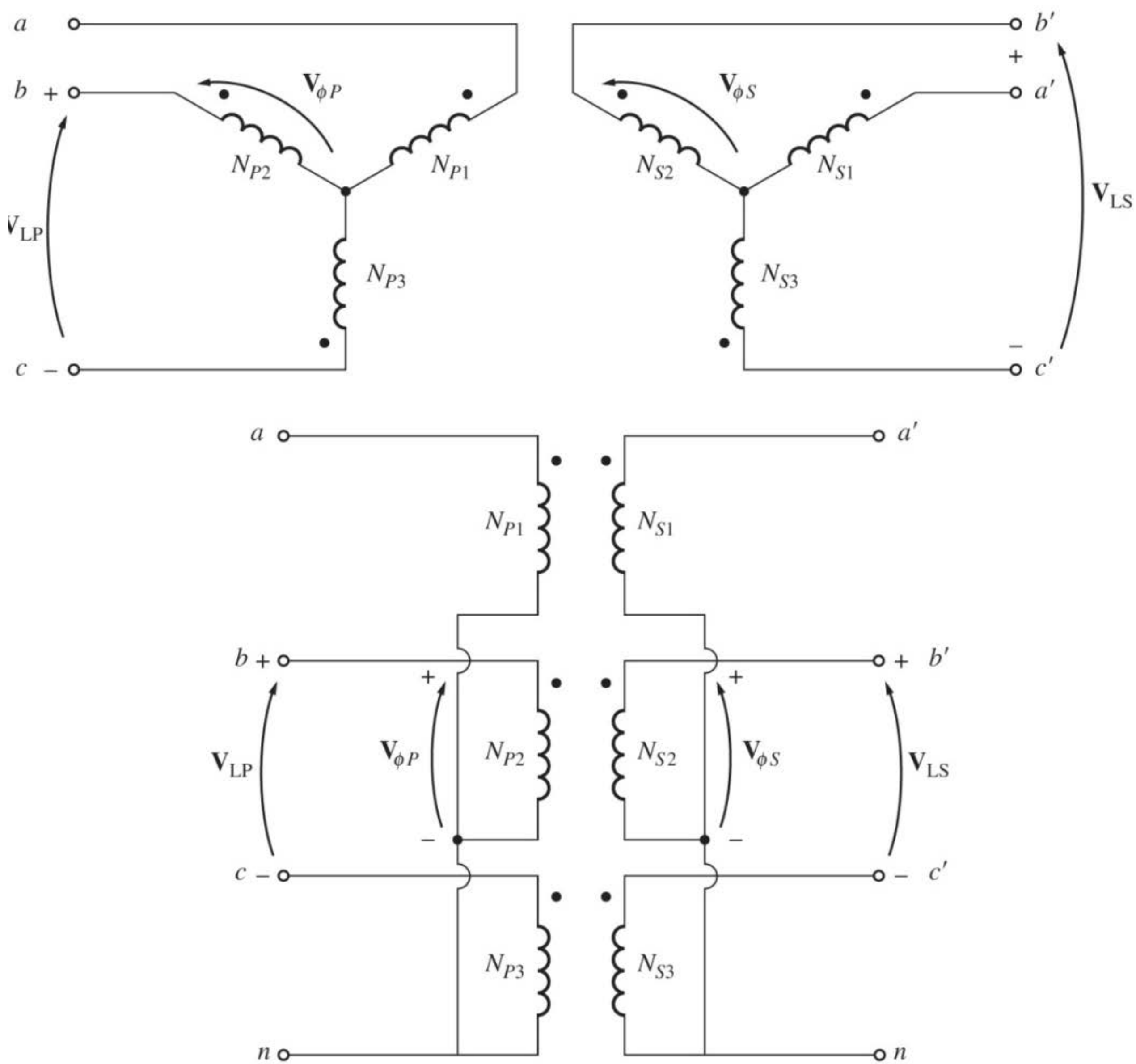


Figure 2-37(a) Three-phase transformer Y-Y connections and wiring diagram.

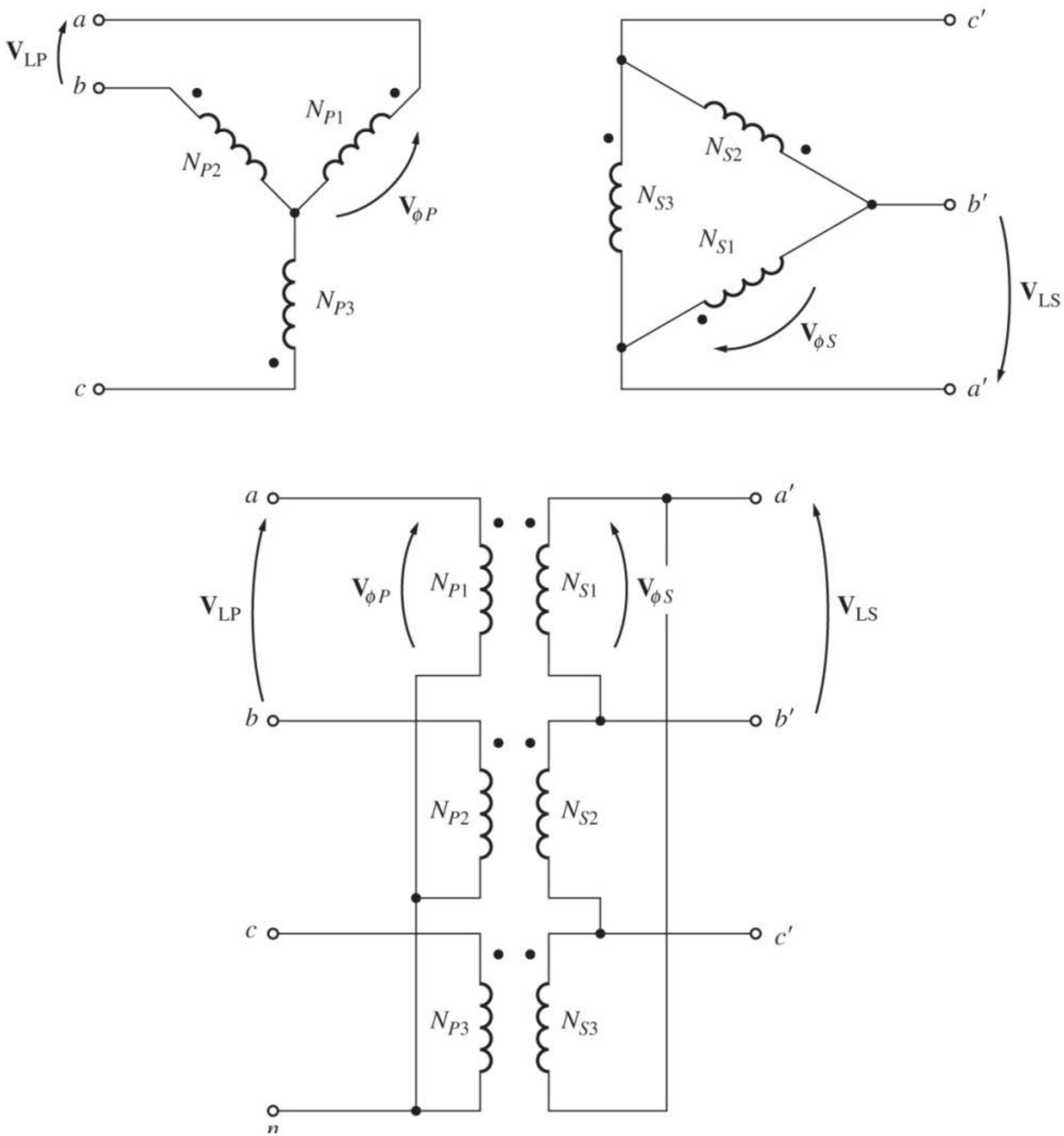


Figure 2-37(b) Three-phase transformer Y-Δ connections and wiring diagram.

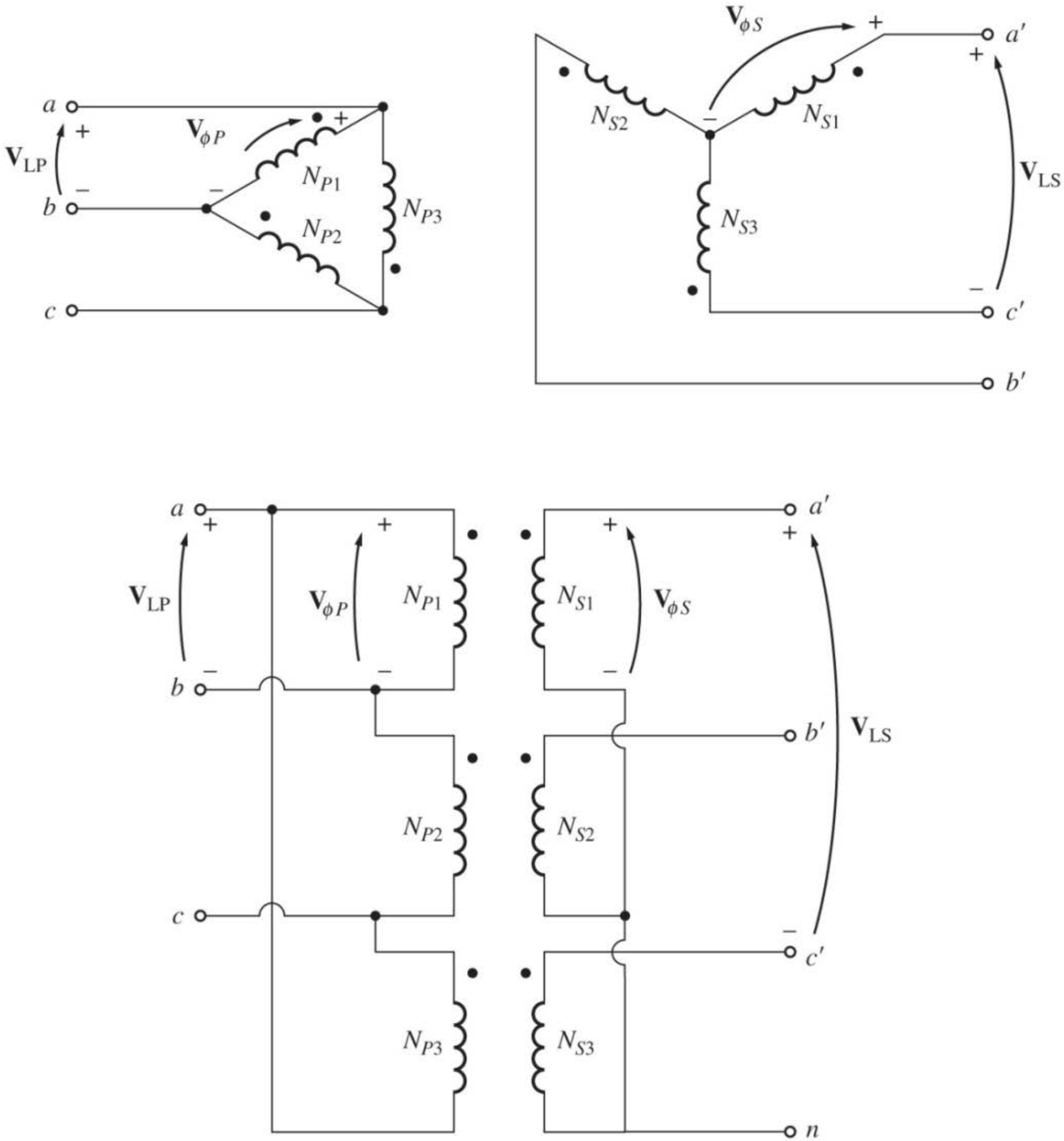


Figure 2-37(c) Three-phase transformer Δ-Y connections and wiring diagram.

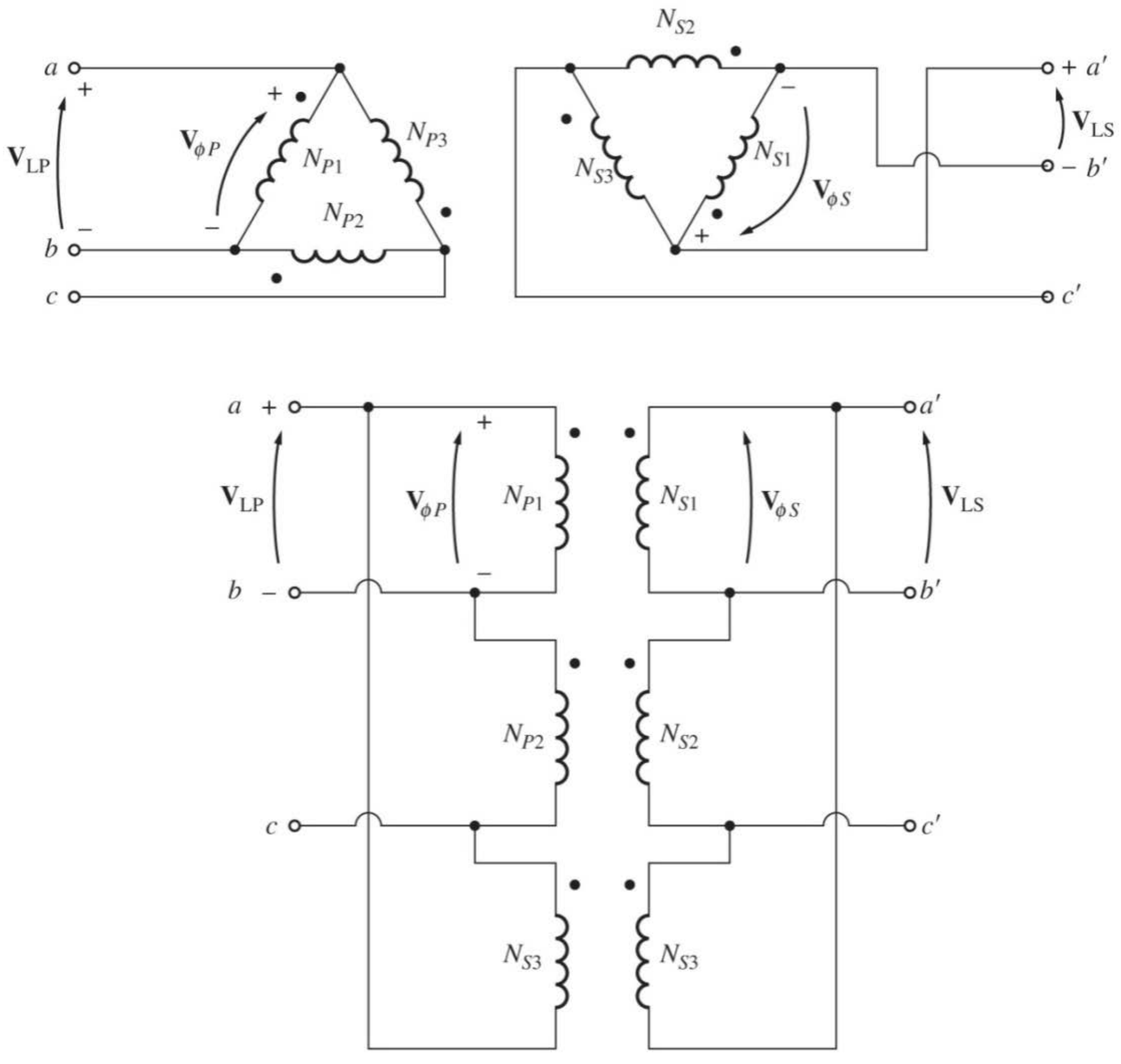
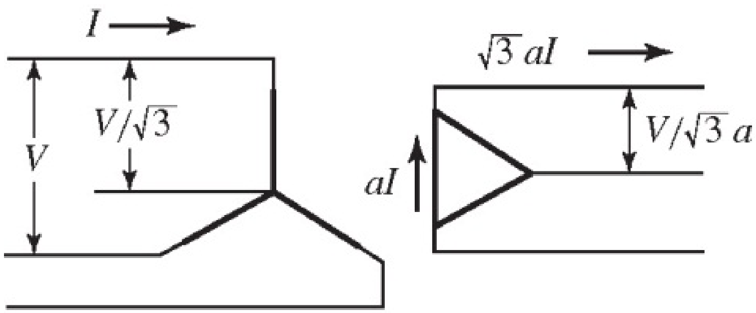
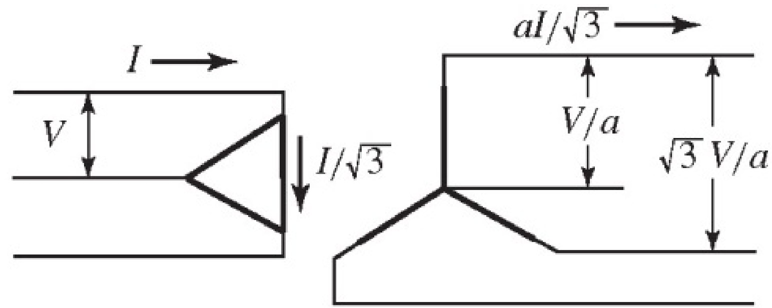


Figure 2-37(d) Three-phase transformer Δ- Δ connections and wiring diagram.

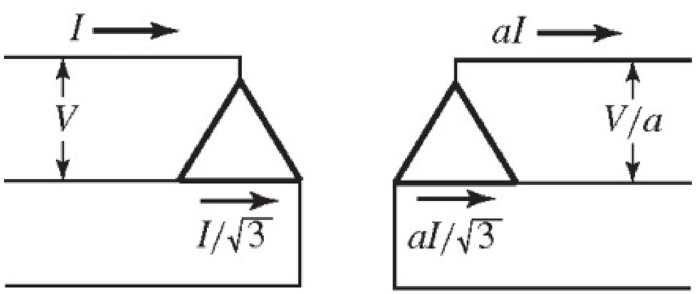
Three-phase transformer connections



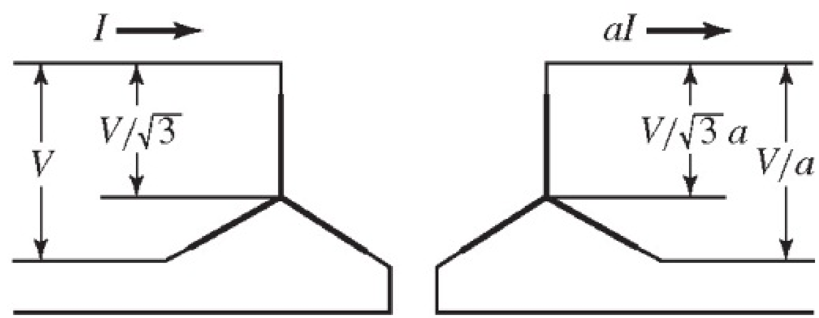
(a) Y-Δ connection



(b) Δ-Y connection



(c) Δ-Δ connection



(d) Y-Y connection

Per Unit System for Three-Phase Transformers

Given $S_{3\phi,b}$, and $V_{\phi,b}$

$$I_{\phi,b} = \frac{S_{3\phi,b}}{3V_{\phi,b}}$$

$$Z_b,\ R_b,\ \text{or}\ X_b = \frac{V_{\phi,b}}{I_{\phi,b}} == \frac{3\left(V_{\phi,b}\right)^2}{S_{3\phi,b}}$$

Note that,

$$I_{L,b} = \frac{S_{3\phi,b}}{\sqrt{3}V_{L,b}}$$