COLORADO SCHOOL OF MINES
ELECTRICAL ENGINEERING DEPARTMENT

EENG 577

ADVANCED ELECTRICAL MACHINE DYNAMICS FOR SMART-GRID SYSTEMS

# M2-P3 Transformer Equivalent Circuit Parameters

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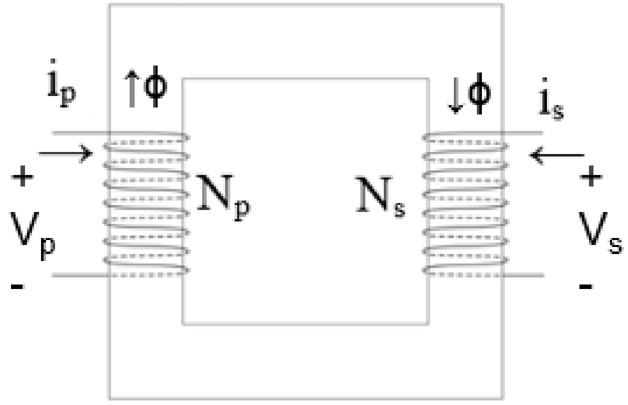
# Transformer-Equivalent Circuit

**Fig. 1**




$$v_j = r_j i_j + \frac{d}{dt}\{\lambda_j\} \qquad \textbf{(1)}$$

$$\lambda_j = \sum_{k=1}^{n} L_{jk}\, i_k \qquad \textbf{(2)}$$

$$v_j = r_j i_j + L_{j1}\frac{di_1}{dt} + \ldots + L_{jn}\frac{di_n}{dt} \qquad \textbf{(3)}$$

**$L_{pp} = N_p^2/\boldsymbol{R}$,**

**$L_{ss} = N_s^2/\boldsymbol{R}$, and**

**$L_{ps} = L_{sp} = N_pN_s/\boldsymbol{R} = \boldsymbol{M}$** **(4)**

$$v_p = r_p \cdot i_p + \frac{d\lambda_p}{dt} \quad (5)$$

$$v_s = r_s \cdot i_s + \frac{d\lambda_s}{dt} \quad (6)$$

$$\lambda_p = L_{pp} \cdot i_p + L_{ps} \cdot i_s \quad (7)$$

$$\lambda_s = L_{ss} \cdot i_s + L_{sp} \cdot i_p \quad (8)$$

Based on the above, and assuming a linear unsaturated magnetic core, and assuming $\mathbf{L_{ps}} = \mathbf{L_{sp}} = \boldsymbol{M}$, the State Space, SS, model equations for the electromagnetic system considered can be represented as follows:

$$v_p = r_p\, i_p + L_{pp} \frac{d}{dt} i_p + M \frac{d}{dt} i_s \quad (9)$$

$$v_s = r_s\, i_s + M \frac{d}{dt} i_p + L_{ss} \frac{d}{dt} i_s \quad (10)$$

Using compact form, the State Space, SS, model equations can be represented as follows:

$$\begin{bmatrix} v_p \\ v_s \end{bmatrix} = \begin{bmatrix} r_p & 0 \\ 0 & r_s \end{bmatrix} \begin{bmatrix} i_p \\ i_s \end{bmatrix} + \begin{bmatrix} L_{pp} & M \\ M & L_{ss} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_p \\ i_s \end{bmatrix} \qquad \textbf{(11)}$$

In addition, it could be represented by the following equivalent circuit:

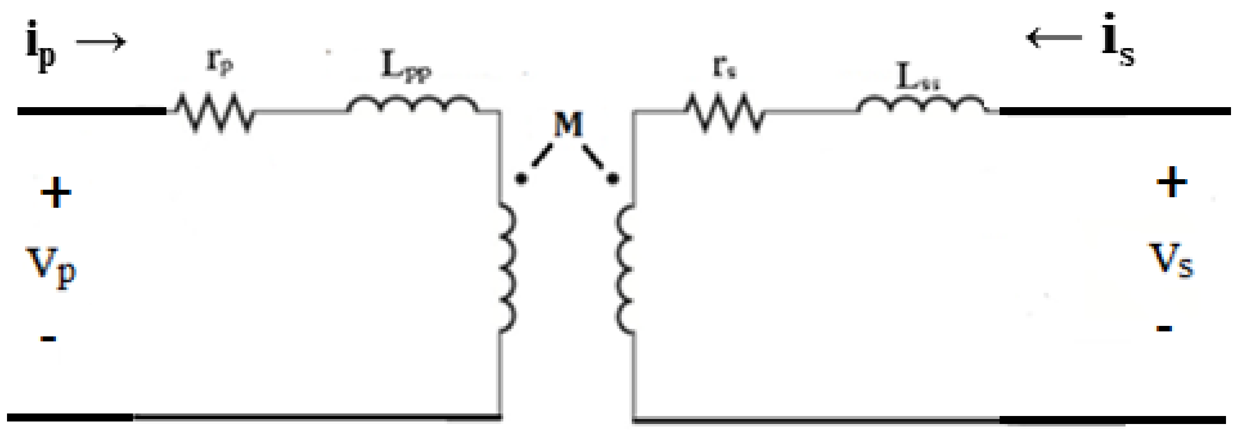
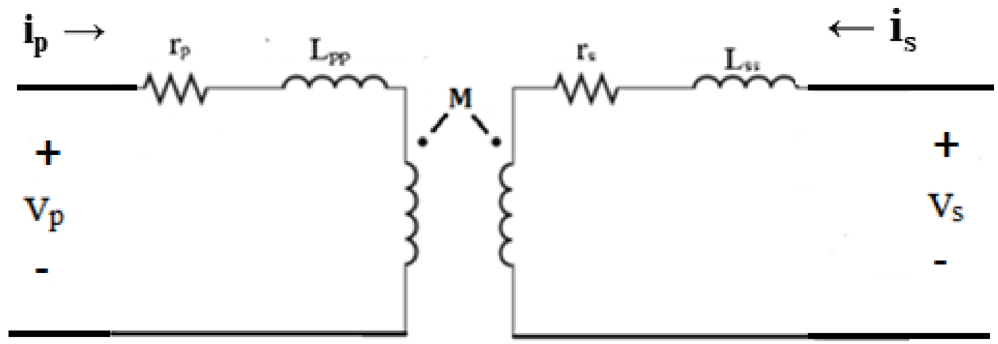



**Fig. 2** Transformer State Space Model Equivalent Circuit

# Relating SS Model & Transformer Cantilever Circuit Parameters

(a)




(b)

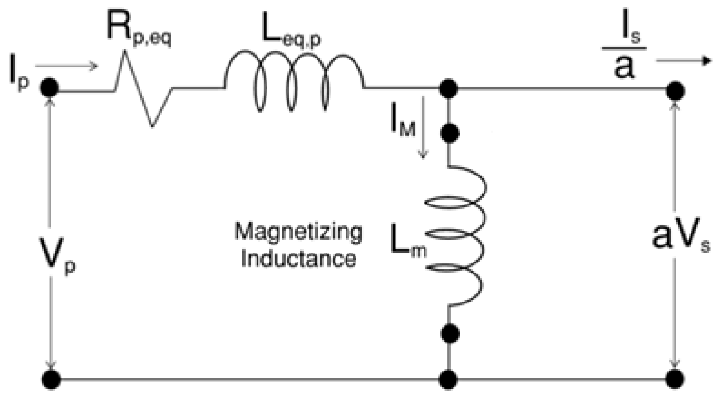



**Fig. 3**: Transformer Equivalent Circuits:

(a) State Space Model Equivalent Circuit

(b) Simplified Cantilever Equivalent Circuit Referred to Primary Side

Using expression (9) and adding the terms in blue, results in the following :

$$v_p = r_p\, i_p + L_{pp}\frac{d}{dt}\, i_p - \frac{N_p}{N_s}\, M\frac{d}{dt}\, i_p + \frac{N_p}{N_s}\, M\frac{d}{dt}\, i_p + M\frac{d}{dt}\, i_s \qquad (12)$$

Also, collecting similar terms would result in the following:

$$\therefore v_p = r_p\, i_p + \left(L_{pp} - \frac{N_p}{N_s}\, M\right)\frac{d}{dt}\, i_p + \frac{N_p}{N_s}\, M\frac{d}{dt}\left(i_p + \frac{N_s}{N_p}\, i_s\right) \qquad (13)$$

Equation (15) can be represented by the circuit of Fig. 4 (a),and can be compared to Fig. 4 (b) of a simplified cantilever equivalent circuit. The two figures relate the self and mutual inductances to the leakage inductance of the cantilever circuit, and thus resulting in the following inductance relationships:

$$L_{eq,p} = L_{lp} + a^2 L_{ls} = L_{pp} - \frac{N_p}{N_s} M \quad \text{and} \quad L_m = \frac{N_p}{N_s} M \qquad \textbf{(14)}$$

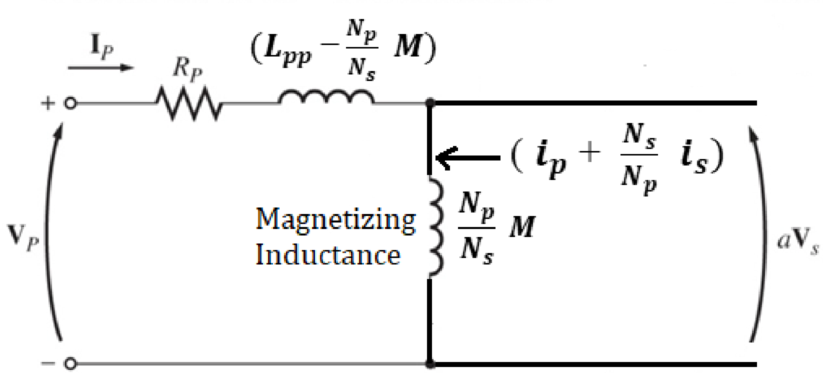
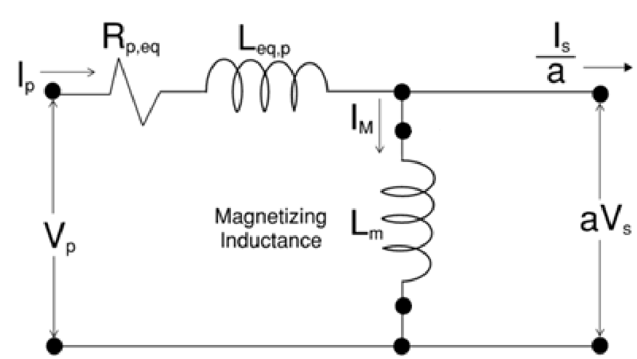



Fig. 4 (a) Transformer SS Model Circuit




(b) Simplified Cantilever Equivalent Circuit