COLORADO SCHOOL OF MINES
ELECTRICAL ENGINEERING DEPARTMENT

EENG577 Advanced Electric Machine Dynamics for Smart-Grid Systems

M3 Project: Synchronous Generator-abc

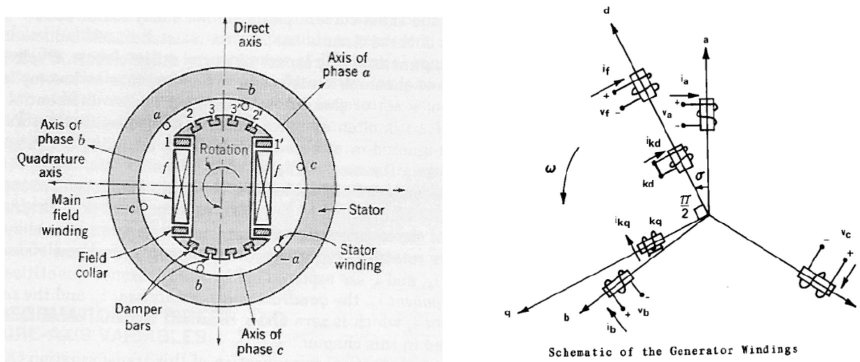
Introduction

A 659 MVA, 4-Pole, 20 kV, 0.9 P.F., Double-Y Connected (with grounded neutral), 60 Hz turbogenerator, whose armature and field winding parameters (neglecting saturation) are summarized as follows:

r_s	=	$7.4 \times 10^{-4}\ \Omega$	r_f	=	$0.0860\ \Omega$
r_{kd1}	=	$1.58 \times 10^{-4}\ \Omega$	L_{sv}	=	0.05×10^{-3} H
r_{kq1}	=	$1.227 \times 10^{-4}\ \Omega$	L_{mv}	=	0.05×10^{-3} H
L_{sa}	=	1.95×10^{-3} H	L_{ff}	=	444×10^{-3} H
L_{ma}	=	0.80×10^{-3} H	L_{akdm1}	=	0.447×10^{-3} H
L_{afm}	=	26.1×10^{-3} H	L_{akqm1}	=	0.670×10^{-3} H
L_{fkd1}	=	5.05×10^{-3} H	L_{kd1kd1}	=	0.1254×10^{-3} H
L_{kd1f}	=	6.29×10^{-3} H	L_{kq1kq1}	=	0.3762×10^{-3} H

PART I

1. Show a cross-section of the synchronous machine and draw corresponding winding schematic diagram and label all axes (a, b, c, f, d, and q) properly.



(a) (b)

Fig-1: (a) A typical synchronous machine cross-section and (b) schematic of synchronous machine windings.

2. Write down the State Space (SS) model in expanded matrix form, use the symbols in the lecture notes (no values) for the following:
   i. Case-1: Neglect the effects of the damping circuits in your SS model. Include only windings a, b, c, and f.

$$V = RI + \frac{d}{dt}(LI)$$

$$V = \begin{bmatrix} v_{aa} \\ v_{bb} \\ v_{cc} \\ v_f \end{bmatrix}, I = \begin{bmatrix} i_{aa} \\ i_{bb} \\ i_{cc} \\ i_f \end{bmatrix}, I = \frac{d}{dt}\begin{bmatrix} i_{aa} \\ i_{bb} \\ i_{cc} \\ i_f \end{bmatrix} \quad R = \begin{bmatrix} r_s & 0 & 0 & 0 \\ 0 & r_s & 0 & 0 \\ 0 & 0 & r_s & 0 \\ 0 & 0 & 0 & r_f \end{bmatrix}, and\ L = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} & L_{af} \\ L_{ba} & L_{bb} & L_{bc} & L_{bf} \\ L_{ca} & L_{cb} & L_{cc} & L_{cf} \\ L_{fa} & L_{fb} & L_{fc} & L_{ff} \end{bmatrix}$$

$$\begin{bmatrix} v_{aa} \\ v_{bb} \\ v_{cc} \\ v_f \end{bmatrix} = \begin{bmatrix} r_s & 0 & 0 & 0 \\ 0 & r_s & 0 & 0 \\ 0 & 0 & r_s & 0 \\ 0 & 0 & 0 & r_f \end{bmatrix}\begin{bmatrix} i_{aa} \\ i_{bb} \\ i_{cc} \\ i_f \end{bmatrix} + \frac{d}{dt}\left(\begin{bmatrix} i_{aa} \\ i_{bb} \\ i_{cc} \\ i_f \end{bmatrix}\begin{bmatrix} L_{aa} & L_{ab} & L_{ac} & L_{af} \\ L_{ba} & L_{bb} & L_{bc} & L_{bf} \\ L_{ca} & L_{cb} & L_{cc} & L_{cf} \\ L_{fa} & L_{fb} & L_{fc} & L_{ff} \end{bmatrix}\right)$$

   ii. Case-2: Account for the effects of the damping circuits in your SS model. Include all windings a, b, c, f, kd1, and kq1.

$$\begin{bmatrix} v_{aa} \\ v_{bb} \\ v_{cc} \\ v_f \\ v_{kd} \\ v_{kq} \end{bmatrix} = \begin{bmatrix} r_s & 0 & 0 & 0 & 0 & 0 \\ 0 & r_s & 0 & 0 & 0 & 0 \\ 0 & 0 & r_s & 0 & 0 & 0 \\ 0 & 0 & 0 & r_f & 0 & 0 \\ 0 & 0 & 0 & 0 & r_{kd} & 0 \\ 0 & 0 & 0 & 0 & 0 & r_{kq} \end{bmatrix}\begin{bmatrix} i_{aa} \\ i_{bb} \\ i_{cc} \\ i_f \\ i_{kd} \\ i_{kq} \end{bmatrix} + \frac{d}{dt}\left(\begin{bmatrix} i_{aa} \\ i_{bb} \\ i_{cc} \\ i_f \\ i_{kd} \\ i_{kq} \end{bmatrix}\begin{bmatrix} L_{aa} & L_{ab} & L_{ac} & L_{af} & L_{akd} & L_{akq} \\ L_{ba} & L_{bb} & L_{bc} & L_{bf} & L_{bkd} & L_{bkq} \\ L_{ca} & L_{cb} & L_{cc} & L_{cf} & L_{ckd} & L_{ckq} \\ L_{fa} & L_{fb} & L_{fc} & L_{ff} & L_{fkd} & L_{fkq} \\ L_{kda} & L_{kdb} & L_{kdc} & L_{kdf} & L_{kdkd} & L_{kdkq} \\ L_{kqa} & L_{kqb} & L_{kqc} & L_{ff} & L_{kqkq} & L_{kqkq} \end{bmatrix}\right)$$

3. Write the state space model in the form using compact matrix notation, given in the course handouts. However, write the expanded form using expressions and numerical values for matrices A and B for:

$$V = RI + \frac{d}{dt}(LI), \qquad V = RI + \ \omega\frac{dL}{d\theta}I + L\frac{dI}{dt}$$

$$\frac{dI}{dt} = \left(-L^{-1}R - L^{-1}\omega\frac{dL}{d\theta}\right)I + L^{-1}V$$

$$A = \left(-L^{-1}R - L^{-1}\omega\frac{dL}{d\theta}\right), \qquad B = L^{-1}$$

i. Case-1: Neglect the effects of the damping circuits in your SS model. Include only windings a, b, c, and f.

$$\frac{d}{dt}\begin{bmatrix} i_{aa} \\ i_{bb} \\ i_{cc} \\ i_f \end{bmatrix} = \left(-\begin{bmatrix} L_{aa}+L_l & L_{ab} & L_{ac} & L_{af} \\ L_{ba} & L_{bb}+L_l & L_{bc} & L_{bf} \\ L_{ca} & L_{cb} & L_{cc+L_l} & L_{cf} \\ L_{fa} & L_{fb} & L_{fc} & L_{ff} \end{bmatrix}^{-1} \begin{bmatrix} r_s+r_l & 0 & 0 & 0 \\ 0 & r_s+r_l & 0 & 0 \\ 0 & 0 & r_s+r_l & 0 \\ 0 & 0 & 0 & r_f \end{bmatrix} \right.$$

$$\left. - \begin{bmatrix} L_{aa}+L_l & L_{ab} & L_{ac} & L_{af} \\ L_{ba} & L_{bb+L_l} & L_{bc} & L_{bf} \\ L_{ca} & L_{cb} & L_{cc+L_l} & L_{cf} \\ L_{fa} & L_{fb} & L_{fc} & L_{ff} \end{bmatrix}^{-1} \omega\frac{d}{d\theta} \begin{bmatrix} L_{aa}+L_l & L_{ab} & L_{ac} & L_{af} \\ L_{ba} & L_{bb+L_l} & L_{bc} & L_{bf} \\ L_{ca} & L_{cb} & L_{cc+L_l} & L_{cf} \\ L_{fa} & L_{fb} & L_{fc} & L_{ff} \end{bmatrix} \right) \begin{bmatrix} i_{aa} \\ i_{bb} \\ i_{cc} \\ i_f \end{bmatrix}$$

$$+ \begin{bmatrix} L_{aa}+L_l & L_{ab} & L_{ac} & L_{af} \\ L_{ba} & L_{bb}+L_l & L_{bc} & L_{bf} \\ L_{ca} & L_{cb} & L_{cc}+L_l & L_{cf} \\ L_{fa} & L_{fb} & L_{fc} & L_{ff} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ v_f \end{bmatrix}$$

Expressions are given in the notes and coefficient values are given in the assignment.

ii. Case-2: Account for the effects of the damping circuits in your SS model. Include all windings a, b, c, f, kd1, and kq1.

$$
\frac{d}{dt}\begin{bmatrix} i_{aa} \\ i_{bb} \\ i_{cc} \\ i_f \\ i_{kd} \\ i_{kq} \end{bmatrix}
= \left(\begin{bmatrix}
L_{aa}+L_l & L_{ab} & L_{ac} & L_{af} & L_{akd} & L_{akq} \\
L_{ba} & L_{bb}+L_l & L_{bc} & L_{bf} & L_{bkd} & L_{bkq} \\
L_{ca} & L_{cb} & L_{cc}+L_l & L_{cf} & L_{ckd} & L_{ckq} \\
L_{fa} & L_{fb} & L_{fc} & L_{ff} & L_{fkd} & L_{fkq} \\
L_{kda} & L_{kdb} & L_{kdc} & L_{kdf} & L_{kdkd} & L_{kdkq} \\
L_{kqa} & L_{kqb} & L_{kqc} & L_{ff} & L_{kqkq} & L_{kqkq}
\end{bmatrix}^{-1}
\begin{bmatrix}
r_s+r_l & 0 & 0 & 0 & 0 & 0 \\
0 & r_s+r_l & 0 & 0 & 0 & 0 \\
0 & 0 & r_s+r_l & 0 & 0 & 0 \\
0 & 0 & 0 & r_f & 0 & 0 \\
0 & 0 & 0 & 0 & r_{kd} & 0 \\
0 & 0 & 0 & 0 & 0 & r_{kq}
\end{bmatrix}
\right.
$$

$$
\left. - \begin{bmatrix}
L_{aa+L_l} & L_{ab} & L_{ac} & L_{af} & L_{akd} & L_{akq} \\
L_{ba} & L_{bb+L_l} & L_{bc} & L_{bf} & L_{bkd} & L_{bkq} \\
L_{ca} & L_{cb} & L_{cc}+L_l & L_{cf} & L_{ckd} & L_{ckq} \\
L_{fa} & L_{fb} & L_{fc} & L_{ff} & L_{fkd} & L_{fkq} \\
L_{kda} & L_{kdb} & L_{kdc} & L_{kdf} & L_{kdkd} & L_{kdkq} \\
L_{kqa} & L_{kqb} & L_{kqc} & L_{ff} & L_{kqkq} & L_{kqkq}
\end{bmatrix}^{-1}
\omega \frac{d}{d\theta}
\begin{bmatrix}
L_{aa}+L_l & L_{ab} & L_{ac} & L_{af} & L_{akd} & L_{akq} \\
L_{ba} & L_{bb+L_l} & L_{bc} & L_{bf} & L_{bkd} & L_{bkq} \\
L_{ca} & L_{cb} & L_{cc+L_l} & L_{cf} & L_{ckd} & L_{ckq} \\
L_{fa} & L_{fb} & L_{fc} & L_{ff} & L_{fkd} & L_{fkq} \\
L_{kda} & L_{kdb} & L_{kdc} & L_{kdf} & L_{kdkd} & L_{kdkq} \\
L_{kqa} & L_{kqb} & L_{kqc} & L_{ff} & L_{kqkq} & L_{kqkq}
\end{bmatrix}
\right)
\begin{bmatrix} i_{aa} \\ i_{bb} \\ i_{cc} \\ i_f \\ i_{kd} \\ i_{kq} \end{bmatrix}
$$

$$
+ \begin{bmatrix}
L_{aa} & L_{ab} & L_{ac} & L_{af} \\
L_{ba} & L_{bb} & L_{bc} & L_{bf} \\
L_{ca} & L_{cb} & L_{cc} & L_{cf} \\
L_{fa} & L_{fb} & L_{fc} & L_{ff}
\end{bmatrix}^{-1}
\begin{bmatrix} 0 \\ 0 \\ 0 \\ v_f \\ 0 \\ 0 \end{bmatrix}
$$

PART II

Assuming that the machine is operating at its rated voltage, current, and power factor (overexcited) as a generator, calculate the necessary excitation field current assuming $v_f = r_f i_f = 338$ V. Also, assume the angle σ, between phase a-axis and the d-axis is equal to zero (meaning in Figs 1 & 2, the a-axis is aligned with the d-axis). In addition, consider the terminal voltage (to neutral) of phase a, Va, to be the reference in the phasor diagram (meaning having a phase angle equal to zero).

At time t=0.0 the machine is connected to an infinite busbar where, a, b, and c phase voltages are given by:

$$v_a = \frac{20 \times 10^3}{\sqrt{3}} \sqrt{2} \cos(\omega t) \text{ Volts}$$

$$v_b = \frac{20 \times 10^3}{\sqrt{3}} \sqrt{2} \cos(\omega t - 2\pi/3) \text{ Volts}$$

$$v_c = \frac{20 \times 10^3}{\sqrt{3}} \sqrt{2} \cos(\omega t - 4\pi/3) \text{ Volts}$$

Assuming that the terminal voltage of the field winding is kept constant by the excitation control system at assuming $v_f = r_f i_f = 338$ V, and that the speed of the turbogenerator is maintained, by its prime mover, constant at its rated synchronous value:

1. Calculate the initial conditions (values at t=0 s) for ia, ib, ic, if, ikd1, ikq1

   Since t=0, $\omega * t = 0$

$$I_a = \frac{S}{3 * |V_a|} \lfloor \cos^{-1}(pf) = 19023.69 \lfloor 25.84^o \ A$$

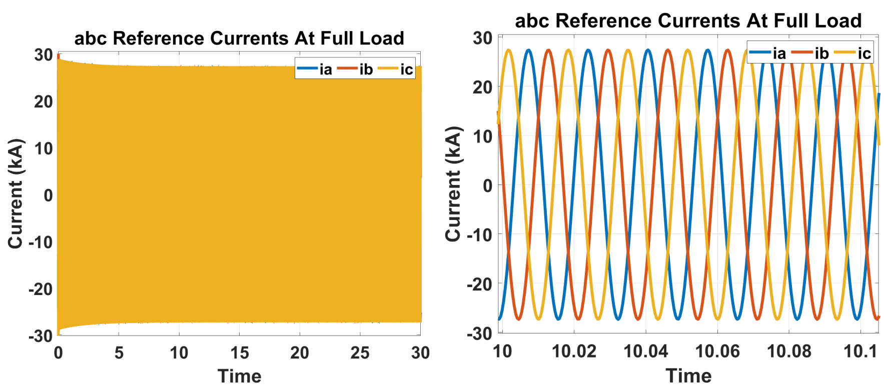
$$i_{ao} = \sqrt{2} * |I_a| \cos(25.84^o)$$

$$i_{bo} = \sqrt{2} * |I_a| \cos(25.84^o - 120^o)$$

$$i_{co} = \sqrt{2} * |I_a| \cos(25.84^o - 240^o)$$

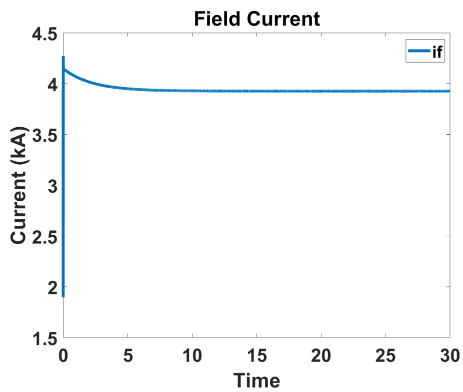
$$i_f = \frac{V_f}{r_f}, \qquad i_{kd} = 0, \qquad and\ i_{kq} = 0$$

2. Perform analysis for the following cases:
   i. Neglect the effects of the damping circuits in your model. Calculate by means of numerical integration (MATLAB or Simulink) and *plot the profiles versus time over the first 30 cycles* for the following variables: ia, ib, ic, if, and Tem.



(a) (b)

Fig-1: Phase currents



(c)

Fig-3: without damping case (a) Currents in abc frame of reference and (b) zoomed in (c) Field current

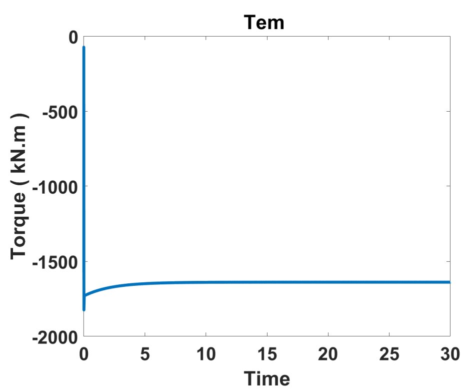
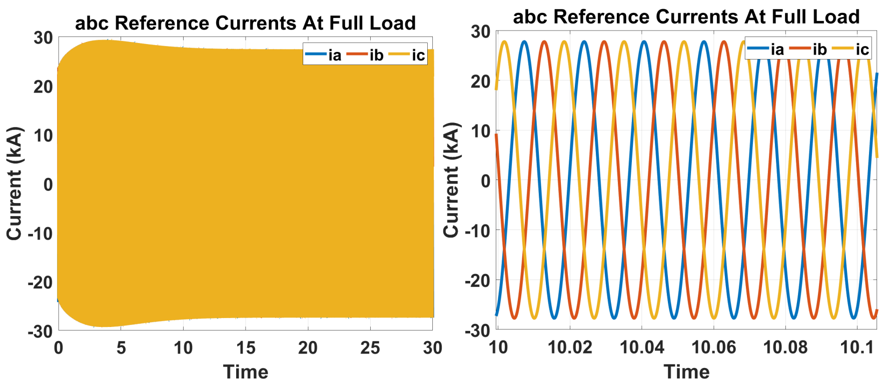


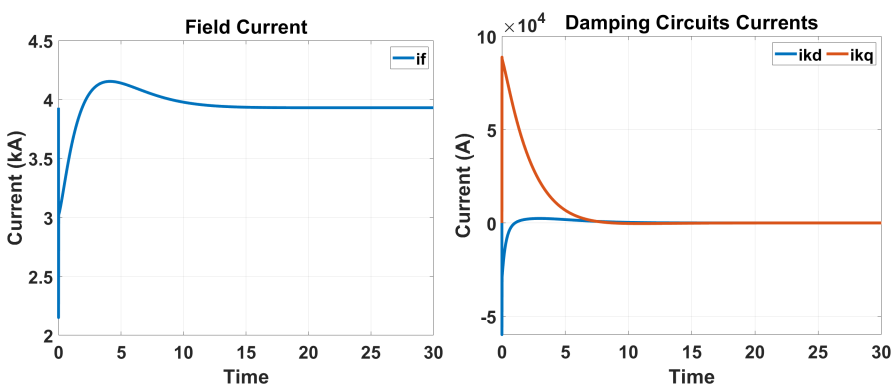
Fig-4: without damping case: Electromagnetic Torque.

ii. Include the effects of the damping circuits in your model. Calculate by means of numerical integration (MATLAB or Simulink) and *plot the profiles versus time over the first 30 cycles* for the following variables: ia, ib, ic, if, ikd1, ikq1, and Tem



(a) (b)

Fig-5 (a) Currents in abc frame of reference and (b) zoomed in



(a) (b)

Fig-6: (a) Field current and (b) Damping currents.

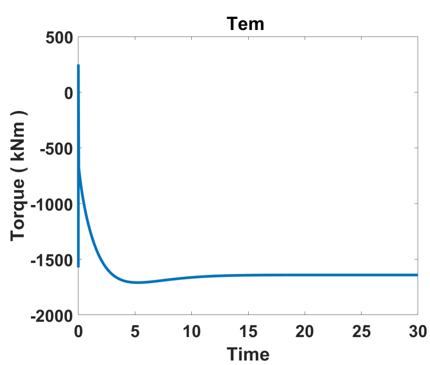


Fig-7: with damping case: T_{EM} Torque.

Appendix

MATLAB Pseudocode: Synchronous Machine State-Space Modeling

- **Define Initial Parameters**
  - Define constants: frequency `f`, angular velocity `w`, voltage values (`Vf`, `VLine`, `Vph`), apparent power `S`, and power factor `pf`.
- **Compute Derived Quantities**
  - Calculate `theta` based on power factor.
  - Determine `Zl`, `rl`, `Xl`, and `Ll`.
  - Compute phase current I_a and phase angle.
  - Calculate initial currents: `ifo`, `iao`, `ibo`, `ico`, `ikdo`, `ikqo`.
- **Set Simulation Parameters**
  - Define initial conditions `iniCon` for the ODE solver.
  - Specify the time span `tspan`.
- **Solve the State-Space Model**
  - Use `ode45` to solve the system of equations with the function `myode`.

```
iniCon = [iao;ibo;ico;ifo;ikdo;ikqo];
tspan = [0 30];
[t,x] = ode78(@myode, tspan, iniCon);  % Solving for State Space Model (Synchronous Machine)
```

ODE Function (myode)

```
function dx = myode(t,x)
```

1. **Calculate angle theta based on time.**
2. **Define Parameters**
   - Define initialization of resistances, inductances, and other constants.
3. **Compute Inductance Matrix (`L`)**
   - Compute self and mutual inductances (`Laa`, `Lbb`, `Lab`, etc.) as functions of angle `theta = wt`.
   - Calculate derivatives of inductances (`dLaa`, `dLab`, etc.).
4. **Define Resistance Matrix (`R`)**
   - Set up the resistance matrix with resistances `rs`, `rl`, `rf`, `rkd`, `rkq`.
5. **Define System Matrices**
   - Compute inductance matrix `L`.
   - Compute derivative of the inductance matrix `dL`.
   - Calculate system matrices `A` and `B`.
6. **Define Input Vector (`u`)**
   - Set input voltages.

7. **Compute State Derivative**
   - Calculate `dx` using the state-space equation: dx=A·x+B·u
   - Return State Derivative (dx)

MATLAB Script solving the Synchronous Machine State-Space with Damping Effect

```
%% M3_SS_with_Damping_effect
clear all; close all; clc;

%% initial values calculations
f= 60;
w = 2*pi*f;
rf = 0.0860;
Vf = 338;
VLine = 20e3; %rms
Vph = 20e3/sqrt(3); %rms
S = 659e+06;
pf = 0.9;
theta = acos(pf); %in radian
IA = (S)/(3*Vph);
% Currents at t = 0
ifo = Vf/rf;
iao = sqrt(2)*IA*cos(-theta1+pi);
ibo = sqrt(2)*IA*cos(-theta1+pi-2*pi/3);
ico = sqrt(2)*IA*cos(-theta1+pi-4*pi/3);
ikdo=0;
ikqo=0;
%% State Space Model (4th-Order Runge Kutta Method for ODEs)
iniCon = [iao;ibo;ico;ifo;ikdo;ikqo];
tspan = [0 30];
[t,x] = ode78(@myode, tspan, iniCon);  % Solving for State Space Model (Synchronous Machine)

%% Plotting the results
Ia =   x(:,1)*1e-3; %in kA
Ib =   x(:,2)*1e-3; %in kA
Ic =   x(:,3)*1e-3; %in kA
If =   x(:,4);
Ikd =   x(:,5);
Ikq =   x(:,6);
figure(1)
plot(t , Ia , t , Ib , t  ,Ic);
legend('ia','ib','ic','NumColumns',3);xlabel('Time')
ylabel('Current (kA)')
title('abc Reference Currents At Full Load')
grid
figure(2)
plot(t ,If);
legend('if');xlabel('Time')
ylabel('Current (A)')
title('Field Current')
grid
figure(3)
plot(t , Ikd , t , Ikq);
legend('ikd','ikq');xlabel('Time')
ylabel('Current (A)')
title('Damping Circuits Currents')
grid
va = Vph*sqrt(2)*cos(w*t);
```

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```
vb = Vph*sqrt(2)*cos(w*t-2*pi/3);
vc = Vph*sqrt(2)*cos(w*t-4*pi/3);
p = va .* x (: ,1) + vb .* x (: ,2) + vc .* x (: ,3) ;
tem = (p) ./(w) ;
figure(4)
plot (t , tem/1e3 )
xlabel (" Time ")
ylabel (" Torque ( kNm ) ")
title (" Tem  ")

function dx = myode(t,x)
    Vf = 338;
    VLine = 20e3; %rms
    S = 659e+06;
    pf = 0.9;
    that = (acos(pf));
    Zl= VLine^2/S;
    rl = Zl*cos(that);
    Xl = Zl*sin(that);
    Ll = Xl/(2*pi*60);
    n_m = (120*60)/4;    %n_m = (120*f)/p
    w_m = n_m*(2*pi)*(1/60);
    theta_r  = (w_m*t);   % Rotor Angle

    rs = 7.4e-4;  rf = 0.0860;
    rkd = 1.58e-4; rkq = 1.227e-4;
    Lsv = 0.05e-3;Lmv = 0.05e-3;
    Lsa = 1.95e-3; Lff = 444e-3;
    Lma = 0.80e-3; Lakdm = 0.447e-3;
    Lafm = 26.1e-3; Lakqm = 0.670e-3;
    Lfkd = 5.05e-3; Lkdkd = 0.1254e-3;
    Lkdf = 6.29e-3; Lkqkq = 0.3762e-3;

    % type the inductance expression
    % function of the rotor angle (theta_r)
    Laa =  Lsa+Lsv*cos(2*theta);
    Lbb =  Lsa+Lsv*cos(2*theta-4*pi/3);
    Lcc =  Lsa+Lsv*cos(2*theta-2*pi/3);
    Lab = -Lma+Lmv*cos(2*theta-2*pi/3);
    Lbc = -Lma+Lmv*cos(2*theta);
    Lac = -Lma+Lmv*cos(2*theta-4*pi/3);
    Laf =  Lafm*cos(2*theta);
    Lbf =  Lafm*cos(2*theta-2*pi/3);
    Lcf =  Lafm*cos(2*theta-4*pi/3);
    Lakd =  Lakdm*cos(2*theta);
    Lbkd =  Lakdm*cos(2*theta-2*pi/3);
    Lckd =  Lakdm*cos(2*theta-4*pi/3);
    Lakq =  Lakqm*cos(2*theta+pi/2);
    Lbkq =  Lakqm*cos(2*theta-2*pi/3+pi/2);
    Lckq =  Lakqm*cos(2*theta-4*pi/3+pi/2);

    %the derivative of the inductances
    % function of the rotor angle (theta_r)
    dLaa = -2*Lsv*sin(2*theta);
    dLbb = -2*Lsv*sin(2*theta-4*pi/3);
    dLcc = -2*Lsv*sin(2*theta-2*pi/3);
    dLab = -2*Lmv*sin(2*theta-2*pi/3);
    dLbc = -2*Lmv*sin(2*theta);
    dLac = -2*Lmv*sin(2*theta-4*pi/3);
    dLaf = -2*Lafm*sin(2*theta);
    dLbf = -2*Lafm*sin(2*theta-2*pi/3);
```

```
    dLcf = -2*Lafm*sin(2*theta-4*pi/3);
    dLakd = -2*Lakdm*sin(2*theta);
    dLbkd = -2*Lakdm*sin(2*theta-2*pi/3);
    dLckd = -2*Lakdm*sin(2*theta-4*pi/3);
    dLakq = -2*Lakqm*sin(2*theta+pi/2);
    dLbkq = -2*Lakqm*sin(2*theta-2*pi/3+pi/2);
    dLckq = -2*Lakqm*sin(2*theta-4*pi/3+pi/2);

    R = [rs+rl     0       0      0    0    0;
          0      rs+rl     0      0    0    0;
          0        0     rs+rl    0    0    0;
          0        0       0      rf   0    0;
          0        0       0      0   rkd   0;
          0        0       0      0    0   rkq];

    L = [Laa+Ll   Lab     Lac     Laf    Lakd    Lakq;
          Lab    Lbb+Ll  Lbc     Lbf    Lbkd    Lbkq;
          Lac      Lbc    Lcc+Ll  Lcf    Lckd    Lckq;
          Laf      Lbf     Lcf     Lff    Lfkd     0;
          Lakd     Lbkd    Lckd    Lkdf   Lkdkd    0;
          Lakq     Lbkq    Lckq     0       0     Lkqkq];

     dL = [ dLaa      dLab      dLac      dLaf    dLakd     dLakq;
            dLab      dLbb      dLbc      dLbf    dLbkd     dLbkq;
            dLac      dLbc      dLcc      dLcf    dLckd     dLckq;
            dLaf      dLbf      dLcf       0        0         0;
            dLakd     dLbkd     dLckd      0        0         0;
            dLakq     dLbkq     dLckq      0        0         0];

   A = -inv(L)*(R+w_m*dL);
    B = inv(L);
    u = [0;0;0; Vf;0;0];
    dx = A*x+ B*u;
end
```

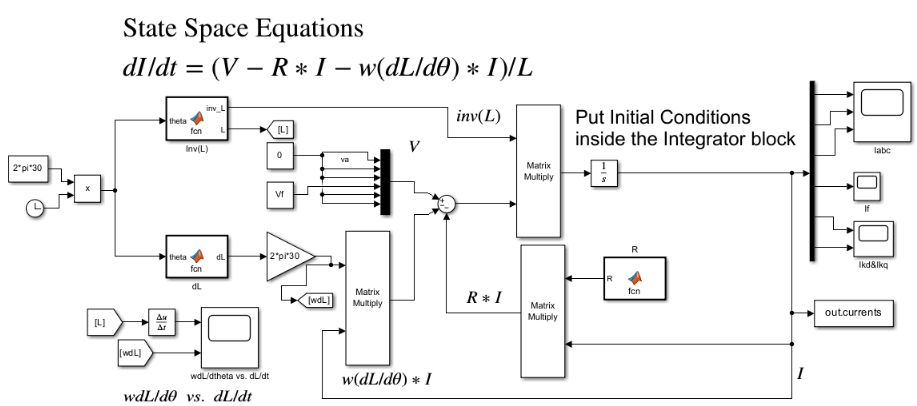

Fig-9: Simulink block diagram used to solve the system.

V. REFERENCES

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