COLORADO SCHOOL OF MINES
ELECTRICAL ENGINEERING DEPARTMENT

EENG 577

ADVANCED ELECTRICAL MACHINE DYNAMICS FOR SMART-GRID SYSTEMS

M3-P1 AC Machinery Fundamentals

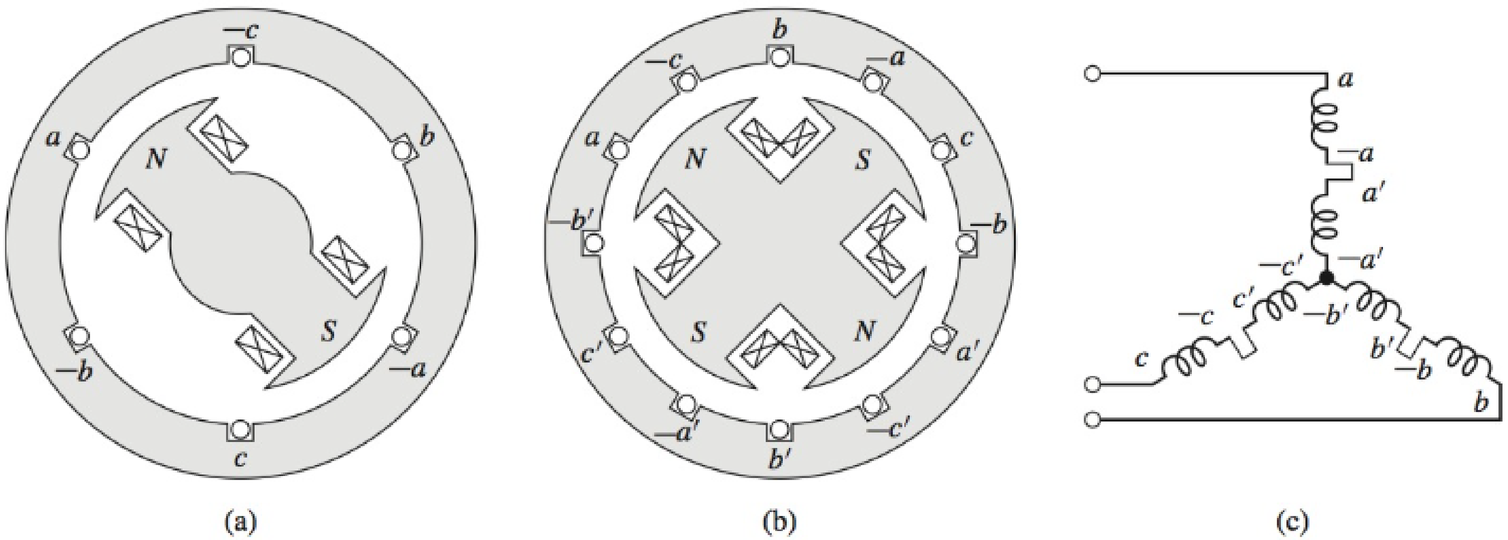
Dr. A.A. Arkadan

LEARNING OBJECTIVES

- Explain how to generate an ac voltage in a loop rotating in a uniform magnetic field.
- Explain how to generate torque in a loop carrying a current in a uniform magnetic field.
- Explain how to create a rotating magnetic field from a three-phase stator.
- Explain how a rotating rotor with a magnetic field induces ac voltages in stator windings.
- Explain and apply the relationship between electrical frequency, the number of poles, and the rotational speed of an electrical machine.
- Explain how torque is induced in an ac machine.
- Explain the types of losses in a machine, and the power flow diagram.

Link: Rotating Magnetic Field & Synchronous Speed

AC Synchronous Machine Principles of Operation



Schematic views of three-phase generators: (a) two-pole, (b) four-pole, and (c) *Y* connection of the windings.

- A three-phase AC synchronous machine consists of two essential elements: a stationary part called the **Stator,** and a rotating part called the **Rotor**.
- The stator of a synchronous machine holds a three-phase winding called the **armature**.
- The rotor holds the **field** winding, which is excited by a dc current – the field current .

Source: A.E. Fitzgerald, C. Kingsley, and S.D. Umans, “Electric Machinery,” *McGraw-Hill Publishing Company, Inc.*, 1990, New York, NY

AC Synchronous Machine Principles of Operation

- In the case of an AC **Generator**,
  - ✓ the rotor is turned by a ***prime mover***, producing a rotating magnetic field within the machine.
  - ✓ This rotating magnetic field induces a three- phase set of voltages within the stator windings.
  - ✓ The stator winding voltages are sinusoidal. When loaded, they produce a rotating mmf at the electrical angular velocity of the rotor.

Input Power:
Output Power:
Direction of Power flow:

Input: Mechanical Power

Output: Electrical Power

AC Synchronous Machine Principles of Operation

- In the case of an AC **Motor**,
  - ✓ The stator is excited by a balanced 3-phase ac voltage source that produces a stator rotating mmf at an electrical angular velocity.
  - ✓ The stator rotating magnetic field (mmf) produces torque in the rotor windings (coils).
  - ✓ the rotor is rotated at a mechanical speed.

Input Power:
Output Power:
Direction of Power flow:

Output: Mechanical Power

Input: Electrical Power

The Voltage Induced in a Rotating Loop

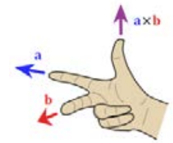
In a conductor, $e_{ind} = (\boldsymbol{v} \times \boldsymbol{B}) \cdot \ell$

where,

$\boldsymbol{v}$ = velocity of the conductor

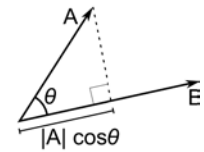
$\boldsymbol{B}$ = Magnetic Flux Density vector

$\boldsymbol{l}$ = Length of the Conductor

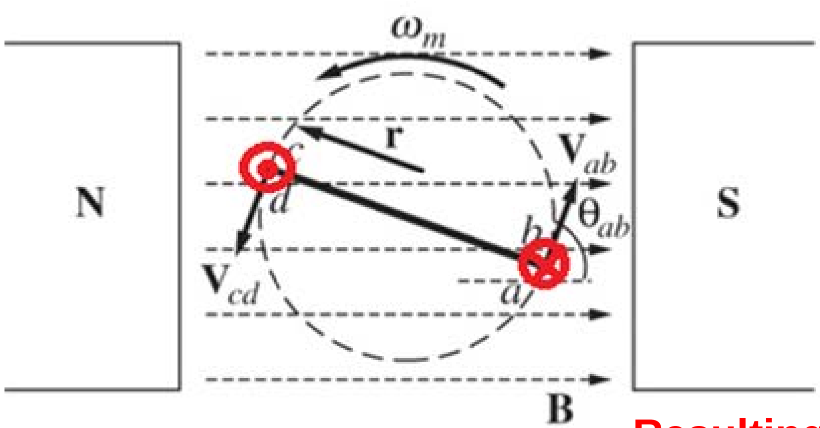


The cross product is defined by the formula

$$\mathbf{a} \times \mathbf{b} = \|\mathbf{a}\| \, \|\mathbf{b}\| \sin(\theta) \, \mathbf{n}$$



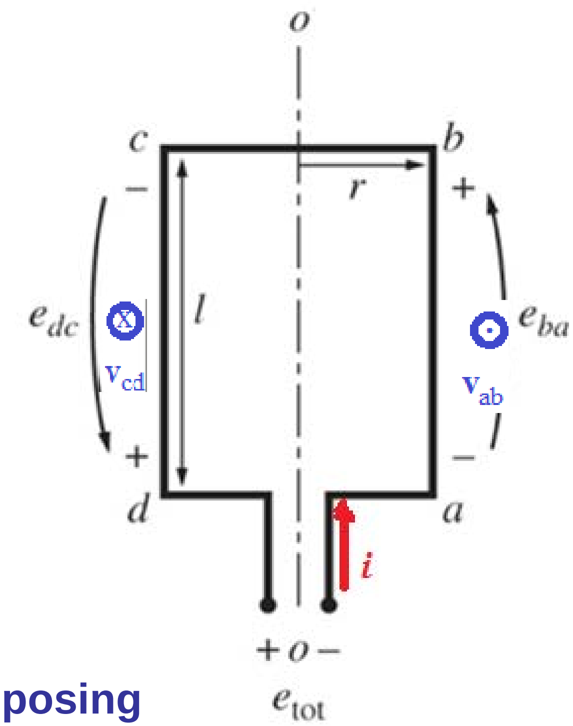
$$\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\| \, \|\mathbf{b}\| \cos(\theta),$$



B is a uniform magnetic field, aligned as shown.

Resulting current direction if coil is loaded

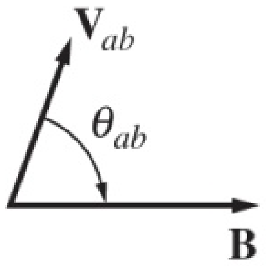
Current Causes Opposing Force!

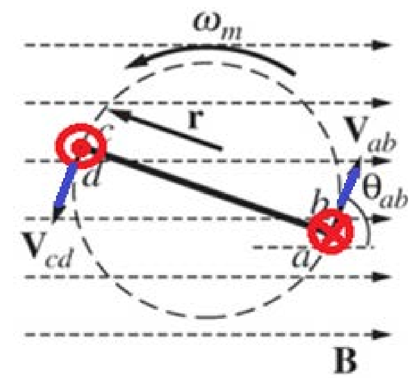


(a) (b)

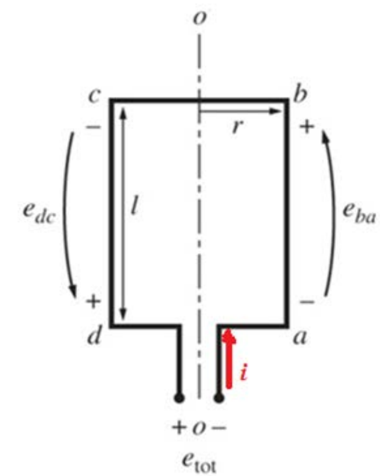
A simple rotating loop in a uniform magnetic field. (a) Front view; (b) view of coil

1. Segment **ab**





Resulting current direction if loaded



- ***v*** is tangential to the path of rotation
- ***B*** points to the right
- (***v*** x ***B***) points into the page
- Hence, $e_{ind} = (\boldsymbol{v} \times \boldsymbol{B}) \cdot \underline{\ell}$ is rising along a,b

2. Segment **bc**.

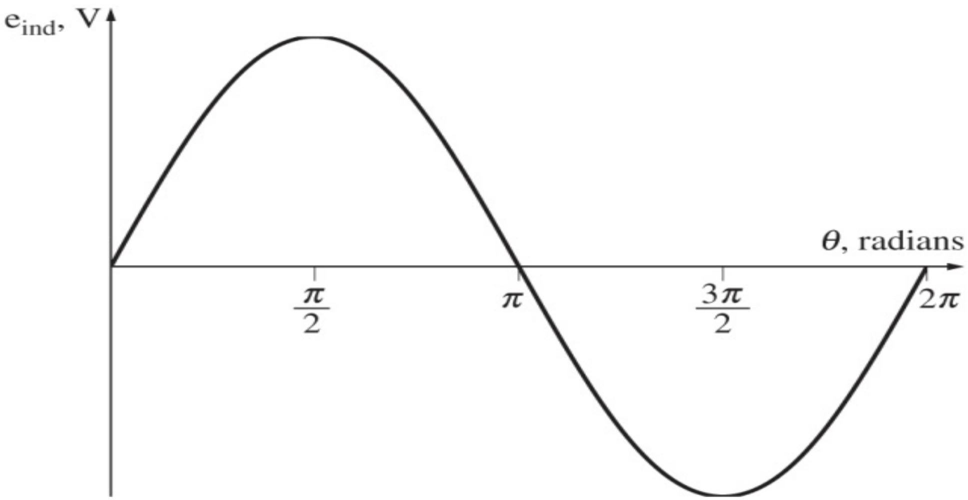
- (***v*** x ***B***) is perpendicular to ℓ, hence e_{ab}=0

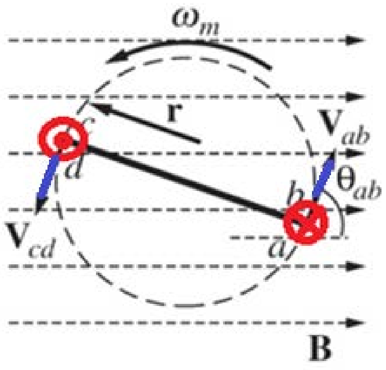
*Also, similar expressions are obtained for the other two segments **cd** and **da**.*

- Total Voltage induced in a loop

$$e_{ind} = e_{ba} + e_{cb} + e_{dc} + e_{ad} = v\ell B sin(\theta_{ab}) + v\ell B sin(\theta_{cd})$$
$$= 2v\ell B sin(\theta_{ab})$$
$$= 2v\ell B sin(\theta)$$

Note: $\theta_{ab} = \theta_{cd}$





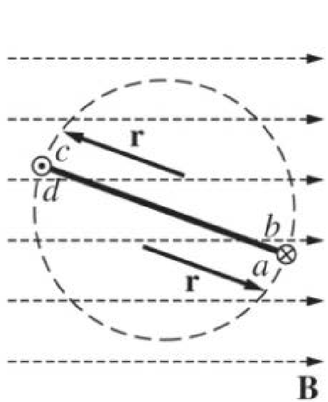
- If loop rotates at a constant angular velocity ω, $\theta = \omega t$ and
- $v = r\omega$, where r is the radius from the axis of rotation to the loop edge, the induced voltage expression is reduced to the following:

$$e_{ind} = 2r\omega\ell B sin(\omega t) = BA\omega sin(\omega t),$$ A=$2r\ell$ is the loop area

Force on a Conductor &Torque Induced in a Current-Carrying Loop

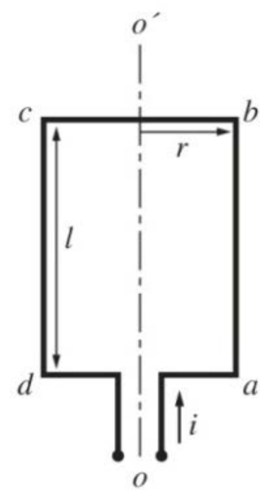
$$\boldsymbol{F} = \boldsymbol{i}\left(\boldsymbol{\ell} \times \boldsymbol{B}\right)$$

- ***F*** = Magnetic force on a segment of a current loop
- *i* = Current in the loop
- ***B*** = magnetic Flux Density
- ℓ = Vector length of the segment its direction is defined to be in the direction of the current flow.

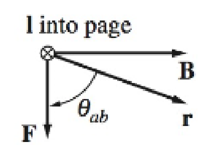


B is a uniform magnetic field, aligned as shown. The × in a wire indicates current flowing into the page, and the • in a wire indicates current flowing out of the page.

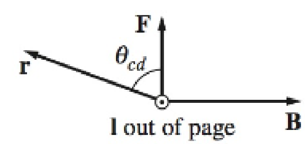
(a)



(b)



(a)



(c)

r **is the radius from the axis of rotation to the loop edge**

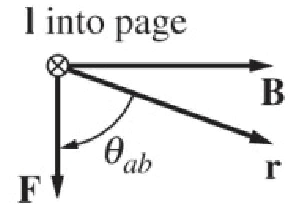
A current-carrying loop in a uniform magnetic field: (a) Front view; (b) view of coil

- The torque on the segment will then be given by

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} \rightarrow |\tau| = rF\sin(\theta)$$

Where θ is the angle between r and F

r is the radius from the axis of rotation to the loop edge



1. Segment **ab:**

$$\tau_{ab} = (F)(r\sin\theta_{ab}) = ri\ell B\sin\theta_{ab} \quad \text{clockwise}$$

2. Segment **bc**

$$\tau_{bc} = (F)(r\sin\theta_{bc}) = 0, \text{since } \theta_{bc} = 0$$

The cross product is defined by the formula

$$\mathbf{a} \times \mathbf{b} = \|\mathbf{a}\|\,\|\mathbf{b}\|\sin(\theta)\,\mathbf{n}$$

*Also, similar expressions are obtained for the other two segments **cd** and **da**.*

Total induced torque on the loop

$$\tau_{ind} = \tau_{ab} + \tau_{bc} + \tau_{cd} + \tau_{da} = Bri\ell\sin\theta_{ab} + Bri\ell\sin\theta_{cd}$$

$$= 2Bri\ell\sin\theta, \text{since } \theta_{ab} = \theta_{cd} = \theta$$

Also,

$$\tau_{ind} = BAi\sin(\omega t) \quad \text{where A=2}r\ell \text{ is the loop area}$$

Summary

- Voltage Induced in a Rotating Loop

$$e_{ind} = BA\omega sin(\omega t)$$

where A=$2r\ell$ is the loop area

- Torque Induced in a Current-Carrying Loop

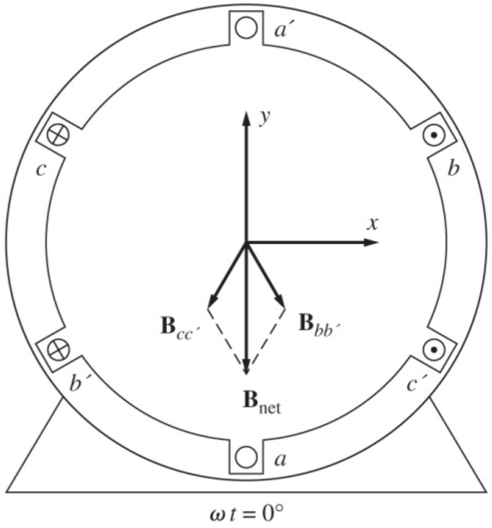
$$\tau_{ind} = BAi sin(\omega t)$$

where A=$2r\ell$ is the loop area

AC Motor Operation

- If two magnetic fields are present in a machine, then a torque will be created which will tend to line up the two magnetic fields.
- If one magnetic field is produced by the stator of an ac machine and the other one is produced by the rotor of the machine, then ***a torque will be induced in the rotor which will cause the rotor to turn and align itself with the stator magnetic field.***
- ***If there were some way to make the stator magnetic field rotate***, then the induced torque in the rotor would cause it to constantly "chase" the stator magnetic field around in a circle. This, in a nutshell, is the ***basic principle of all ac motor operation***.

How can stator magnetic field be made to rotate?



- Three coils spaced 120 electrical degrees apart.

- A balanced 3-phase currents are applied to them.

$$i_{aa'}(t) = I_M \sin(\omega t) \quad \text{A}$$
$$i_{bb'}(t) = I_M \sin(\omega t - 120^o) \quad \text{A}$$
$$i_{cc'}(t) = I_M \sin(\omega t - 240^o) \quad \text{A}$$

- The field intensity produced by these currents are

$$H_{aa'}(t) = H_M \sin(\omega t)\angle 0^o \quad \text{A.turns/m}$$
$$H_{bb'}(t) = H_M \sin(\omega t - 120^o)\angle 120^o \quad \text{A.turns/m}$$
$$H_{cc'}(t) = H_M \sin(\omega t - 240^o)\angle 240^o \quad \text{A.turns/m}$$

- The flux densities resulting from these magnetic fields are give by:

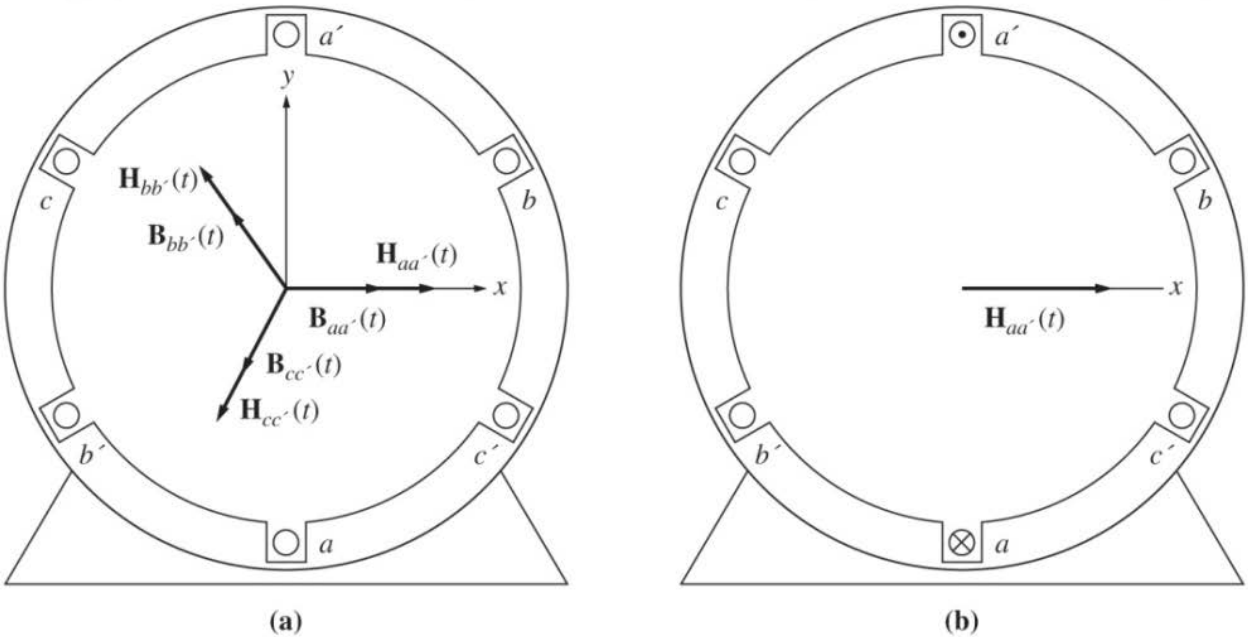
$$B_{aa'}(t) = B_M \sin(\omega t)\angle 0^o \quad \text{T}$$

$$B_{bb'}(t) = B_M \sin(\omega t - 120^o)\angle 120^o \quad \text{T}$$

$$B_{cc'}(t) = B_M \sin(\omega t - 240^o)\angle 240^o \quad \text{T}$$

where,

$$B_M = \mu H_M$$

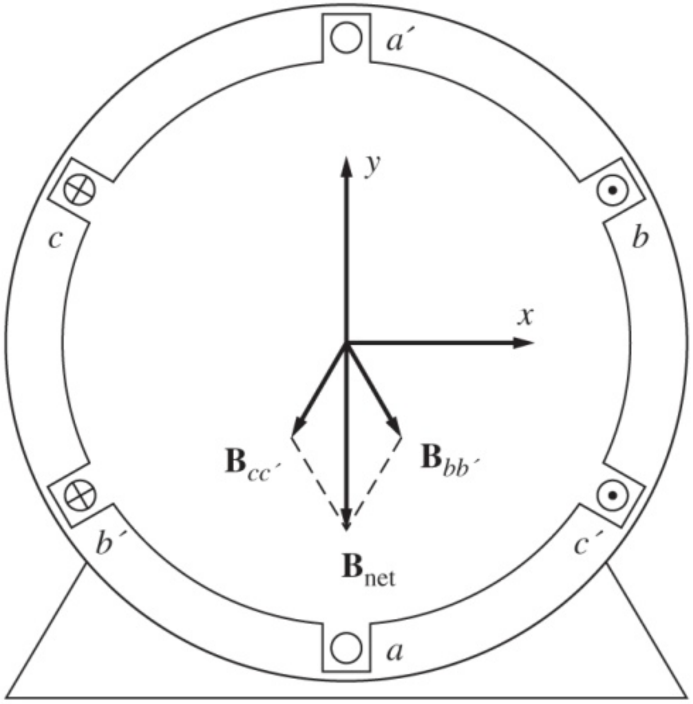


(a) A simple three-phase stator. Currents are assumed positive if they flow into unprimed end of the coil. (b) The field intensity Haa'(t) produced by a current flowing in coil aa'

At $\omega t = 0^o$

$$B_{aa'} = 0, \quad B_{bb'} = B_M \sin(-120^o)\angle 120^o, \; B_{cc''} = B_M \sin(-240^o)\angle 240^o$$

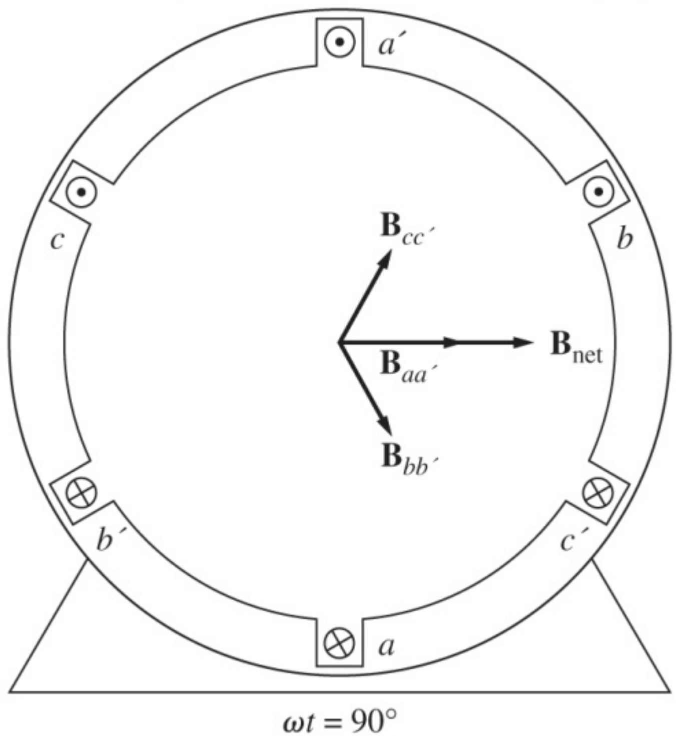
$$B_{net} = B_{aa'} + B_{bb'} + B_{cc'} = 1.5 B_M \angle -90^o$$



At $\omega t = 90^o$

$$B_{net} = B_M\angle 0^o + \left(-\frac{1}{2}B_M\right)\angle 120^o + \left(-\frac{1}{2}B_M\right)\angle 240^o$$

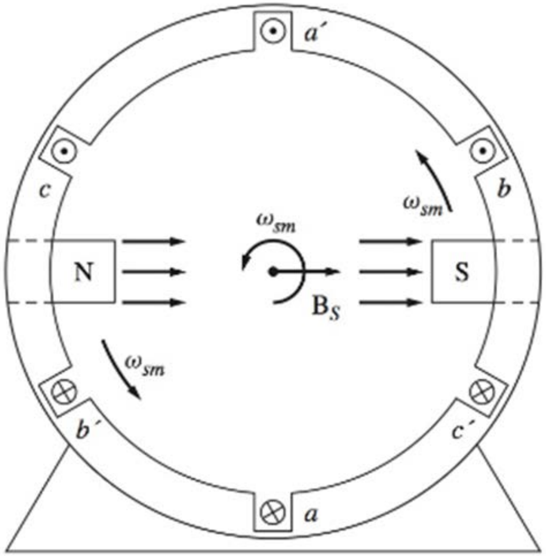
$$B_{net} = 1.5B_M\angle 0^o$$



- The resulting magnetic field has moved in CCW.
- The magnitude of the magnetic field remains constant.
- At any time t, It can be shown the resultant magnetic field has a form given by:

$$B_{net} = \left(1.5B_M \sin\omega t\right)\hat{x} - \left(1.5B_M \cos\omega t\right)\hat{y}$$

  - The magnitude of the field is a constant $1.5B_M$.
  - The position changes at an angular velocity ω.
  - The angular position of the field is $\alpha = 90 - \omega t$.


a′
c
b
ω_{sm}
ω_{sm}
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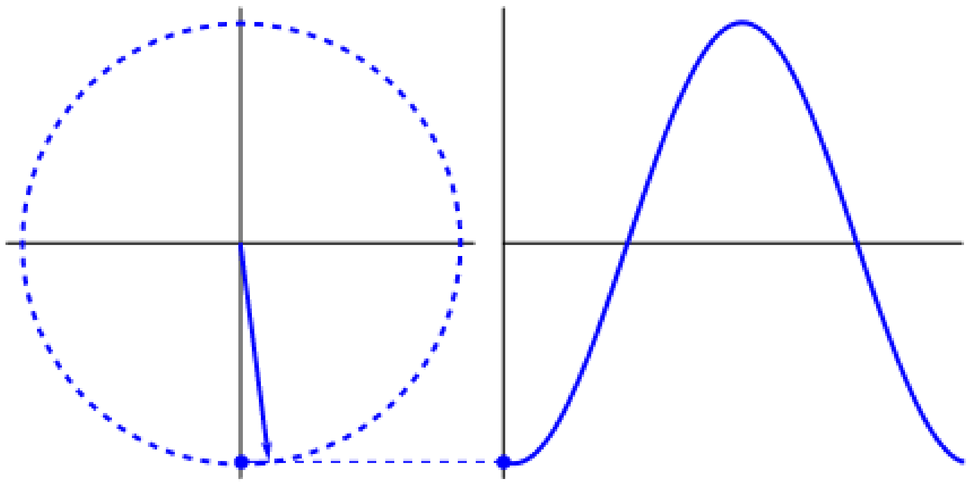
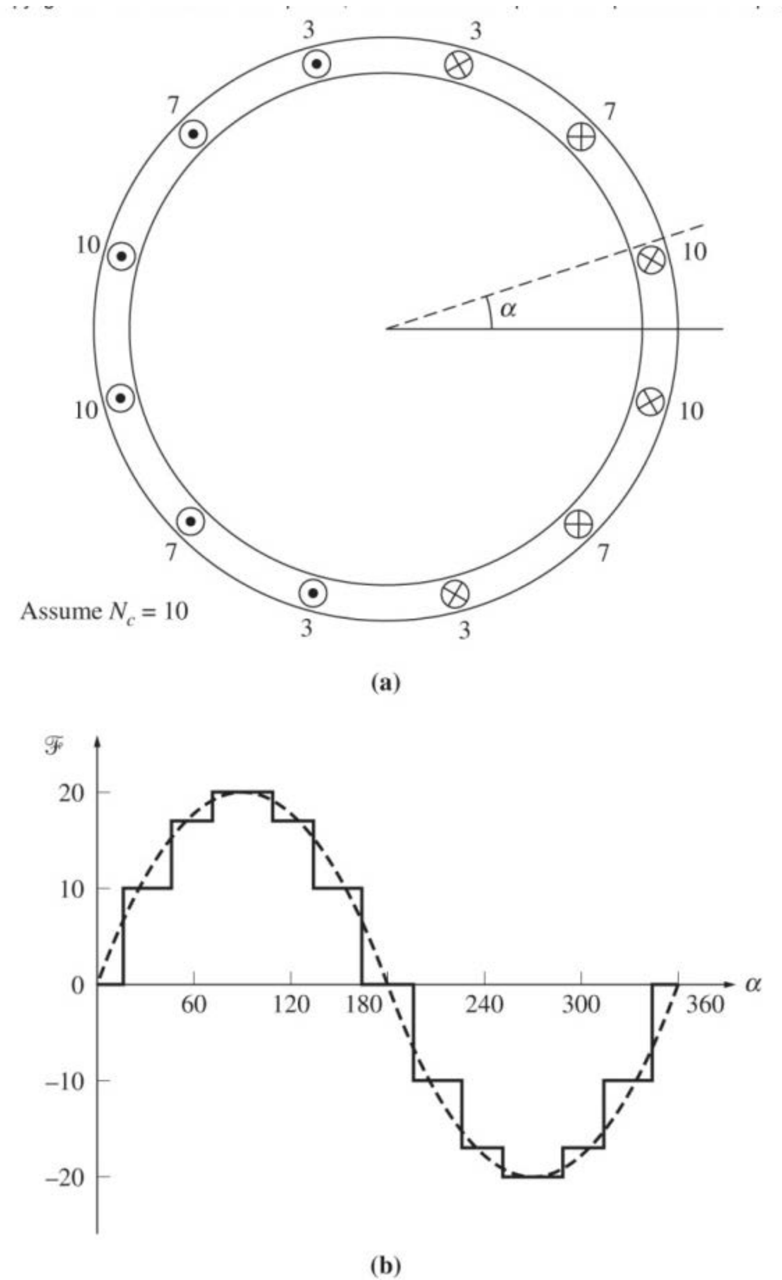
ω_{sm}
b′
c′
a



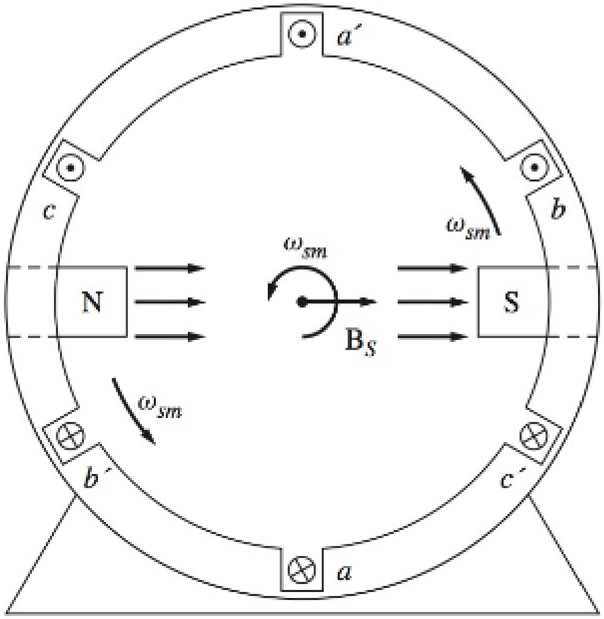
Stator MMF

A sinusoidal air-gap flux density may be achieved by arranging the **stator conductors** in closely spaced slots in a sinusoidal manner given by $n_c = N_c Cos(\alpha)$



(a) An ac machine with a distributed stator winding to produce a sinusoidally varying air-gap flux density. The mmf resulting from the winding, compared to an ideal distribution.

Frequency of Stator Quantities

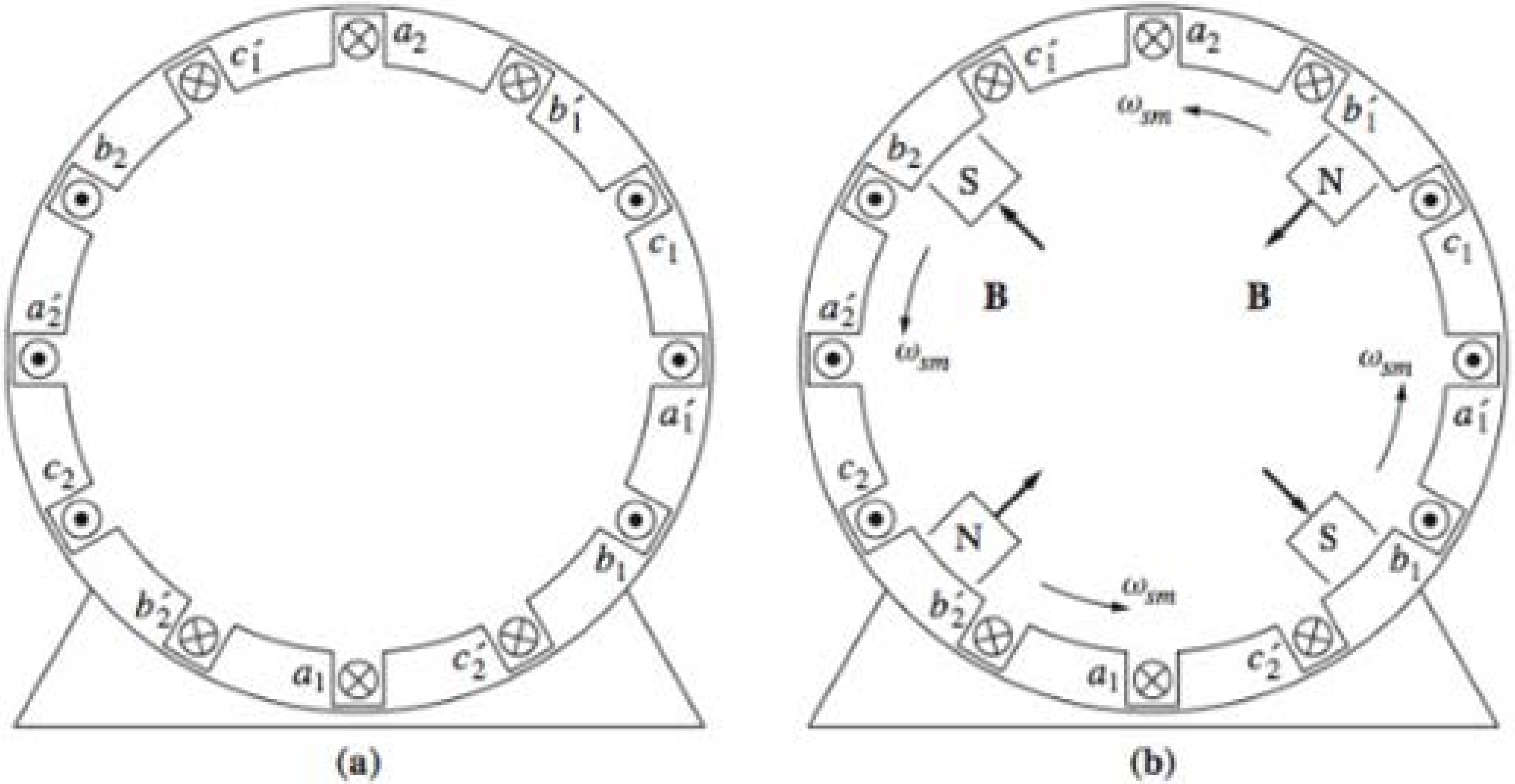


The rotating magnetic field in a stator represented as moving north and south stator poles.

In a 2-pole ac machine, the stator electrical, **se**, and stator mechanical, **sm,** quantities are related as:

$$f_{se} = f_{sm} \qquad \text{two poles}$$

$$\omega_{se} = \omega_{sm} \qquad \text{two poles}$$



(a) A simple four-pole stator winding. (b) The resulting stator magnetic poles. Notice that there are moving poles of alternating polarity every 90° around the stator surface.

In a P-poles ac machine, the stator electrical, **se**, and stator mechanical, **sm**, quantities are related as:

$$\theta_{se} = \frac{P}{2}\theta_{sm}$$

$$f_{se} = \frac{P}{2}f_{sm}$$

$$\omega_{se} = \frac{P}{2}\omega_{sm}$$

$$f_{se} = \frac{n_{sm}P}{120}$$

In Generation Mode:

- The rotation of the rotor coupled with the field current I_f produce a rotating magnetic field that **induces currents in the stator windings**.

- The electrical frequency, **f**, is related to the rotor synchronous mechanical speed, **n**, as follows:

$$f = \frac{p}{2} \times \frac{n}{60}$$

where: n is the rotor speed in *rev/min* and p is the number of rotor poles.

In Motoring Mode:

- **A three-phase set of currents**, each of equal magnitude and differing in phase by 120°, **flowing in three-phase distributed windings**, **produces a magnetic field of constant magnitude rotating at synchronous speed.**
- The **interaction** between the **synchronously rotating magnetic fields** of the rotor and the stator **creates the developed torque** of the machine.

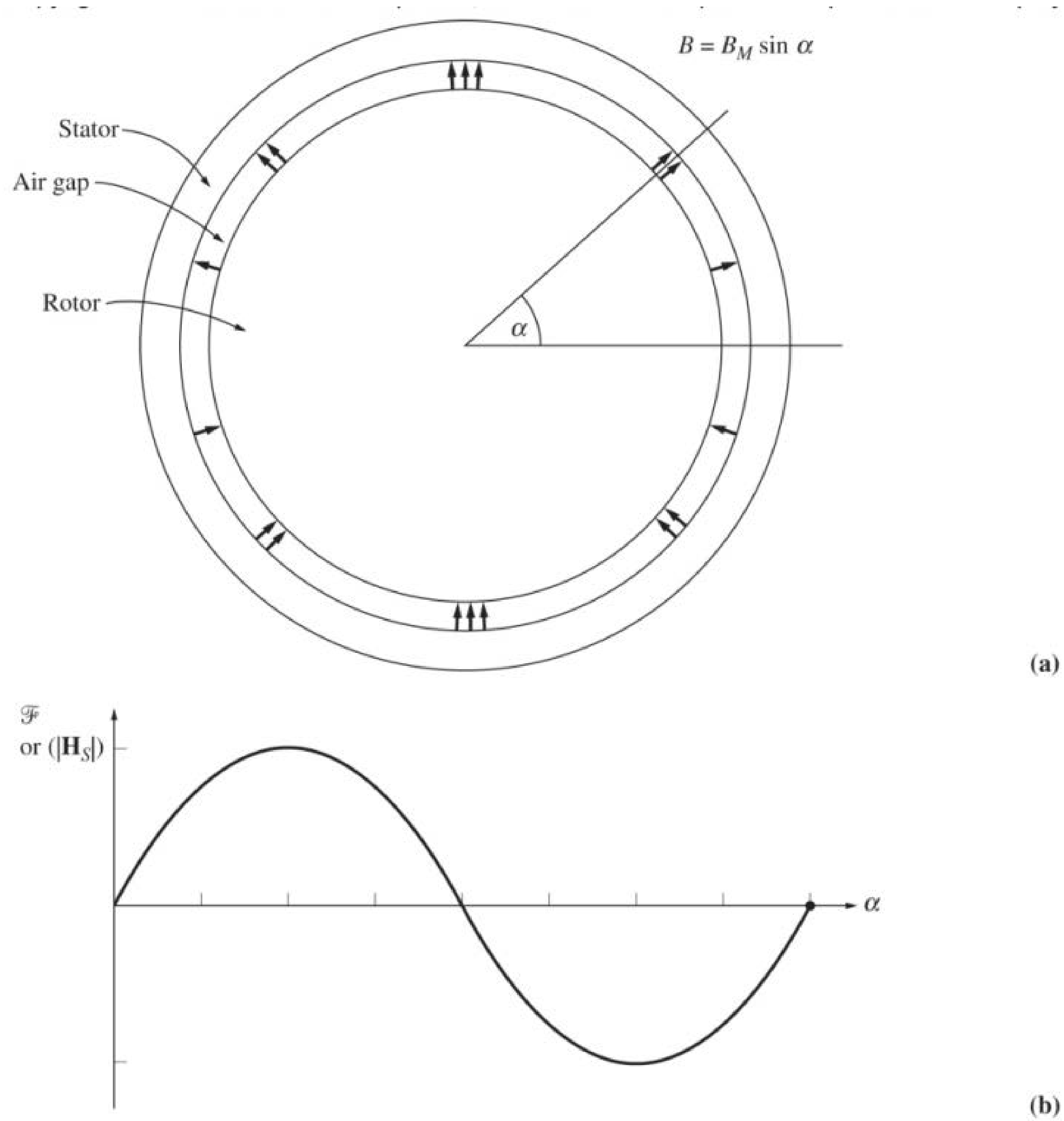
The Voltage Induced by the Rotating Magnetic Field

Assumptions:

- Air gap flux density is radial.
- Flux density is distributed sinusoidally in tangential direction.

(a) A **cylindrical rotor** with sinusoidally varying air-gap flux density.

(b) (b) The mmf or field intensity as a function of angle α in the air gap.



- The rotor rotating field induces voltages in the stator three-phase winding having N_c turns per phase as,

$$e_{aa'}(t) = N_C \phi\omega \sin\omega t$$

$$e_{bb'}(t) = N_C \phi\omega \sin(\omega t - 120^o)$$

$$e_{cc'}(t) = N_C \phi\omega \sin(\omega t - 240^o)$$

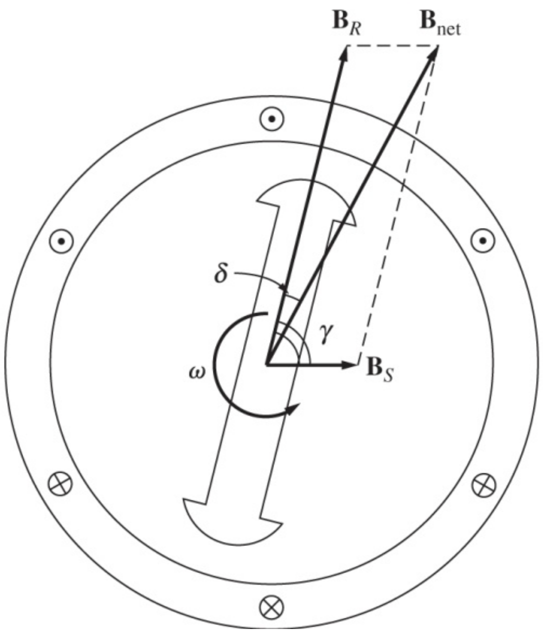
where,

$$\phi = \left(2r\ell\right)B_M$$

- The rms voltage induced in any given phase of the stator is given by $E_A = \sqrt{2}\pi N_C \phi f$

where, $\omega = 2\pi f$

Induced Torque in an AC Machine



- For a simple loop, the generated torque can be expressed as:

$$\tau_{ind} = KB_RB_S \sin\theta$$

- If B_R represents the rotor flux density and B_S represents the stator rotating magnetic field, as shown

$$\tau_{ind} = KB_RB_S \sin\gamma$$
$$\underline{\tau}_{ind} = K\underline{B}_R x \underline{B}_S = K\underline{B}_R x(\underline{B}_{net} - \underline{B}_R) = K\underline{B}_R x \underline{B}_{net}$$

Hence,

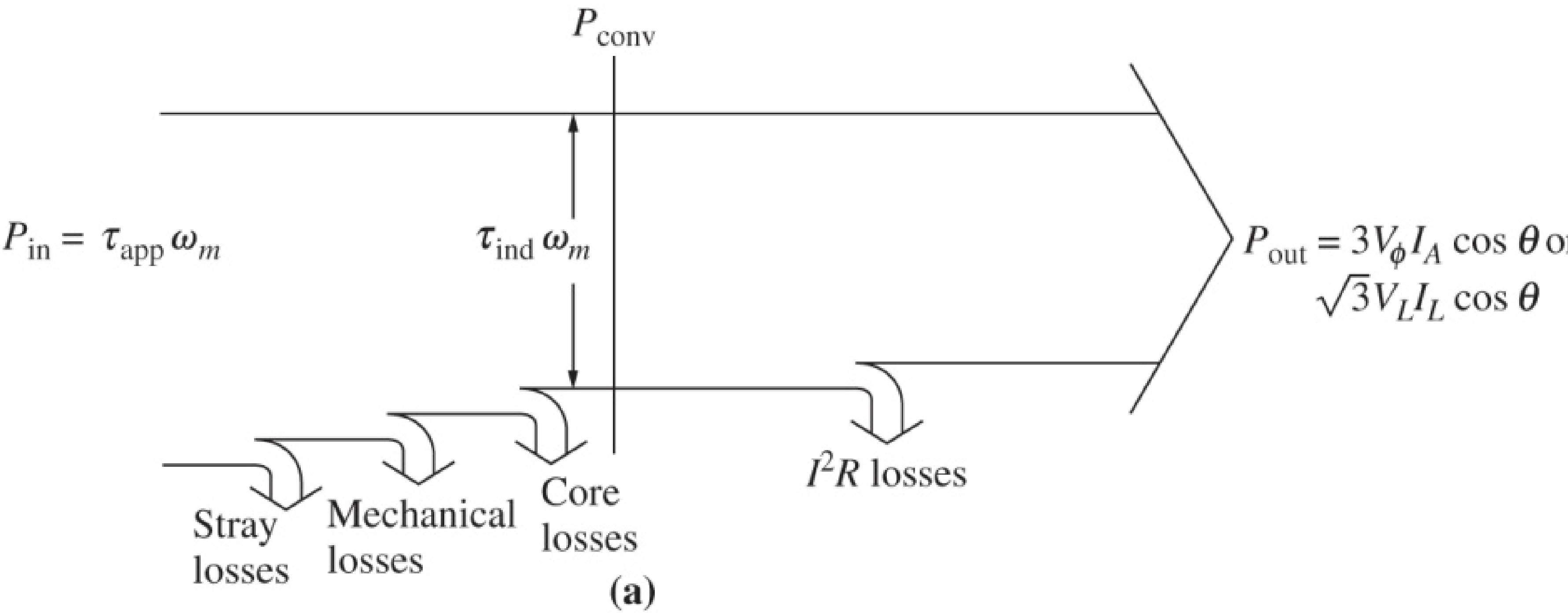
$$\tau_{ind} = KB_RB_{net} \sin\delta$$

where δ is the angle between $\underline{B}_R and \underline{B}_{net}$ and is called torque angle.

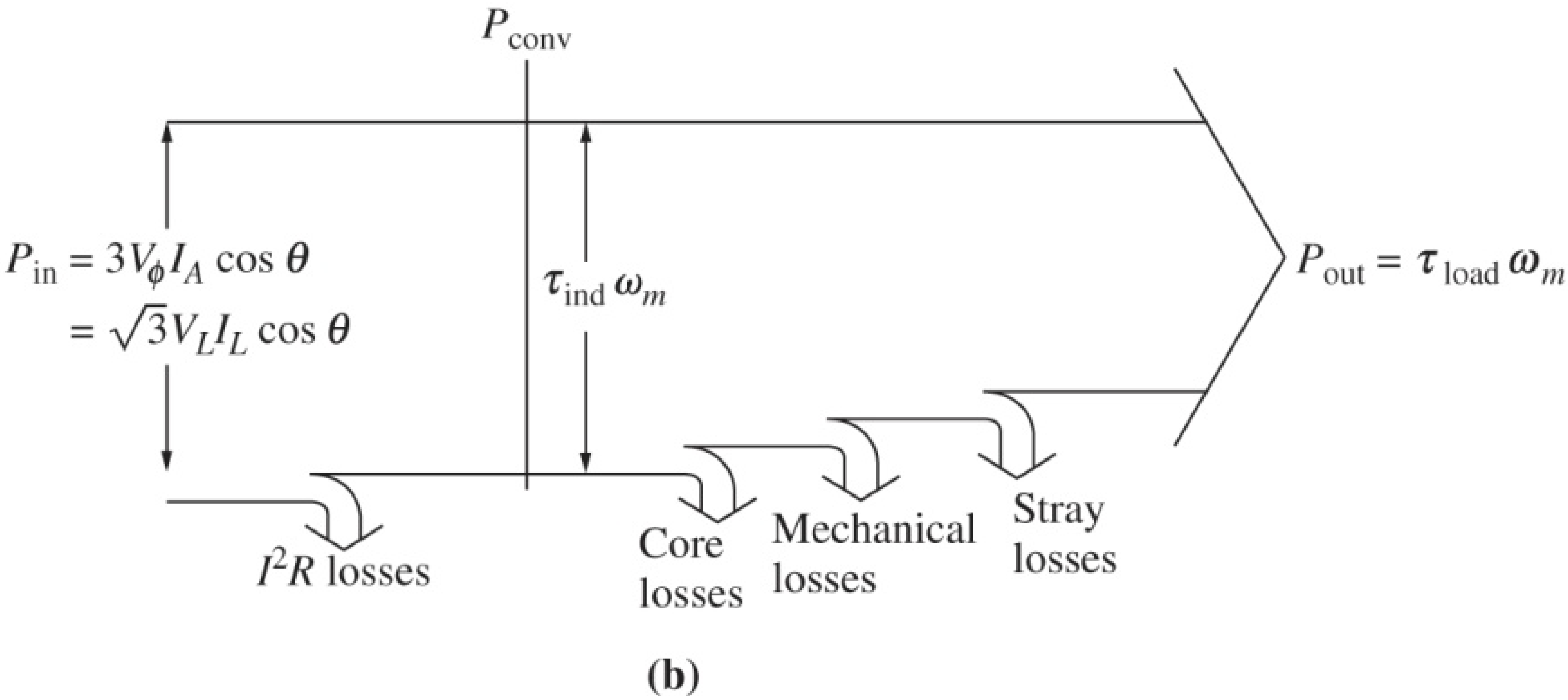
The Power-Flow Diagram

- For an AC generator, mechanical power is input to the machine, and then stray, mechanical, and core losses are subtracted. The remaining power is what can be converted to electrical form and is called converted power (labeled P_{CONV}). Other losses are copper (I^2R) losses in stator and rotor.

- For an AC motor, the electrical power is input to the machine, and then the stator and rotor copper losses are subtracted. The remaining power is $P_{CONV.}$



(a) The power-flow diagram of a three-phase AC generator.



(b) The power flow diagram of a three-phase AC motor.

LEARNING OUTCOMES

- Able to explain how ac voltage is generated in a loop rotating in a uniform magnetic field.
- Able to explain how torque is generated in a loop carrying a current in a uniform magnetic field.
- Able to explain how a rotating magnetic field is created from a three-phase stator.
- Able to explain how a rotating rotor with a magnetic field induces ac voltages in stator windings.
- Able to explain the relationship between electrical frequency, the number of poles, and the rotational speed of an electrical machine.
- Able to explain how torque is induced in an ac machine.
- Able to explain the types of losses in a machine, and the power flow diagram.