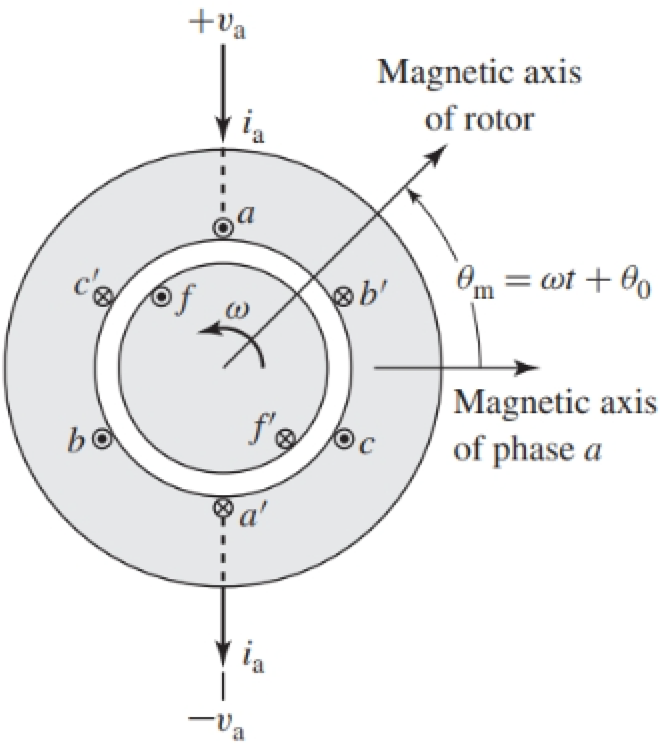
COLORADO SCHOOL OF MINES
ELECTRICAL ENGINEERING DEPARTMENT

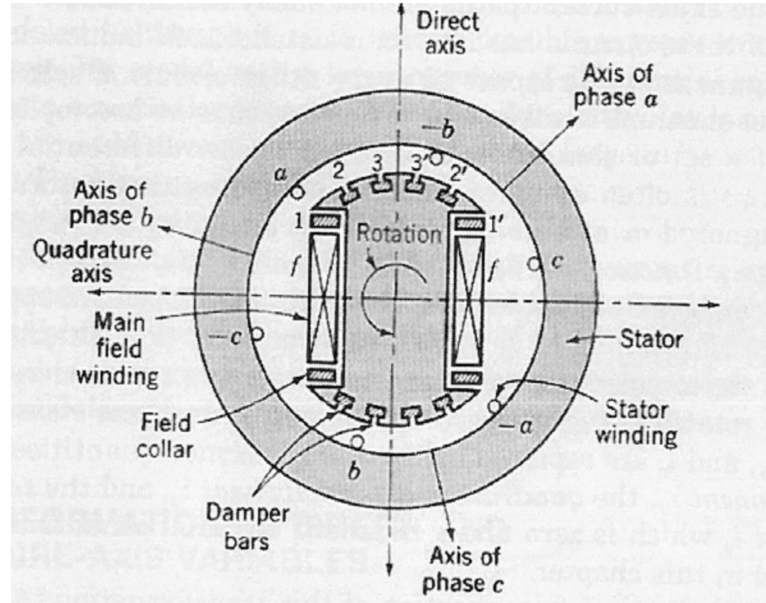
EENG 577

ADVANCED ELECTRICAL MACHINE DYNAMICS FOR SMART-GRID SYSTEMS

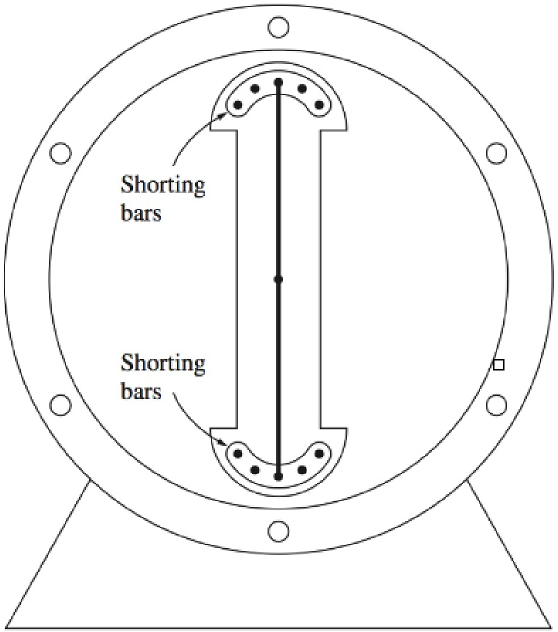
M3-P3 Synchronous Machine State Space Model

Dr. A.A. Arkadan



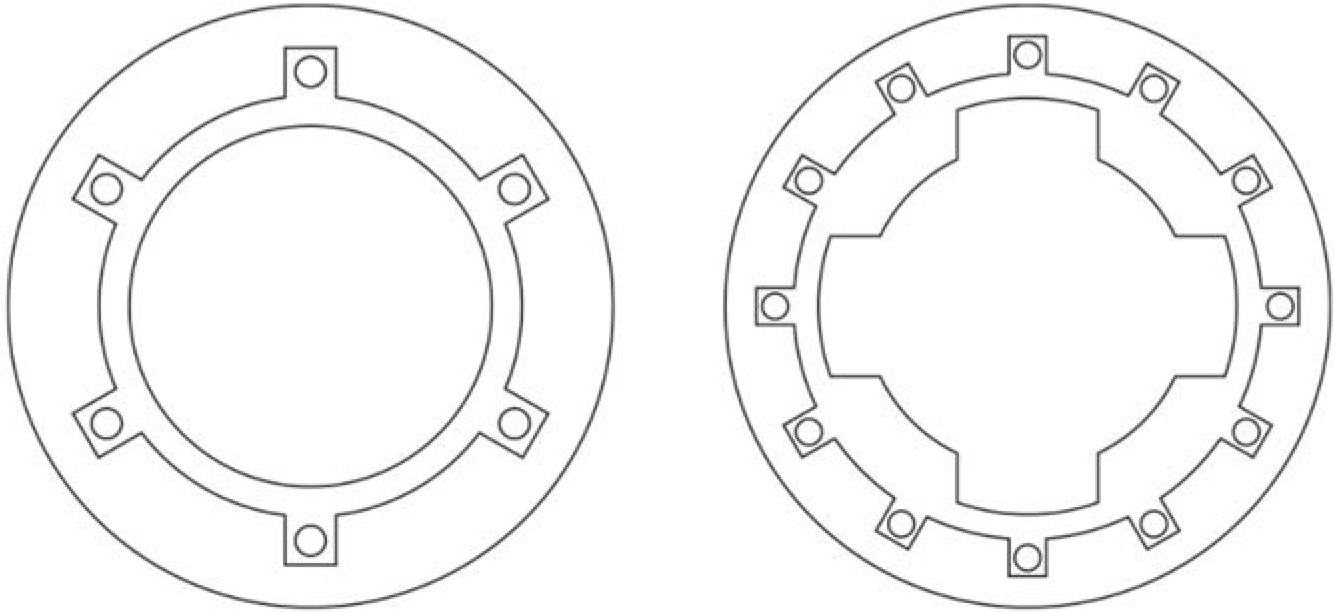


Schematic diagram of a two-pole, three-phase synchronous machine
(a) cylindrical-rotor without damper bars
(b) Salient pole rotor with damper bars



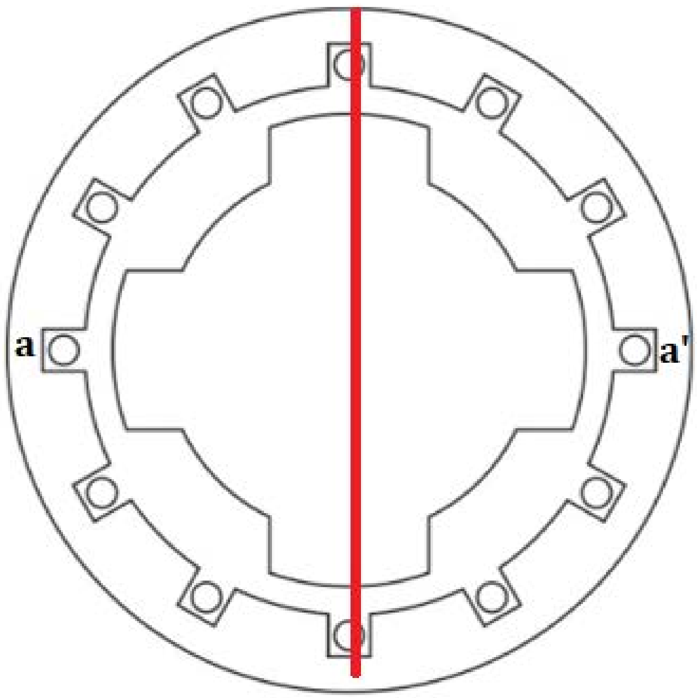
A simplified diagram of a salient two-pole machine showing amortisseur windings.

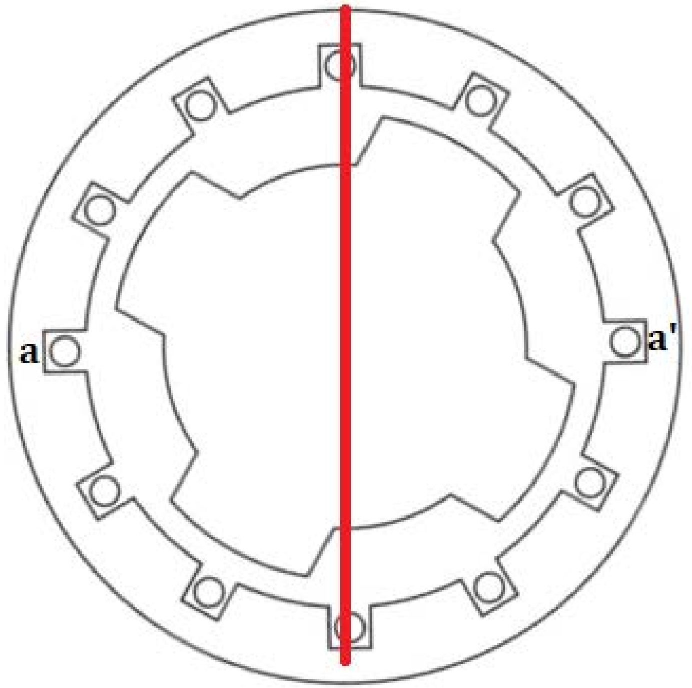
A rotor field pole for a synchronous machine showing amortisseur windings in the pole face.



(a) (b)

(a) An ac machine with a cylindrical or nonsalient-pole rotor.
rotor.
(b) An ac machine with a salient-pole


a
a'


a
a'

Synchronous Machine State Space Modeling Approach

- The differential equations used to model the dynamic performance of electrical machines in general and three-phase machines are derived from the interaction between the armature windings, field, and damping circuits.

- For each of these machines we can write an equation in terms of flux linkages. Hence, a general expression of the following form can be used to express the terminal voltage of winding j :

$$v_j = r_j i_j + \frac{d}{dt}\{\lambda_j\}$$

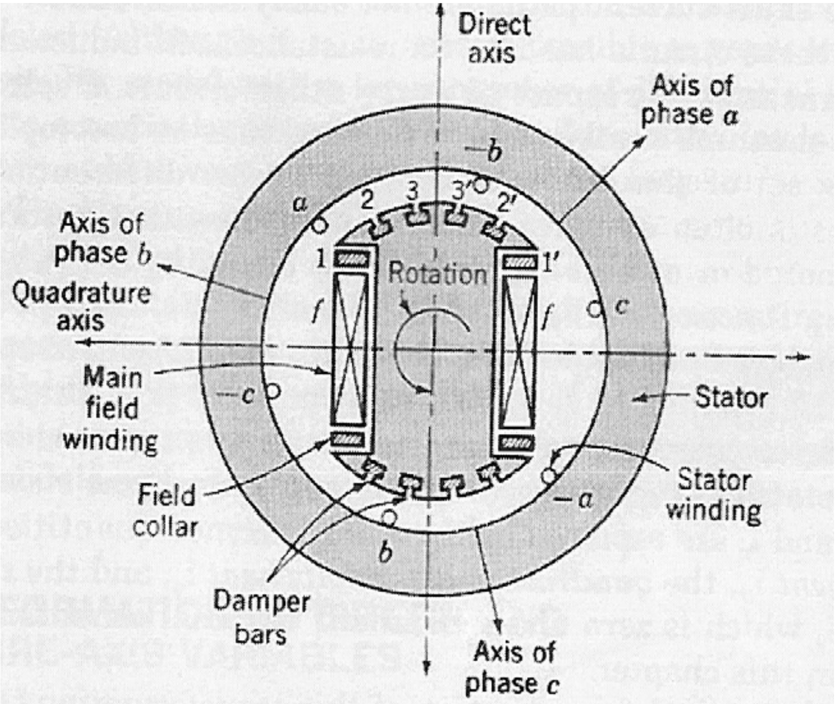
  where, r_j is the winding Ohmic resistance, i_j is the winding instantaneous current, and λ_j is the flux linkage of the j^{th} winding.

- The flux linkage of a coil, j, can be expressed in terms of the coil self inductance, its current, as well as the mutual inductance and currents associated with all coupled windings.

Synchronous Machine Inductances

For the 2-pole synchronous generator shown, the machine is represented by six windings:

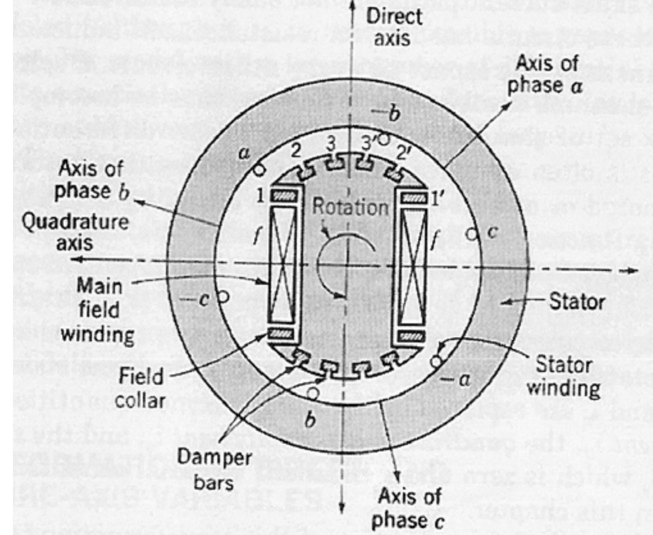
- the windings **a**, **b** and **c** represent the armature three phase a, b and c, respectively,
- the winding **f** represents the rotor field winding, and
- the equivalent windings $\mathbf{k_d}$ and $\mathbf{k_q}$ represent the squirrel cage type damper bars which are embedded in the pole faces of the rotor structure.



Consider a synchronous generator with the following windings:

- Three armature phases: a, b, c
- Field winding: f
- Equivalent damper bars windings: kd and kq

The machine winding inductances can be defined as follows:



The stator inductances:

- Stator Self Inductances

$$L_{aa} = L_{aa0} + L_{al} + L_g \cos(2\theta)$$

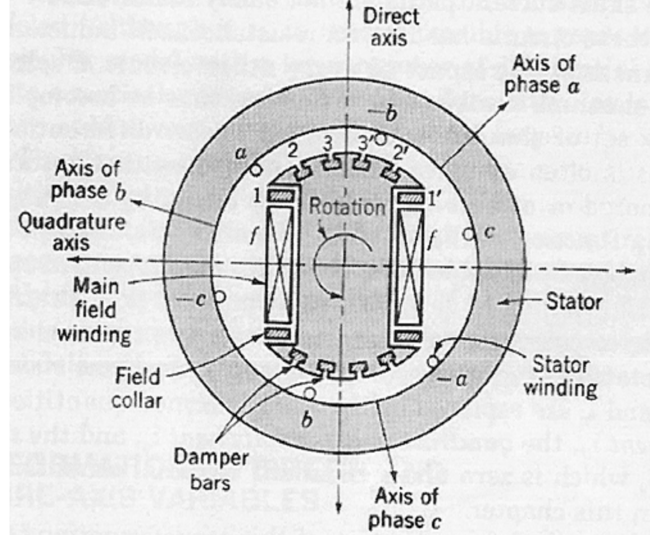
Here L_{aa0} is the component of the self inductance due to the space fundamental of the air-gap flux and L_{al} is the component due to armature leakage flux. Also, θ is the rotor angle.

Similar expressions apply to L_{bb}, L_{cc}

$$L_{bb} = L_{aa0} + L_{al} + L_g \cos(2\theta + \frac{2\pi}{3})$$

$$L_{cc} = L_{aa0} + L_{al} + L_g \cos(2\theta - \frac{2\pi}{3})$$

- Stator Mutual Inductances



The armature phase-to-phase mutual inductances:

$$L_{ab} = L_{ba} = -\frac{L_{aa0}}{2} + L_g \cos(2\theta - \frac{2\pi}{3})$$

$$L_{bc} = L_{cb} = -\frac{L_{aa0}}{2} + L_g \cos(2\theta)$$

$$L_{ac} = L_{ca} = -\frac{L_{aa0}}{2} + L_g \cos(2\theta + \frac{2\pi}{3})$$

Again, L_{aa0} is the inductance due to the space fundamental of the air-gap flux. The -1/2 is because the armature phases are displaced by $2\pi/3$ and $\cos(2\pi/3) = \cos(-2\pi/3) = -1/2$

- The Rotor Inductances

The field self inductance

$L_{ff} = L_{ff0} + L_{fl}$

Here L_{ff0} is the component of the self inductance due to the space fundamental of the air-gap flux and L_{fl} is the component due to field winding leakage flux.

The rotor field to stator phase winding mutual inductance

$$L_{af} = L_{fa} = L_{afm} \cos(\theta)$$

Similar expressions apply to phases b and c, that is L_{bf}, L_{fb} and, $L_{cf,}$ L_{fc}, with (θ) replaced by $(\theta-2\pi/3)$ and $(\theta+2\pi/3)$, respectively. That is

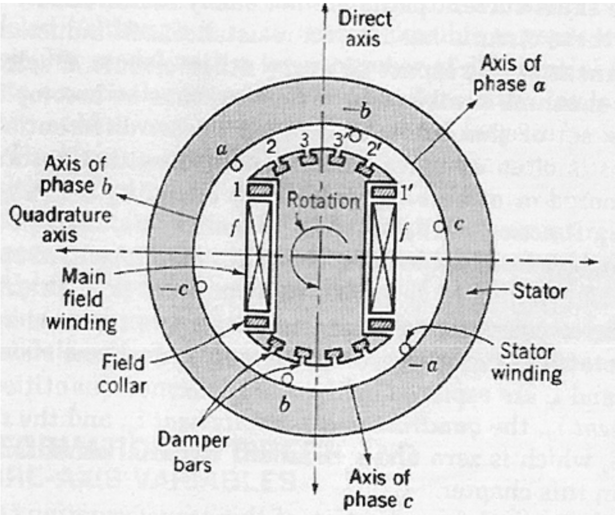
$$L_{bf} = L_{fb} = L_{afm} \cos(\theta - \frac{2\pi}{3})$$

$$L_{cf} = L_{fc} = L_{afm} \cos(\theta + \frac{2\pi}{3})$$

The rotor damper bars are represented by two windings, kd and kq. L_{kdkd} and L_{kqkq} are the damper bars equivalent self inductances:

$L_{kdkd}=L_{kdkd}=$ constant

$L_{kqkq}=L_{kqkq}=$ constant



The stator to damper bars equivalent winding mutual inductances are given as:

Mutuals with kd

$$L_{akd} = L_{kda} = L_{akdm}\cos(\theta)$$

Similar expressions apply to phases b and c, that is L_{bkd}, L_{kdb} and, L_{ckd}, L_{kdc}, with (θ) replaced by (θ-2π/3) and (θ+2π/3), respectively.

$$L_{bkd} = L_{kdb} = L_{akdm}\cos(\theta - \frac{2\pi}{3})$$

$$L_{ckd} = L_{kdc} = L_{akdm}\cos(\theta + \frac{2\pi}{3})$$

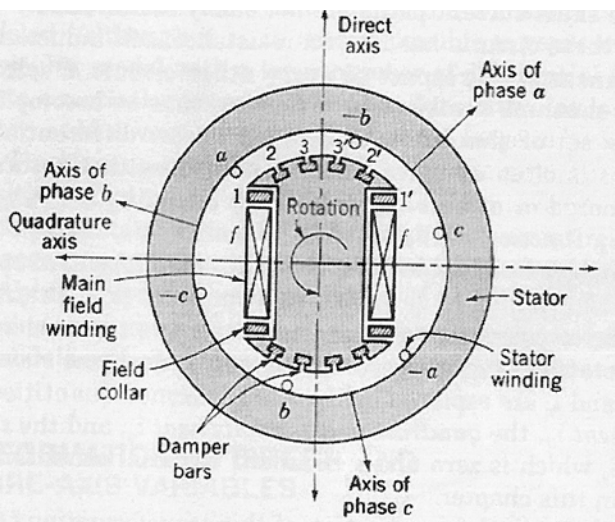
Mutuals with kq

The equivalent winding kq has similar expressions to those of kd except for a phase shift of $\pi/2$. That is

$$L_{akq} = L_{kqa} = L_{akqm}\cos(\theta + \frac{\pi}{2})$$

$$L_{bkq} = L_{kqb} = L_{akqm}\cos(\theta + \frac{\pi}{2} - \frac{2\pi}{3})$$

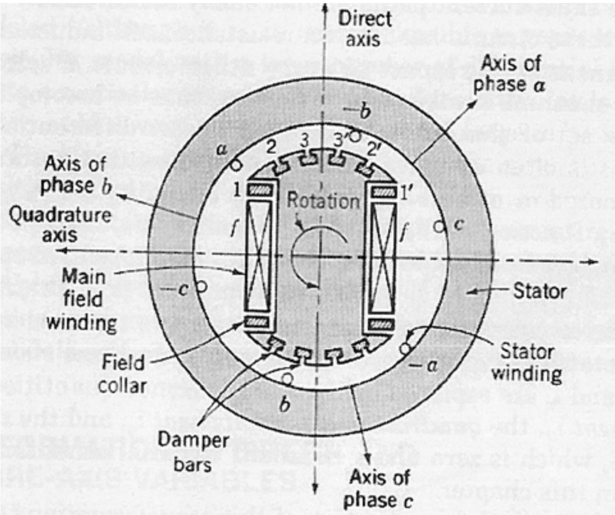
$$L_{ckq} = L_{kqc} = L_{akqm}\cos(\theta + \frac{\pi}{2} + \frac{2\pi}{3})$$



As for the rotor to rotor mutual inductances:
$L_{fkd} = L_{kdf}$ = Constant
$L_{fkq} = L_{kqf} = L_{kdkq} = L_{kqkd} = 0$ Due to the $\pi/2$ displacement between them.

The flux linkages can be expressed in terms of the inductances and currents as follows:

$$\lambda_a = L_{aa}i_a + L_{ab}i_b + L_{ac}i_c + L_{af}i_f + L_{akd}i_{kd} + L_{akq}i_{kq}$$
$$\lambda_b = L_{ba}i_a + L_{bb}i_b + L_{bc}i_c + L_{bf}i_f + L_{bkd}i_{kd} + L_{bkq}i_{kq}$$
$$\lambda_c = L_{ca}i_a + L_{cb}i_b + L_{cc}i_c + L_{cf}i_f + L_{ckd}i_{kd} + L_{ckq}i_{kq}$$
$$\lambda_f = L_{fa}i_a + L_{fb}i_b + L_{fc}i_c + L_{ff}i_f + L_{fkd}i_{kd} + L_{fkq}i_{kq}$$
$$\lambda_{kd} = L_{kda}i_a + L_{kdb}i_b + L_{kdc}i_c + L_{kdf}i_f + L_{kdkd}i_{kd} + L_{kqkq}i_{kq}$$
$$\lambda_{kq} = L_{kqa}i_a + L_{kqb}i_b + L_{kqc}i_c + L_{kqf}i_f + L_{kqkd}i_{kd} + L_{kqkq}i_{kq}$$



State Space Equations

- A general expression to express the terminal voltage of winding j :

  $$v_j = r_j i_j + \frac{d}{dt}\{\lambda_j\}$$

  where, r_j is the winding Ohmic resistance, i_j is the winding instantaneous current, and λ_j is the flux linkage of the j^{th} winding

- In the above equation, the flux linkage in the coil j is related to the currents in the coupled coils through the self and mutual inductances as follows:

  $$\lambda_j = \sum_{k=1}^{n} L_{jk}\, i_k$$

  where the subscript j denotes the coils of interest, n is the number of coils, and the subscript k ranges from 1 to n. In this case, n=6 and it covers the armature phases a, b, and c, the field winding f, and the equivalent damper windings kd and kq.

- The SS model stated above can be transformed using compact matrix notation as follows:

$\underline{V} = \underline{RI} + \frac{d}{dt}\{\underline{\Lambda}\}$ where $\underline{\Lambda} = \underline{LI}$ thus resulting in:

$$\underline{V} = \underline{RI} + \frac{d}{dt}\{\underline{LI}\}$$

- Accordingly, the synchronous generator with three armature phases a, b, c, the field winding f, and rotor equivalent damping circuits k_d and k_q can be represented by a 6[th] order SS model as follows:

$$\begin{bmatrix} v_a \\ v_b \\ v_c \\ v_f \\ v_{kd} \\ v_{kq} \end{bmatrix} = \begin{bmatrix} r_s & 0 & 0 & 0 & 0 & 0 \\ 0 & r_s & 0 & 0 & 0 & 0 \\ 0 & 0 & r_s & 0 & 0 & 0 \\ 0 & 0 & 0 & r_f & 0 & 0 \\ 0 & 0 & 0 & 0 & r_{kd} & 0 \\ 0 & 0 & 0 & 0 & 0 & r_{kq} \end{bmatrix} \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_f \\ i_{kd} \\ i_{kq} \end{bmatrix}$$

$$+ \frac{d}{dt}\left\{ \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} & L_{af} & L_{akd} & L_{akq} \\ L_{ba} & L_{bb} & L_{bc} & L_{bf} & L_{bkd} & L_{bkq} \\ L_{ca} & L_{cb} & L_{cc} & L_{cf} & L_{ckd} & L_{ckq} \\ L_{fa} & L_{fb} & L_{fc} & L_{ff} & L_{fkd} & L_{fkq} \\ L_{kda} & L_{kdb} & L_{kdc} & L_{kdf} & L_{kdkd} & L_{kdkq} \\ L_{kqa} & L_{kqb} & L_{kqc} & L_{kqf} & L_{kqkd} & L_{kqkq} \end{bmatrix} \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_f \\ i_{kd} \\ i_{kq} \end{bmatrix} \right\}$$

In the above equation, the currents are used as state variables. The last term in the above equation can be expressed as follows:

$$\frac{d}{dt}\{\underline{LI}\} = \left\{\frac{d\underline{\underline{L}}}{dt}\right\}\underline{I} + \underline{L}\left\{\frac{d\underline{I}}{dt}\right\}$$

For the unsaturated and slightly saturated machines, the above equation can be expanded as:

$$\frac{d}{dt}\{\underline{LI}\} = \omega\left\{\frac{d\underline{\underline{L}}}{d\theta}\right\}\underline{I} + \underline{L}\left\{\frac{d\underline{I}}{dt}\right\}$$

where θ is the electrical rotor position angle, measured from a fixed reference, and ω is the angular speed in electrical rad/s.

Combining the above equations, results the following SS model:

$$\underline{V} = \underline{RI} + \omega\left\{\frac{d\underline{L}}{d\theta}\right\}\underline{I} + \underline{L}\left\{\frac{d\underline{I}}{dt}\right\}$$

- The array $\underline{I}$ represents the current of the six windings.
- The array $\underline{V}$ represents the terminal voltages of the six windings with $v_{kd} = 0$ and $v_{kq} = 0$ as they represent the terminal voltages of the equivalent shorted windings representing the damper bars.
- The diagonal matrix $\underline{R}$ represents the resistances of the machine equivalent windings, including the damper bars equivalent windings, and
- The matrix $\underline{L}$ represents the machine self and mutual inductances.
- The values of the currents can be determined numerically for any set of initial conditions and terminal voltages, and .

In addition, it should be noted that the circuits representing the three-phase loads are linked to the machine model through the terminal voltages. In the case of a three-phase Y-connected RL load, the armature terminal voltages can be expressed as:

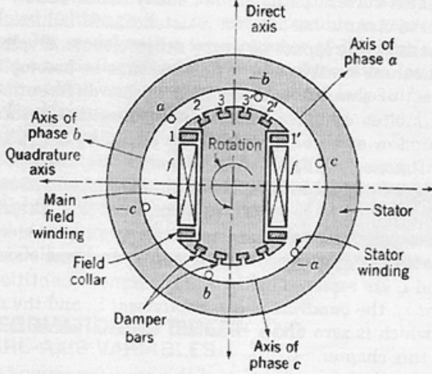
$$v_a = -r_l i_a - L_l \frac{di_a}{dt} \qquad v_b = -r_l i_b - L_l \frac{di_b}{dt} \quad \text{and} \quad v_c = -r_l i_c - L_l \frac{di_c}{dt}$$

where r_l and L_l are the load per phase Ohmic resistance and inductance, respectively. Using the equations above, the SS model for a single generator feeding a three-phase Y connected RL load can be written as follows:

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ v_f \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_s + r_l & 0 & 0 & 0 & 0 & 0 \\ 0 & r_s + r_l & 0 & 0 & 0 & 0 \\ 0 & 0 & r_s + r_l & 0 & 0 & 0 \\ 0 & 0 & 0 & r_f & 0 & 0 \\ 0 & 0 & 0 & 0 & r_{kd} & 0 \\ 0 & 0 & 0 & 0 & 0 & r_{kq} \end{bmatrix} \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_f \\ i_{kd} \\ i_{kq} \end{bmatrix}$$

$$+ \frac{d}{dt} \left\{ \begin{bmatrix} L_{aa} + L_l & L_{ab} & L_{ac} & L_{af} & L_{akd} & L_{akq} \\ L_{ba} & L_{bb} + L_l & L_{bc} & L_{bf} & L_{bkd} & L_{bkq} \\ L_{ca} & L_{cb} & L_{cc} + L_l & L_{cf} & L_{ckd} & L_{ckq} \\ L_{fa} & L_{fb} & L_{fc} & L_{ff} & L_{fkd} & L_{fkq} \\ L_{kda} & L_{kdb} & L_{kdc} & L_{kdf} & L_{kdkd} & L_{kdkq} \\ L_{kqa} & L_{kqb} & L_{kqc} & L_{kqf} & L_{kqkd} & L_{kqkq} \end{bmatrix} \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_f \\ i_{kd} \\ i_{kq} \end{bmatrix} \right\}$$

Synchronous Machine Inductance Expressions



Consider the synchronous machine shown, which has:

- ✓ Three armature phases: a, b, c
- ✓ Field winding: f
- ✓ Damper bars equivalent circuits: kd and kq

The flux linkages are expressed in terms of inductances and currents as:

$$\lambda_a = L_{aa}i_a + L_{ab}i_b + L_{ac}i_c + L_{af}i_f + L_{akd}i_{kd} + L_{akq}i_{kq}$$
$$\lambda_b = L_{ba}i_a + L_{bb}i_b + L_{bc}i_c + L_{bf}i_f + L_{bkd}i_{kd} + L_{bkq}i_{kq}$$
$$\lambda_c = L_{ca}i_a + L_{cb}i_b + L_{cc}i_c + L_{cf}i_f + L_{ckd}i_{kd} + L_{ckq}i_{kq}$$
$$\lambda_f = L_{fa}i_a + L_{fb}i_b + L_{fc}i_c + L_{ff}i_f + L_{fkd}i_{kd} + L_{fkq}i_{kq}$$
$$\lambda_{kd} = L_{kda}i_a + L_{kdb}i_b + L_{kdc}i_c + L_{kdf}i_f + L_{kdkd}i_{kd} + L_{kqkq}i_{kq}$$
$$\lambda_{kq} = L_{kqa}i_a + L_{kqb}i_b + L_{kqc}i_c + L_{kqf}i_f + L_{kqkd}i_{kd} + L_{kqkq}i_{kq}$$

Where:

- ✓ L_{aa}, L_{bb} and L_{cc} are the Stator Self Inductances
- ✓ L_{ff} is the Rotor Self Inductance
- ✓ L_{kd} and L_{kq} are the Damper Self Inductances
- ✓ L_{ab}, L_{ac}, L_{ba}, L_{bc}, L_{ca} and L_{cb} are the Stator-to-Stator Mutual Inductances
- ✓ L_{af}, L_{bf}, L_{cf} and L_{fa}, L_{fb}, L_{fc} are the Stator and Rotor Mutual Inductances
- ✓ L_{akd}, L_{akq}, L_{bkd}, L_{bkq}, L_{ckd}, L_{ckq} and L_{kda}, L_{kqa}, L_{kdb}, L_{kqb}, L_{kdc}, L_{kqc} are the Stator-to-Damper Bars Mutual Inductances
- ✓ L_{fkd}, L_{fkq} and L_{kdf}, L_{kqf} are the Rotor-to-Damper/Damper-to-Rotor Mutual Inductances, respectively

Typically, the inductances for a 4-pole machine have the following relationships:

$$L_{aa} = L_{sa} + L_{sv}\cos(2\theta)$$

$$L_{bb} = L_{sa} + L_{sv}\cos(2\theta - \frac{4\pi}{3})$$

$$L_{cc} = L_{sa} + L_{sv}\cos(2\theta - \frac{2\pi}{3})$$

$$L_{ab} = L_{ba} = -L_{ma} + L_{mv}\cos(2\theta - \frac{2\pi}{3})$$

$$L_{cb} = L_{bc} = -L_{ma} + L_{mv}\cos(2\theta)$$

$$L_{ac} = L_{ca} = -L_{ma} + L_{mv}\cos(2\theta - \frac{4\pi}{3})$$

$$L_{af} = L_{fa} = L_{afm}\cos(2\theta)$$

$$L_{bf} = L_{fb} = L_{afm}\cos(2\theta - \frac{2\pi}{3})$$

$$L_{cf} = L_{fc} = L_{afm}\cos(2\theta - \frac{4\pi}{3})$$

where θ is the rotor angle.

$$L_{akd} = L_{kda} = L_{akdm}\cos(2\theta)$$

$$L_{bkd} = L_{kdb} = L_{akdm}\cos(2\theta - \frac{2\pi}{3})$$

$$L_{ckd} = L_{kdc} = L_{akdm}\cos(2\theta - \frac{4\pi}{3})$$

$$L_{akq} = L_{kqa} = -L_{akqm}\cos(2\theta)$$

$$L_{bkq} = L_{kqb} = -L_{akqm}\cos(2\theta - \frac{2\pi}{3})$$

$$L_{ckq} = L_{kqc} = -L_{akqm}\cos(2\theta - \frac{4\pi}{3})$$

- **A Case Study**

- Consider the synchronous generator shown in Fig. 3. It is a three-phase, field wound, 4-pole *ac* generator with brushless exciter, rated at 90 kVA, 208 V, and 400 Hz.
- The main field coil of the machine has four windings connected in series that form four poles on the rotor structure.
- The rotor assembly includes a damping circuit made from squirrel cage type bars located on the rotor pole surfaces. There are five rotor dampers per pole face (20 rotor bars total).
- The stator houses the generator three-phase armature winding, which is distributed in 48 stator slots as shown in *Figure 3*.

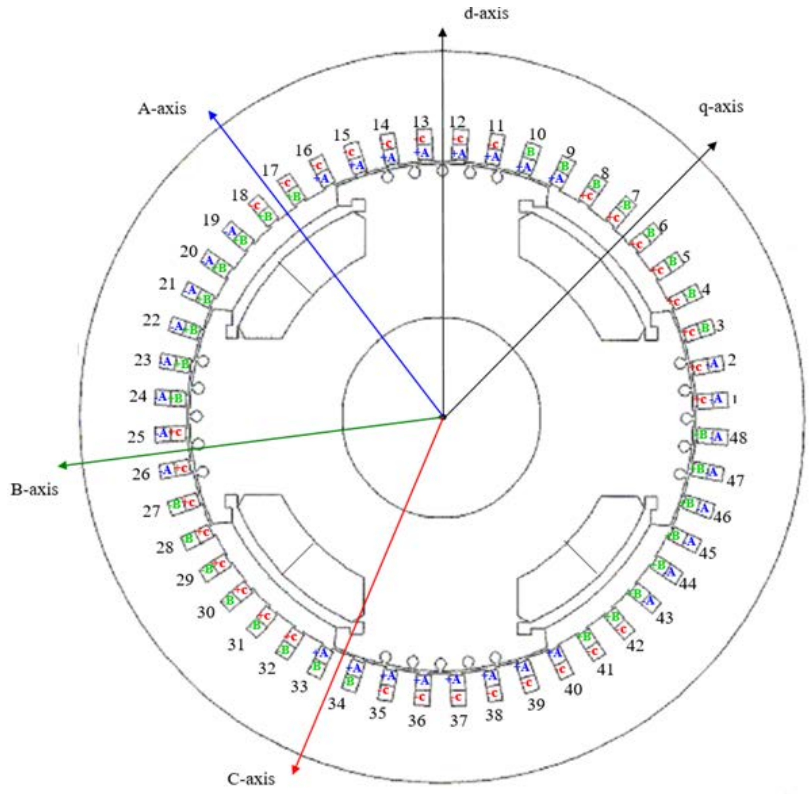


Fig. 3: Cross Section of a 4-pole distributed windings Synchronous Generator

- **A Case Study (Continued)**
- Figure 3 also shows the conductor distribution of the three phases where the phase windings are indicated with *A, B* and *C*, as well as (+) and (-) symbols which represent positive and negative current directions, respectively.
- The current distribution of the phase windings as well as the associated mmf distribution of the three phases is depicted in Figure 4, where the mmf waveforms of the three phases are shifted by 120° relative to each other and each waveform is closer to a sinusoidal waveform.

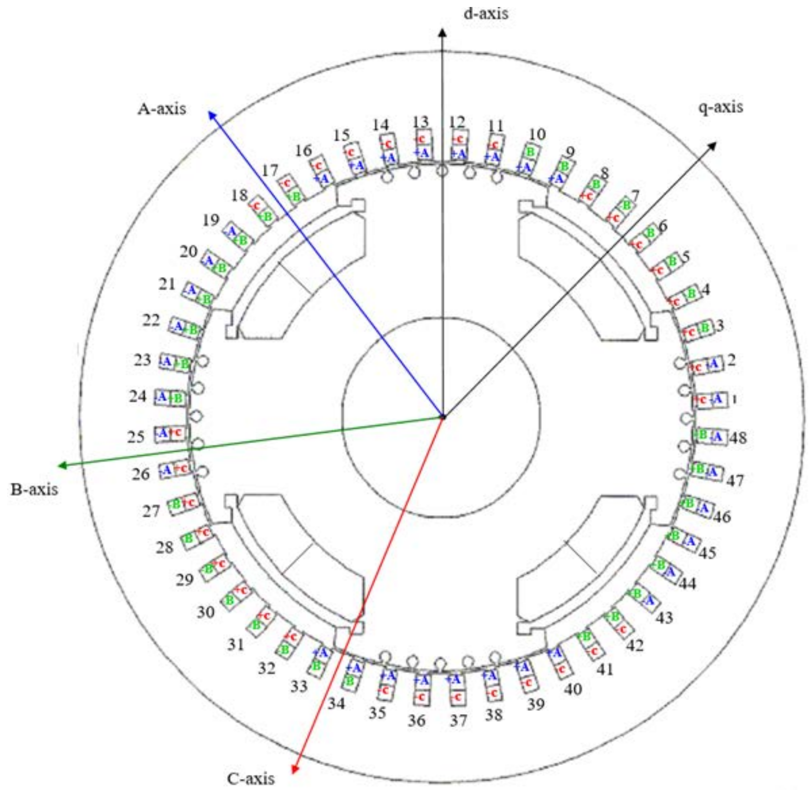


Fig. 3: Cross Section of a 4-pole distributed windings Synchronous Generator

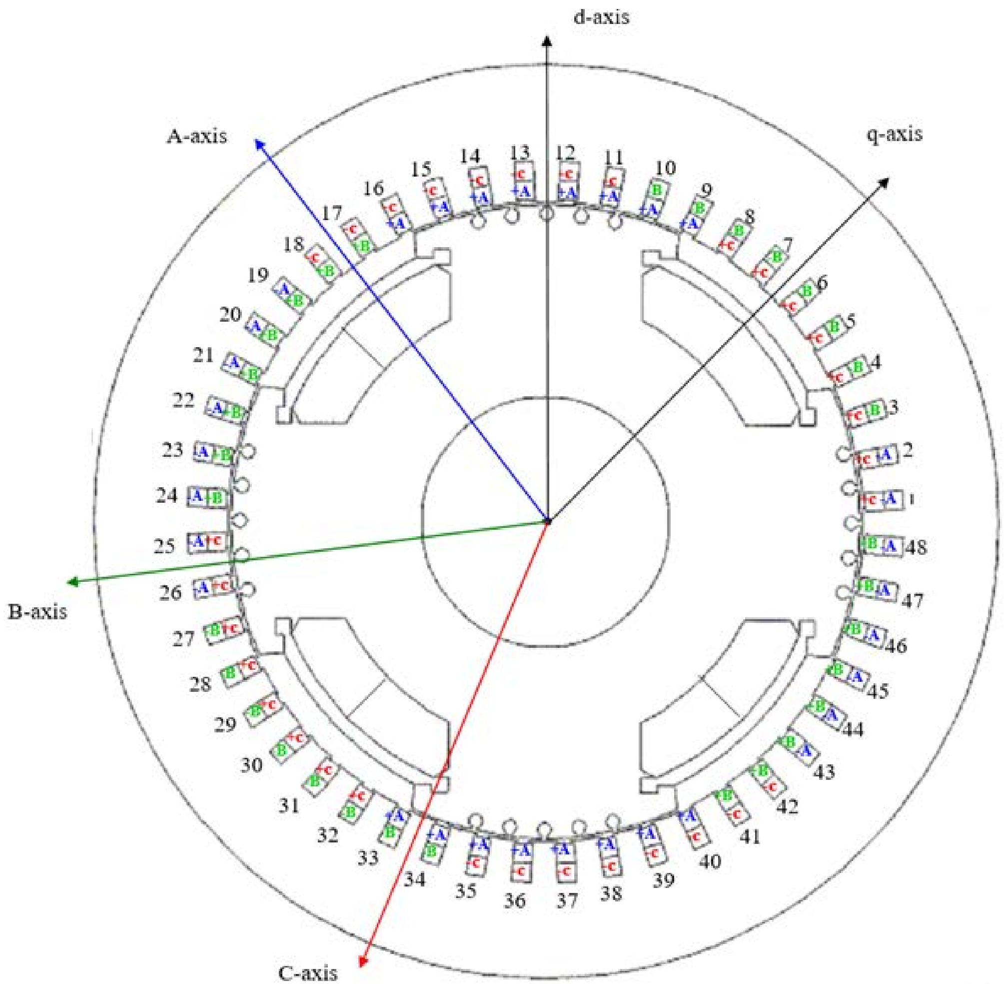


Fig. 3: Cross Section of a 4-pole distributed windings Synchronous Generator

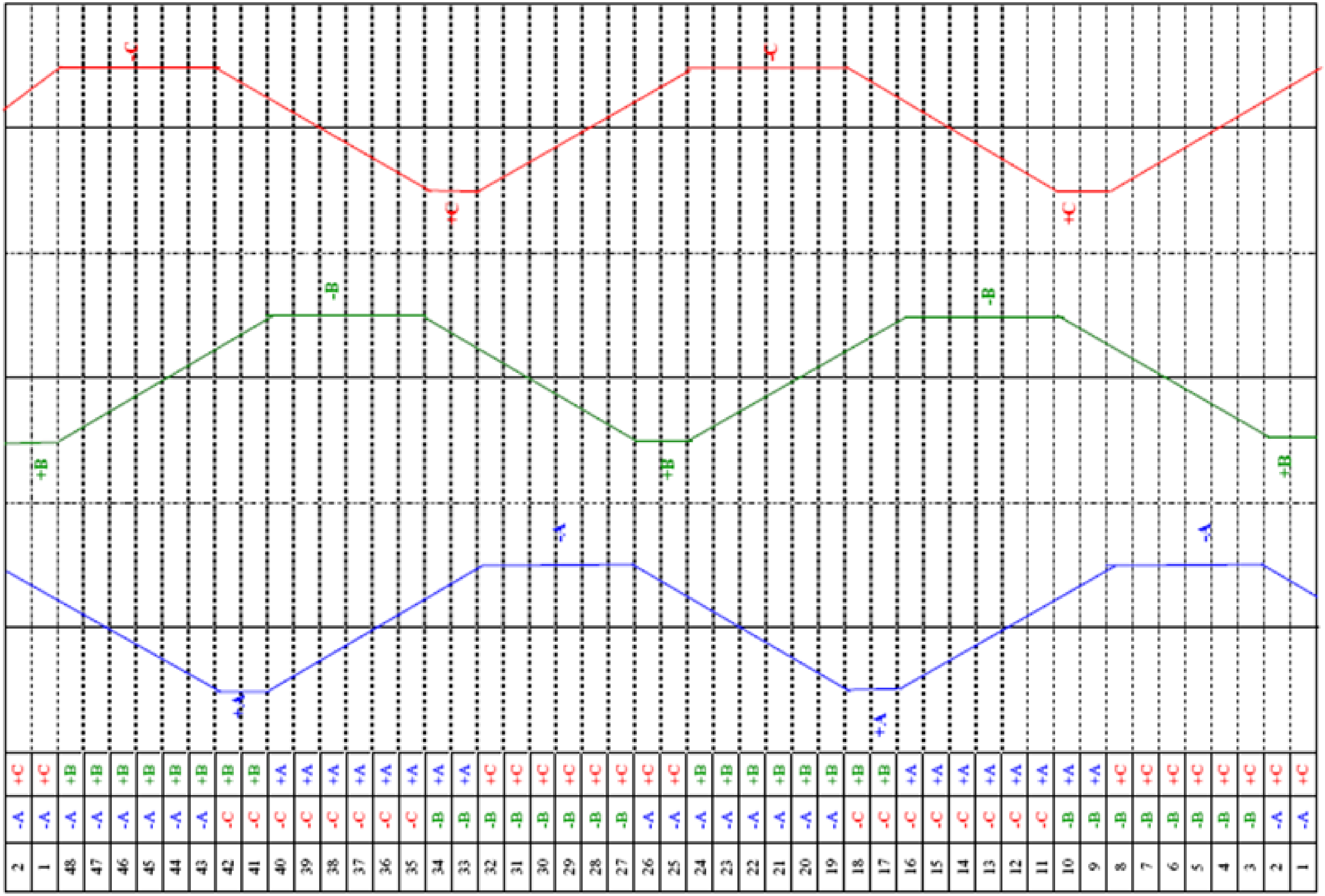


Fig. 4: Synchronous Generator Current and MMF Distributions

Inductance Family of Curves Approach

- The machine winding self and mutual inductances represent the main parameters of the state space, SS model.
- Furthermore, inductances are functions of the rotor position, as well as the load current, due to the magnetic material nonlinearity and saturation effects.
- In order to account for these effects, as well as the machine complex geometry and conductor distribution layout, a coupled Finite Element-State Space, FE-SS, model approach which makes use of the Family of Curves, FC, technique can be implemented to predict the performance characteristics of a machine.
- A flow chart describing this approach is given in *Fig. 5*.

- Generally, this approach involves computing sets of inductances corresponding to a range of load conditions, usually represented by a range of currents (spanning the range from no-load condition to full load condition, in increments.
- The range of excitation currents is chosen to cover all possible operating conditions.
- For each rotor position, the field solutions corresponding to the range of the excitation current are computed.

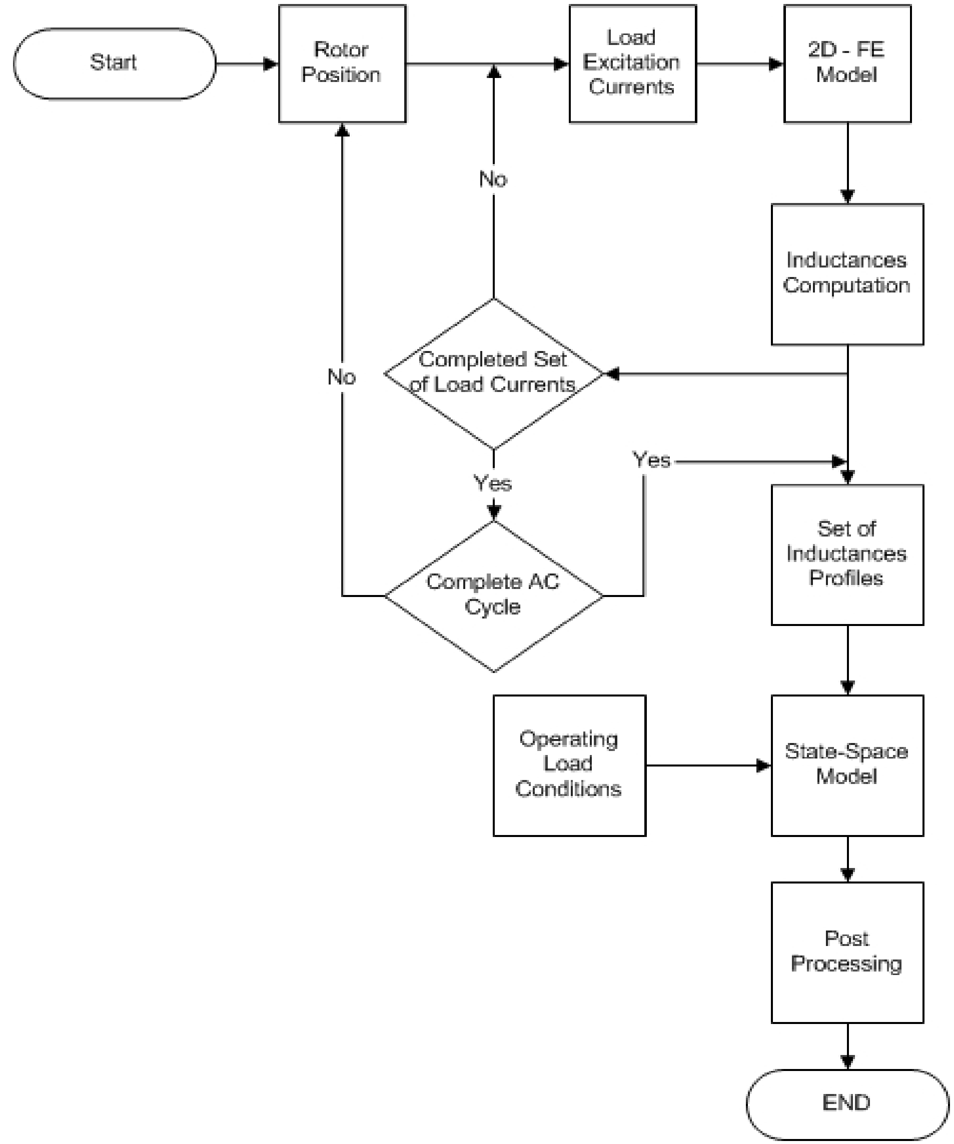


Fig. 5: Flow Chart of Family of Curves Approach

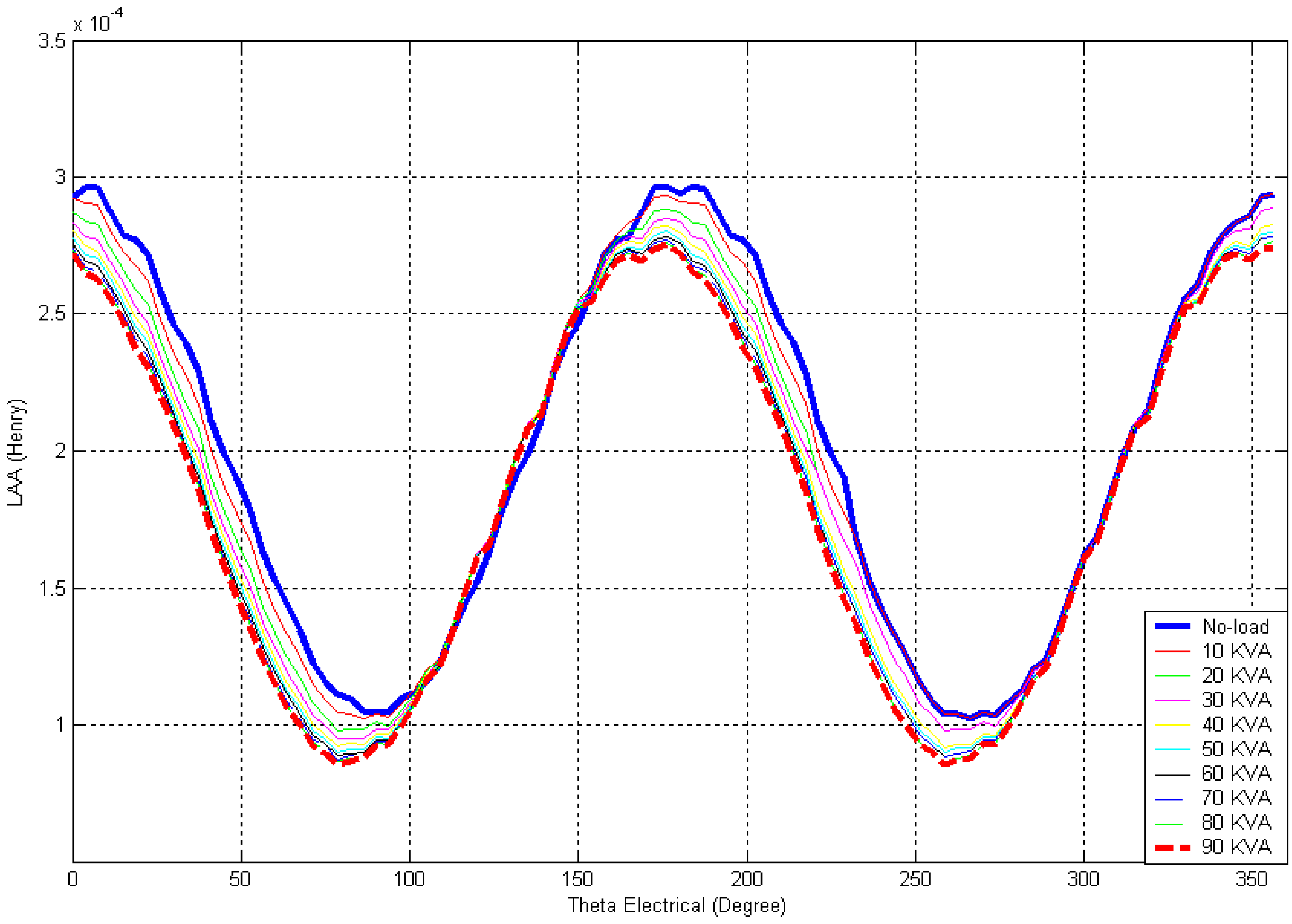


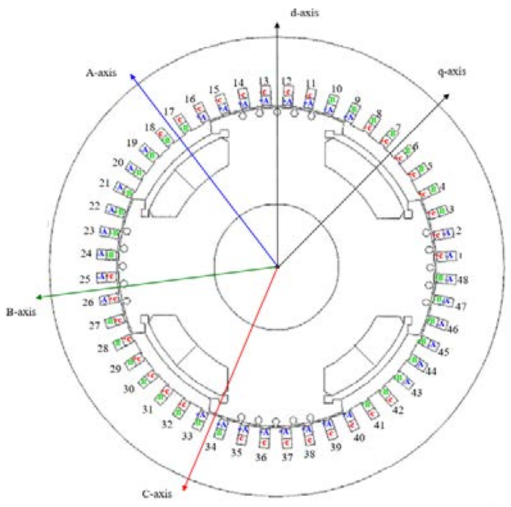
Fig. 6 Family of Curves Representing L_{aa}

Case Study Details

Consider a three-phase, field wound, 4-pole *ac* synchronous generator with brushless exciter, rated at 90 kVA, 208 V, and 400 Hz, whose cross-section is shown.

For full details on the application of the state space model developed above, see the following paper:

Arkadan, A.A., Abou-Samra, Y. and Al-Aawar, N., "Characterization of Stand Alone AC Generators during No-Break Power Transfer using Radial Basis Networks," *IEEE Trans. on Magnetics,* Vol. 43, No. 12, pp. 1821-1824, April 2007.



Cross-section of a 4-pole distributed windings Synchronous Generator