EEENG577

W6-M5 *PM Synchronous Machine Formulations*

INTRODUCTION

In this report, a permanent magnet synchronous machine "PMSM" is developed. First a cross sectional drawing of the machine is represented along with the schematic diagram that shows the windings of such machine. Then, writing the voltage expression for this "PMSM" and corresponding State Space Models using lecture notes [1].

MACHINE'S PARAMETERS

A 6-pole, three-phase magnet type "synchronous" machine has a phase self-inductance of 150 µH and a phase-to-phase mutual inductance of 15 µH. At an electrical angular speed of 1,337 rad/sec, the rotor radially mounted permanent magnets induced the following back emf's in the a, b, and c phases of the stationary armature.

$e_a = E_m \cos(\omega t - 0.46)$ Volts
$e_b = E_m \cos(\omega t - 0.46 - 2\pi/3)$ Volts
$e_c = E_m \cos(\omega t - 0.46 - 4\pi/3)$ Volts
where, $E_m = 63$ Volts.

The three armature phases are Y-connected with an isolated neutral, that is ia+ib+ic=0, and has a per-phase resistance of 9.4 mΩ

MACHINE'S CROSS SECTION AND SCHEMATIC DIAGRAM

The cross-section of the permanent magnet synchronous machine and the corresponding winding schematic diagram are given in Figures 1 and 2, respectively. The machine under consideration does not include any damping circuits. However, Fig. 2 show two sets of damping circuits that are added for illustration.

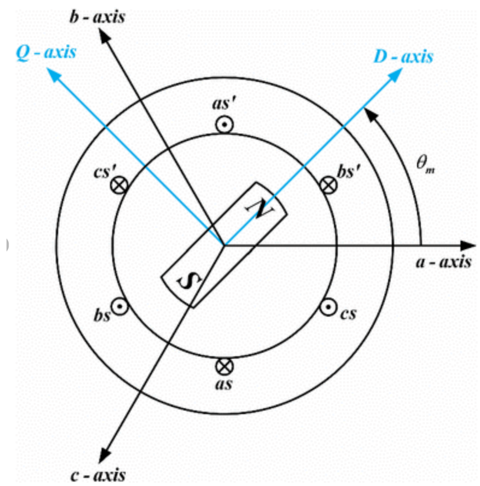

Fig. 1: Two-pole, 3-phase, Salient Pole PM Synchronous Machine Cross-section [2]

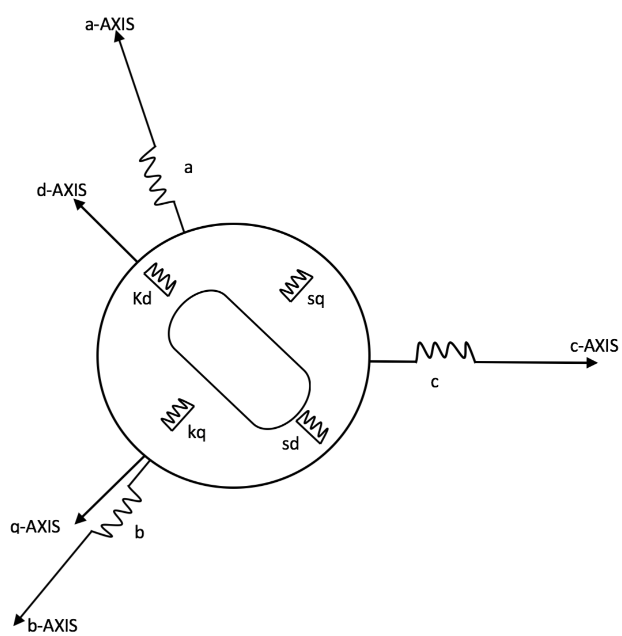

Fig. 2: The winding representation in the synchronous machine [3]

FORMULATIONS

A- Voltage expression:

The voltage expression for the permanent magnet synchronous machine is as follows [1,3]:

$$\underline{V}_{abc} = \underline{R}.\underline{I}_{abc} + \underline{L}.\underline{\dot{I}}_{abc} + \underline{E}_{abc} \tag{1}$$

Equation (1) can be expanded in matrix form in the following equation:

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} + \begin{bmatrix} L_{sa} & L_{ma} & L_{ma} \\ L_{ma} & L_{sa} & L_{ma} \\ L_{ma} & L_{ma} & L_{sa} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} + E_m \begin{bmatrix} \cos(\omega t - 0.46) \\ \cos(\omega t - 0.46 - \tfrac{2}{3}\pi) \\ \cos(\omega t - 0.46 - \tfrac{4}{3}\pi) \end{bmatrix} \tag{2}$$

B- State-Space Model

C- The state space equations that represent the machine can be expressed as follows [1,3]:

$$\dot{X} = AX + BU \tag{3}$$

Or

$$\underline{\dot{I}}_{abc} = \left(-\underline{L}^{-1}.R\right).\underline{I}_{abc} + \underline{L}^{-1}.\left(\underline{V}_{abc} - \underline{E}_{abc}\right) \tag{4}$$

$$\frac{d}{dt}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \left(-\begin{bmatrix} L_{sa} & L_{ma} & L_{ma} \\ L_{ma} & L_{sa} & L_{ma} \\ L_{ma} & L_{ma} & L_{sa} \end{bmatrix}^{-1} \cdot \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \right) \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} + \left(\begin{bmatrix} L_{sa} & L_{ma} & L_{ma} \\ L_{ma} & L_{sa} & L_{ma} \\ L_{ma} & L_{ma} & L_{sa} \end{bmatrix}^{-1} \right) \left(\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} - \begin{bmatrix} E_m \cos(\omega t - 0.46) \\ E_m \cos(\omega t - 0.46 - \tfrac{2}{3}\pi) \\ E_m \cos(\omega t - 0.46 - \tfrac{4}{3}\pi) \end{bmatrix} \right) \tag{5}$$

$$\underline{A} = -\begin{bmatrix} L_{sa} & L_{ma} & L_{ma} \\ L_{ma} & L_{sa} & L_{ma} \\ L_{ma} & L_{ma} & L_{sa} \end{bmatrix}^{-1} \cdot \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} = \begin{bmatrix} -63.8272 & 5.8025 & 5.8025 \\ 5.8025 & -63.8272 & 5.8025 \\ 5.8025 & 5.8025 & -63.8272 \end{bmatrix} \tag{6}$$

$$\underline{B} = \begin{bmatrix} L_{sa} & L_{ma} & L_{ma} \\ L_{ma} & L_{sa} & L_{ma} \\ L_{ma} & L_{ma} & L_{sa} \end{bmatrix}^{-1} = \begin{bmatrix} 6790.1 & -617.2840 & -617.2840 \\ -617.2840 & 6790.1 & -617.2840 \\ -617.2840 & -617.2840 & 6790.1 \end{bmatrix} \tag{7}$$

REFERENCES

[1] A.A. Arkadan EENG577 Class Notes, Colorado School of Mines.

[2] Xin Wang and C. Steve Suh, "Time-frequency based field oriented control of permanent magnet synchronous motors," June 2018, International Journal of Dynamics and Control6(6):1-16, DOI: 10.1007/s40435-017-0327-5

[3] A. A. Arkadan, and N. A. Demerdash, "Modeling of Transients in Permanent Magnet Generators with Multiple Damping Circuits Using the Natural abc Frame of Reference," IEEE Trans. on Energy Conversion, Vol. EC-3, No. 3, pp. 722-731, September 1988.