

Colorado School of Mines

Electrical Engineering Department

EENG577

W6-M5 PM Synchronous Machine

INTRODUCTION

In this report, a permanent magnet synchronous machine “PMSM” is developed. First a cross-sectional drawing of the machine is represented along with the schematic diagram that shows the windings of such machine. Then, writing the voltage expression for this “PMSM” and corresponding State Space Models using lecture notes [1].

MACHINE’S PARAMETERS

A 6-pole, three-phase magnet type synchronous machine has a phase self-inductance of 150 μH and a phase-to-phase mutual inductance of 15 μH . At an electrical angular speed of 1,337 rad/sec, the rotor radially mounted permanent magnets induced the following back emf’s in the a, b, and c phases of the stationary armature.

$$\begin{aligned} e_a &= E_m \cos(\omega t - 0.46) \text{ Volts} \\ e_b &= E_m \cos(\omega t - 0.46 - 2\pi/3) \text{ Volts} \\ e_c &= E_m \cos(\omega t - 0.46 - 4\pi/3) \text{ Volts} \end{aligned}$$

where, $E_m = 63$ Volts.

The three armature phases are Y-connected with an isolated neutral, that is $i_a + i_b + i_c = 0$, and has a per-phase resistance of 9.4 m Ω

MACHINE’S CROSS SECTION AND SCHEMATIC DIAGRAM

The cross-section of the permanent magnet synchronous machine and the corresponding winding schematic diagram are given in Figures 1 and 2, respectively. The machine under consideration does not include any damping circuits. However, Fig. 2 show two sets of damping circuits that are added for illustration.

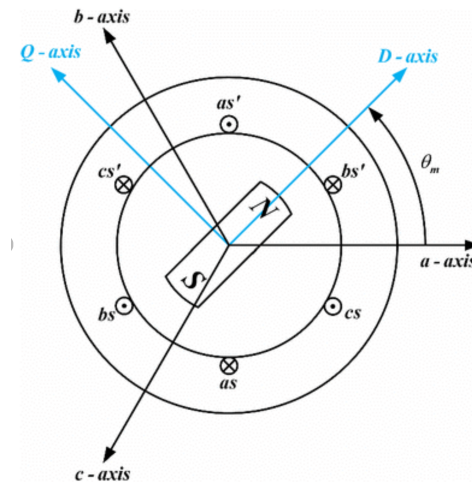


Fig. 1: Two-pole, 3-phase, Salient Pole PM Synchronous Machine Cross-section [2]

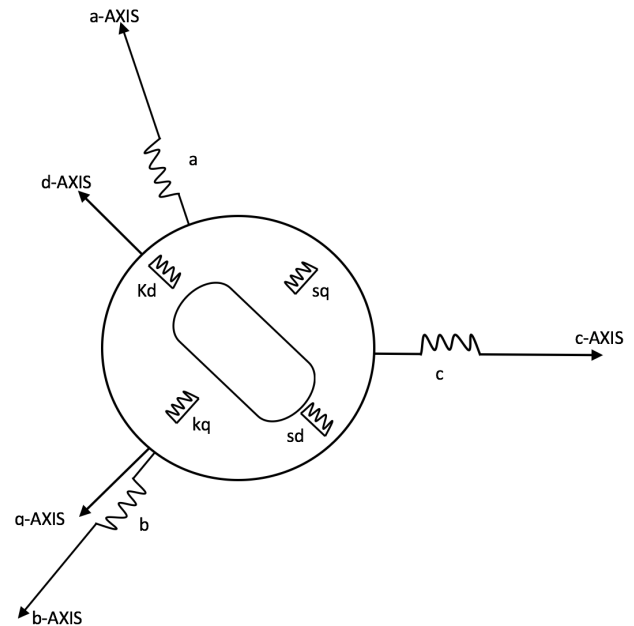


Fig. 2: The winding representation in the synchronous machine [3]

FORMULATIONS

A- Voltage expression:

The voltage expression for the permanent magnet synchronous machine is as follows [1,3]:

$$\underline{V}_{abc} = \underline{R} \cdot \underline{I}_{abc} + \underline{L} \cdot \dot{\underline{I}}_{abc} + \underline{E}_{abc} \quad (1)$$

Equation (1) can be expanded in matrix form in the following equation:

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} + \begin{bmatrix} L_{sa} & L_{ma} & L_{ma} \\ L_{ma} & L_{sa} & L_{ma} \\ L_{ma} & L_{ma} & L_{sa} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} + E_m \begin{bmatrix} \cos(\omega t - 0.46) \\ \cos(\omega t - 0.46 - \frac{2}{3}\pi) \\ \cos(\omega t - 0.46 - \frac{4}{3}\pi) \end{bmatrix} \quad (2)$$

B- State-Space Model

The state space equation that represent the machine should be as follows [1,3]:

$$\dot{\underline{X}} = \underline{A}\underline{X} + \underline{B}\underline{U} \quad (3)$$

Or

$$\dot{\underline{I}}_{abc} = (-\underline{L}^{-1} \cdot \underline{R}) \cdot \underline{I}_{abc} + \underline{L}^{-1} \cdot (\underline{V}_{abc} - \underline{E}_{abc}) \quad (4)$$

$$\frac{d}{dt} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \left(- \begin{bmatrix} L_{sa} & L_{ma} & L_{ma} \\ L_{ma} & L_{sa} & L_{ma} \\ L_{ma} & L_{ma} & L_{sa} \end{bmatrix}^{-1} \cdot \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \right) \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} + \left(\begin{bmatrix} L_{sa} & L_{ma} & L_{ma} \\ L_{ma} & L_{sa} & L_{ma} \\ L_{ma} & L_{ma} & L_{sa} \end{bmatrix}^{-1} \right) \left(\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} - \begin{bmatrix} E_m \cos(\omega t - 0.46) \\ E_m \cos(\omega t - 0.46 - \frac{2}{3}\pi) \\ E_m \cos(\omega t - 0.46 - \frac{4}{3}\pi) \end{bmatrix} \right) \quad (5)$$

$$\underline{A} = - \begin{bmatrix} L_{sa} & L_{ma} & L_{ma} \\ L_{ma} & L_{sa} & L_{ma} \\ L_{ma} & L_{ma} & L_{sa} \end{bmatrix}^{-1} \cdot \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} = \begin{bmatrix} -63.8272 & 5.8025 & 5.8025 \\ 5.8025 & -63.8272 & 5.8025 \\ 5.8025 & 5.8025 & -63.8272 \end{bmatrix} \quad (6)$$

$$\underline{B} = \begin{bmatrix} L_{sa} & L_{ma} & L_{ma} \\ L_{ma} & L_{sa} & L_{ma} \\ L_{ma} & L_{ma} & L_{sa} \end{bmatrix}^{-1} = \begin{bmatrix} 6790.1 & -617.2840 & -617.2840 \\ -617.2840 & 6790.1 & -617.2840 \\ -617.2840 & -617.2840 & 6790.1 \end{bmatrix} \quad (7)$$

COMPUTATIONAL MODELING

A. Part 1

At time, $t=0$, the machine was connected to a three-phase voltage source, and a, b, c phase currents were all zero while the rotor was rotating at the speed of ω of 1,337 electrical rad/sec. The currents a, b, and c transient current profiles will be found after decoupling the external mechanical prime mover in addition to the torque throughout the first 20 cycles.

Using MATLAB/Simulink, A Simulink block diagram can be built based on the A and B matrix above. Ea, Eb, Ec, Va, Vb and Vc are given. The Simulink block diagram for this case is shown in figure 3.

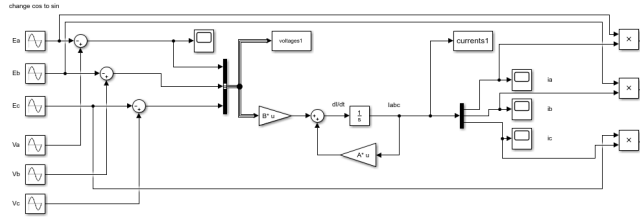


Figure 1: Simulink Block Diagram of a permanent magnet type synchronous machine

The transient currents a, b and c for the case above are shown in figures 4, 5 and 6 respectively.

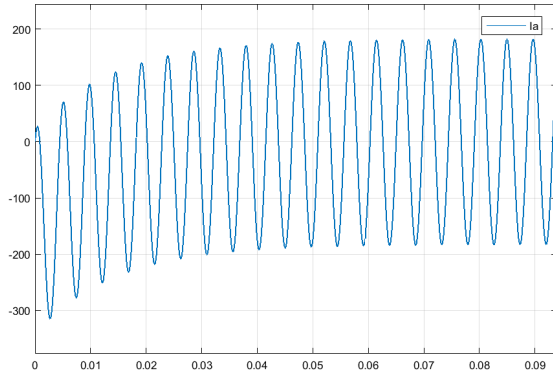


Figure 2: transient current of phase a (unit: Ampere)

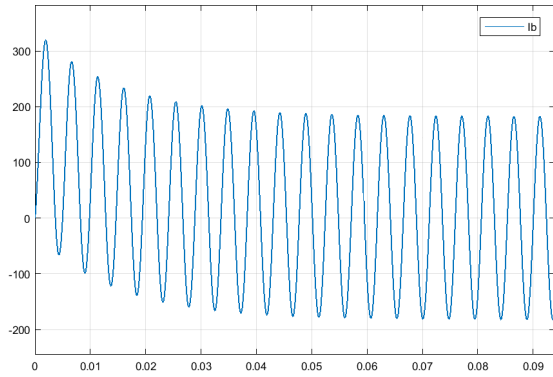


Figure 3: transient current of phase b (unit: Ampere)

Note that after decoupling the external mechanical source, the currents has different profiles due to the position of the stator winding. For a and b it has a maximum current of about 320 amp while in current c the current profile is stable.

The torque curve for this case is shown in figure 7.

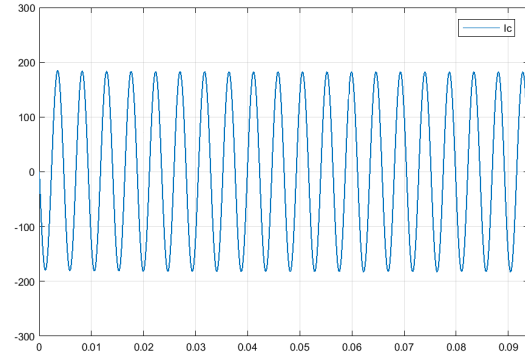


Figure 4: transient current of phase c (unit: Ampere)

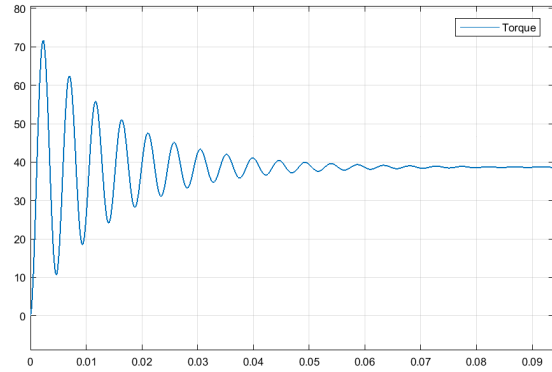


Figure 5: torque curve (unit: N.m)

Note that the starting torque is high with high oscillation in this case until it reaches to steady state.

B. Part 2

At $t=0$, with the external mechanical source running the machine at a speed $\omega=1,337$ electrical rad/sec, the three phase terminals were all shorted to the neutral point of the Y-connected armature. the a, b, and c transient current profiles are found using Simulink block in addition to the torque, and the plots throughout the transient period (20 ac cycles).

First, the Simulink block diagram is shown in figure 8. A Simulink block diagram can be built based on the A and B matrix above. Ea, Eb, and Ec are given, Va, Vb, and Vc are shorted so they become zeros.

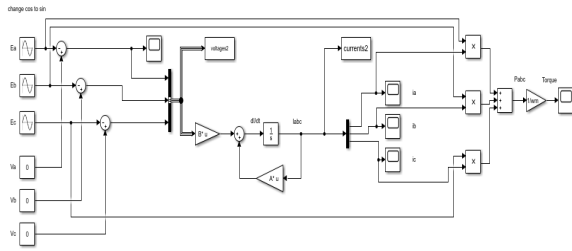


Figure 6: Simulink Block Diagram of a permanent magnet type synchronous machine

The transient currents a, b and c for the case above are shown in figures 9, 10 and 11 respectively.

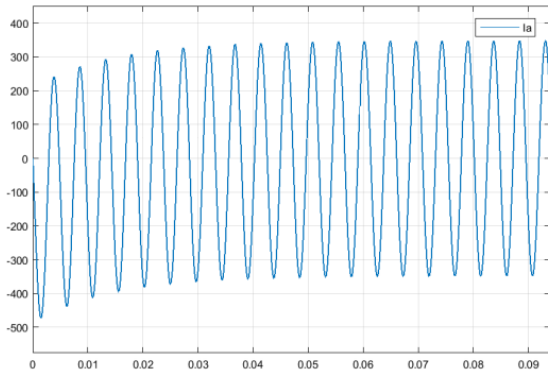


Figure 7: transient current of phase a (unit: Ampere)

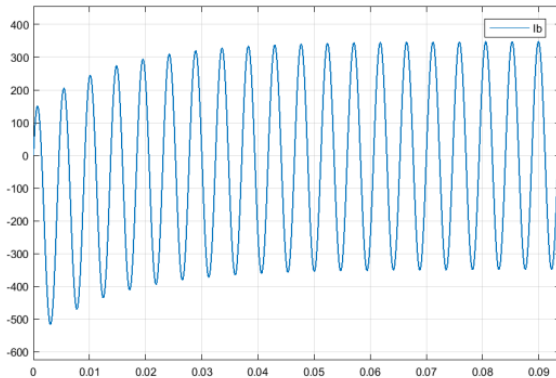


Figure 8: transient current of phase b (unit: Ampere)

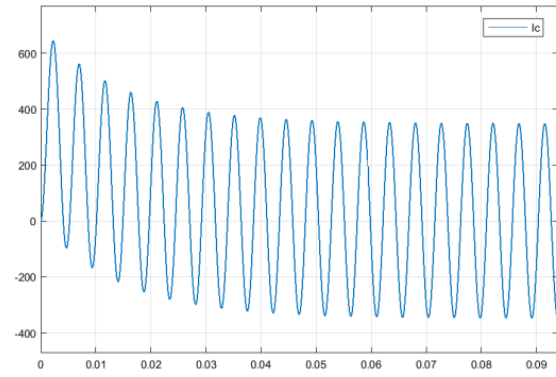


Figure 9: transient current of phase c (unit: Ampere)

Note that because of the shorted armature voltage, the current in this case is high at the beginning of the transient “almost three times the steady state”.

The torque curve for this case is shown in figure 12.

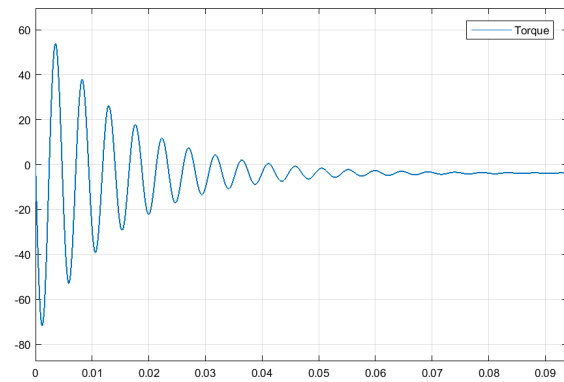


Figure 10: torque curve (unit: N.m)

Note that the torque due to the fault is disrupted and then get to a stable point at the end.

REFERENCES

- [1] A.A. Arkadan EENG577 Class Notes, Colorado School of Mines.
- [2] Xin Wang and C. Steve Suh, "Time-frequency based field oriented control of permanent magnet synchronous motors," June 2018, International Journal of Dynamics and Control 6(6):1-16, DOI: 10.1007/s40435-017-0327-5
- [3] A. A. Arkadan, and N. A. Demerdash, "Modeling of Transients in Permanent Magnet Generators with Multiple Damping Circuits Using the Natural abc Frame of Reference," IEEE Trans. on Energy Conversion, Vol. EC-3, No. 3, pp. 722-731, September 1988.