COLORADO SCHOOL OF MINES
ELECTRICAL ENGINEERING DEPARTMENT

EENG 577

ADVANCED ELECTRICAL MACHINE DYNAMICS FOR SMART-GRID SYSTEMS

M5-1 Permanent Magnet Machines

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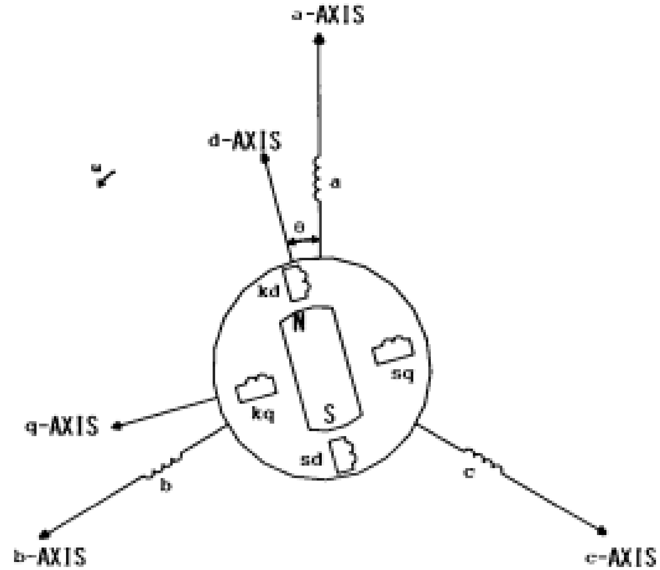
- Consider the case of a Permanent Magnet (PM) machine, whose schematic diagram is shown,
- The magnets as well as two sets of equivalent damper windings are mounted on the rotor, while the 3-phase armature is mounted on the stator.
- If we neglect the damping circuits for now and treat the PM excitation as an electromagnet (filed winging) excitation, the following state space (SS) model, which governs the dynamics of the machine, can be written:



PM Machine Schematic Diagram

$$\begin{bmatrix} v_a \\ v_b \\ v_c \\ v_f \end{bmatrix} = \begin{bmatrix} r_s & 0 & 0 & 0 \\ 0 & r_s & 0 & 0 \\ 0 & 0 & r_s & 0 \\ 0 & 0 & 0 & r_f \end{bmatrix} \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_f \end{bmatrix} + \frac{d}{dt} \left\{ \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} & L_{af} \\ L_{ba} & L_{bb} & L_{bc} & L_{bf} \\ L_{ca} & L_{cb} & L_{cc} & L_{cf} \\ L_{fa} & L_{fb} & L_{fc} & L_{ff} \end{bmatrix} \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_f \end{bmatrix} \right\} \quad (1)$$

Equation (1) can be rewritten for the armature windings in expended form as:

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} r_s & 0 & 0 & 0 \\ 0 & r_s & 0 & 0 \\ 0 & 0 & r_s & 0 \end{bmatrix} \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \frac{d}{dt}\left\{\begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc} \end{bmatrix} \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}\right\}$$

$$+ \frac{d}{dt}\left\{\begin{bmatrix} L_{af} . i_f \\ L_{bf} . i_f \\ L_{cf} . i_f \end{bmatrix}\right\} \qquad (2)$$

Consider the last term in equation (2). The mutual inductances between the field and the armature phase windings, are functions of the rotor angle, θ, as was shown earlier for a synchronous machine:

$L_{af} = L_{afm} \cos(\theta)$

$L_{bf} = L_{afm} \cos(\theta - 2\pi/3)$

$L_{cf} = L_{afm} \cos(\theta - 4\pi/3)$ (3)

Inspection of equation (3) reveals that the induced voltage in the phase winding due to the rotor PM is proportional to the following quantities:

- The field current used to represent the equivalent PM mmf, i_f
- the rate of change of the mutual inductances between the stator and the rotor at a given rotor position angle, θ
- the speed of the rotor, *ω*, which is related to the rotor position angle as $\theta=\omega t$ and,
- the time, t

Consequently, this vector term, on the right-hand side of equation (2), is the no-load phase to neutral *back emf* vector with components e_a, e_b, and e_c. A such, the last term in equation (2) is the vector representing the armature windings a, b, and c no-load back emf:

$$E_{abc} = \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} \qquad (4)$$

Using equation (4) in equation (2), and expressing it in expended matrix notation, one can write the following:

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} r_s & 0 & 0 & 0 \\ 0 & r_s & 0 & 0 \\ 0 & 0 & r_s & 0 \end{bmatrix} \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \frac{d}{dt}\left\{ \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc} \end{bmatrix} \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \right\} + \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} \quad (5)$$

Permanent Magnets

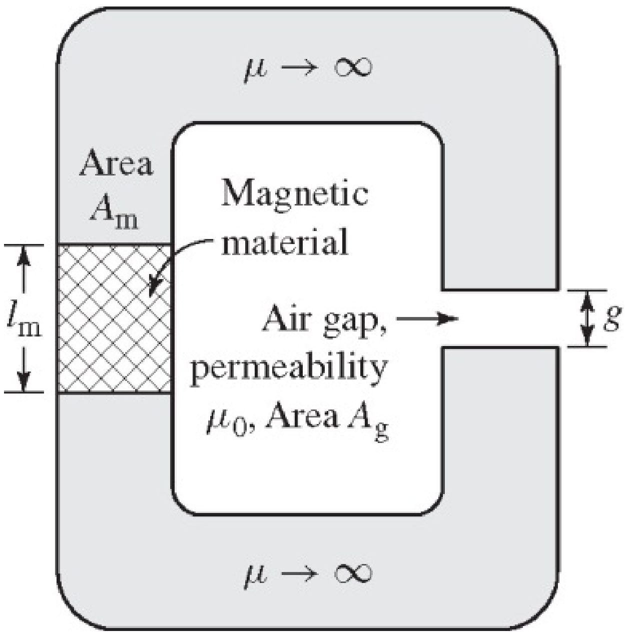
Let us consider the magnetic circuit shown below.

Applying Ampere's law:

$$H_m l_m + H_g g = 0 \qquad H_g = -\frac{l_m}{g} H_m$$

$$\phi = A_g B_g = A_m B_m \qquad B_g = \frac{A_m}{A_g} B_m$$

$$B_m = -\mu_0 \left(\frac{A_g}{A_m}\right)\left(\frac{l_m}{g}\right) H_m$$



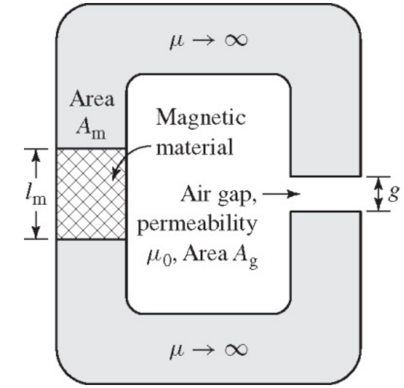
Magnetic Circuit with PM

Also, neglecting leakage and air gap fringing, and considering $A_m = A_g$, one gets $B_m = B_g$ and

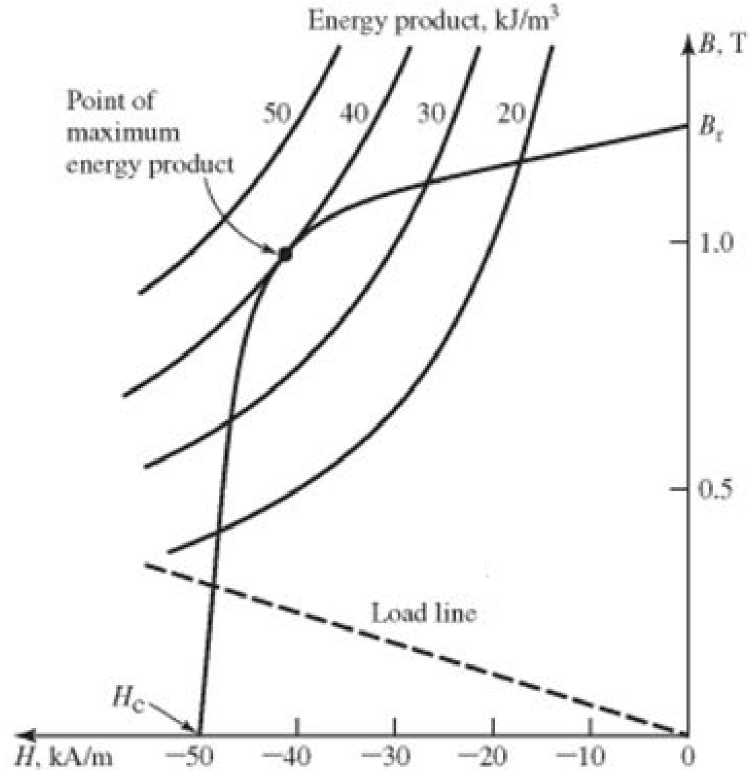
$$B_m = -\mu_0 \left(\frac{l_m}{g}\right) H_m$$

Source: Electric Machinery, by Fitzgerald, Kinsley, and Umans, 7th Edition, McGraw Hill.

Permanent Magnets



- Permanent magnet residual magnetism B_r and coercivity H_c are shown in the figure
  - B_r gives the flux density with no demagnetizing mmf
  - H_c gives the demagnetizing field needed to reduce B to zero
  - The load line shown illustrates the effect of cutting an air gap in the magnetic core
  - For details, check: *Electric Machinery, by Fitzgerald, Kinsley, and Umans, 7th Edition, McGraw Hill.*



Second quadrant hysteresis loop for Alnico 5

Source: Electric Machinery, by Fitzgerald, Kinsley, and Umans, 7th Edition, McGraw Hill.

- Based on the above, it can be shown that the operating flux obtained by a permanent magnet can be obtained by a constant excitation mmf of $l_m H_c$.

- In other words, a rotor filed permanent magnet (PM excitation) system is equivalent to a rotor field electromagnet (winding excitation) supplied by a *constant* excitation current, i_f, whose ampere-turns are proportional to the coercivity, H_c, of the permanent magnet times its length in the direction of Magnetization, l_m.

- As such, the PM can be replaced by an mmf of an equivalent electromagnet winding whose field current, i_f, is constant and the number of turns N=1, as follows:

$$I_f = H_c\, l_m \text{ Amps} \qquad (6)$$

As a result, equation (2) can be rewritten as:

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} r_s & 0 & 0 & 0 \\ 0 & r_s & 0 & 0 \\ 0 & 0 & r_s & 0 \end{bmatrix} \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \frac{d}{dt}\left\{ \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc} \end{bmatrix} \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \right\}$$

$$+(\mathrm{H_c\, l_m})\frac{d}{dt}\left\{ \begin{bmatrix} L_{af} \\ L_{\boldsymbol{b}f} \\ L_{\boldsymbol{c}f} \end{bmatrix} \right\} \quad (7)$$

The last term in equation (7) is the armature windings back emf vector:

$$E_{abc} = \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} = (\mathrm{H_c\, l_m})\frac{d}{dt}\left\{ \begin{bmatrix} L_{af} \\ L_{\boldsymbol{b}f} \\ L_{\boldsymbol{c}f} \end{bmatrix} \right\} \quad (8)$$

Now, consider the field to armature windings' mutual inductance expressions:

$L_{af} = L_{afm} \cos(\theta)$

$L_{\boldsymbol{bf}} = L_{afm} \cos(\theta - 2\pi/3)$

$L_{\boldsymbol{cf}} = L_{afm} \cos(\theta - 4\pi/3)$ (9)

The last term in equation (7) can be rewritten as:

$$(H_c\, l_m)\, \frac{d}{dt}\left\{\begin{bmatrix} L_{af} \\ L_{bf} \\ L_{\boldsymbol{c}f} \end{bmatrix}\right\} = (H_c\, l_m)\, \frac{d}{dt}\left\{\begin{bmatrix} L_{afm} \cos(\theta) \\ L_{afm} \cos(\theta - 2\pi/3) \\ L_{afm} \cos(\theta - 4\pi/3) \end{bmatrix}\right\}$$

$$= (-\frac{d\theta}{dt}\, H_c\, l_m)\begin{bmatrix} L_{afm} \sin(\theta) \\ L_{afm} \sin(\theta - 2\pi/3) \\ L_{afm} \sin(\theta - 4\pi/3) \end{bmatrix}$$

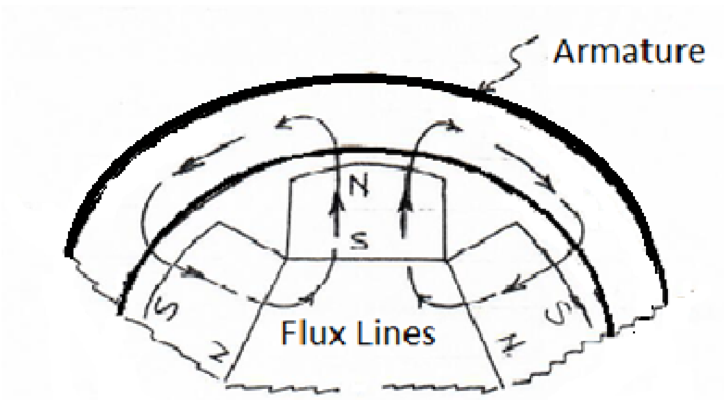
where, $\frac{d\theta}{dt} = \omega_e$, the electrical angular speed. As such, equation (4) is as follows:

$$E_{abc} = \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} = (-\omega_e H_c\, l_m)\, \frac{d}{dt}\left\{\begin{bmatrix} L_{af} \\ L_{bf} \\ L_{\boldsymbol{c}f} \end{bmatrix}\right\} = E_{max}\begin{bmatrix} \sin(\theta) \\ \sin(\theta - 2\pi/3) \\ \sin(\theta - 4\pi/3) \end{bmatrix} \quad (10)$$

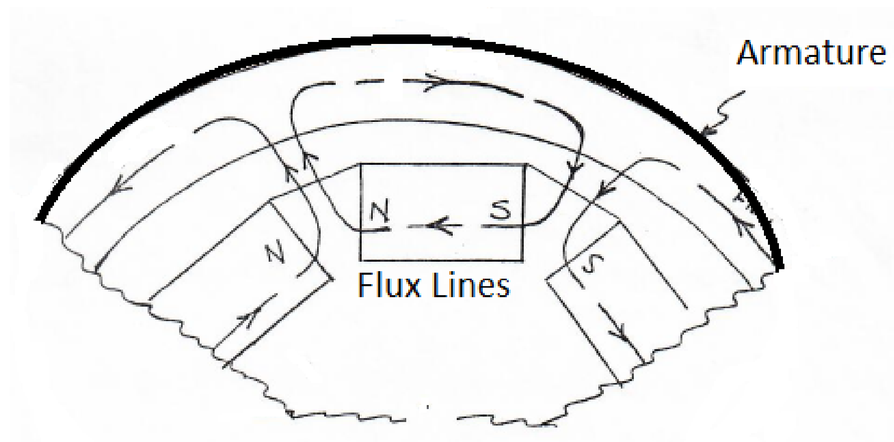
Permanent Magnet Machine – Rotor Design

In the figure below, two types of PM machine designs are shown. The first is a radially oriented rotor PM structure, where PM flux flows in the radial direction. The second is a tangentially oriented PM rotor structure where PM flux flows in the tangential, or circumferential, direction.

In the radial design, the effects of rotor saliency is minimal and can be neglected as permanent magnets made from rare earth material such as Samarium Cobalt, whose permeability is like that of air.



Radial PM Machine Design



Tangential PM Machine Design

As such, one can write the following for the armature windings (a, b, and c) self and mutual inductances, which are independent of rotor position:

$$L_{aa} = L_{bb} = L_{cc} = L_{sa}$$
$$L_{ab} = L_{ba} = L_{bc} = L_{cb} = L_{ac} = L_{ca} = L_{ma} \quad (11)$$

where, L_{sa} as well as L_{ma} are constants, as was defined earlier for a synchronous machine.

As a result, the armature windings self and mutual inductances are as follows:

$$\begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc} \end{bmatrix} = \begin{bmatrix} L_{sa} & L_{ma} & L_{ma} \\ L_{ma} & L_{sa} & L_{ma} \\ L_{ma} & L_{ma} & L_{sa} \end{bmatrix} \quad (12)$$

Using equations (10) and (12) in (7), the PM generator SS model is as follow:

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} r_s & 0 & 0 & 0 \\ 0 & r_s & 0 & 0 \\ 0 & 0 & r_s & 0 \end{bmatrix} \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} L_{sa} & L_{ma} & L_{ma} \\ L_{ma} & L_{sa} & L_{ma} \\ L_{ma} & L_{ma} & L_{sa} \end{bmatrix} \cdot \frac{d}{dt}\left\{\left(\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}\right)\right\} + \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix}$$

(13)

In compact matrix form, equation (13) can be expressed as:

$$V_{abc} = R.I_{abc} + L.\dot{I}_{abc} + E_{abc} \qquad (14)$$

where, the dot (·) at the top of the current vector, I_{abc}, represents the time derivative operator $\frac{d}{dt}$.

Equation (14) can be rewritten as:

$$\dot{I}_{abc} = (-L^{-1}.R).I_{abc} + L^{-1}.(V_{abc} - E_{abc}) \qquad (15)$$

where the superscript (-1) is the matrix inverse operator.

Also, equation (15) can be written in expanded matrix form as follows:

$$\begin{bmatrix} \dot{i}_a \\ \dot{i}_b \\ \dot{i}_c \end{bmatrix} = -\begin{bmatrix} L_{sa} & L_{ma} & L_{ma} \\ L_{ma} & L_{sa} & L_{ma} \\ L_{ma} & L_{ma} & L_{sa} \end{bmatrix}^{-1} . \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} . \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} +$$

$$\begin{bmatrix} L_{sa} & L_{ma} & L_{ma} \\ L_{ma} & L_{sa} & L_{ma} \\ L_{ma} & L_{ma} & L_{sa} \end{bmatrix}^{-1} . \begin{bmatrix} v_a - E_{max}.\sin(\theta) \\ v_b - E_{max}.\sin(\theta - {}^{2\pi}/_{3}) \\ v_c - E_{max}.\sin(\theta - {}^{4\pi}/_{3}) \end{bmatrix} \qquad (16)$$

In the general form, equation (16) can be expressed as:

$$\dot{X}=AX+BU \tag{17}$$

In equation (17), the matrices A and B and vector U and X, are given as follows:

$$A=\left(-\underline{L}^{-1}.R\right)=-\begin{bmatrix} L_{sa} & L_{ma} & L_{ma} \\ L_{ma} & L_{sa} & L_{ma} \\ L_{ma} & L_{ma} & L_{sa} \end{bmatrix}^{-1}.\begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \tag{18}$$

$$B=\underline{L}^{-1}=\begin{bmatrix} L_{sa} & L_{ma} & L_{ma} \\ L_{ma} & L_{sa} & L_{ma} \\ L_{ma} & L_{ma} & L_{sa} \end{bmatrix}^{-1} \tag{19}$$

$$U=\begin{bmatrix} v_a - E_{max}.\sin(\theta) \\ v_b - E_{max}.\sin(\theta - {}^{2\pi}/_{3}) \\ v_c - E_{max}.\sin(\theta - {}^{4\pi}/_{3}) \end{bmatrix} \tag{20}$$

And

$$X=\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \tag{21}$$

Solving the set of deferential equations given in (17) using in MATLAB/SIMULINK, as was shown earlier, would result in the values of the PM machine current vector $\underline{I}_{abc}$.

It should be noted also that the developed Power, *P*, of the machine can be obtained as follows:

$$P=e_a i_a + e_b i_{b+} e_c i_c \qquad (22)$$

Also, developed Torque, *T*, is related to *P* as follows:

$$T=P/\ \omega_m \qquad (23)$$

Or,

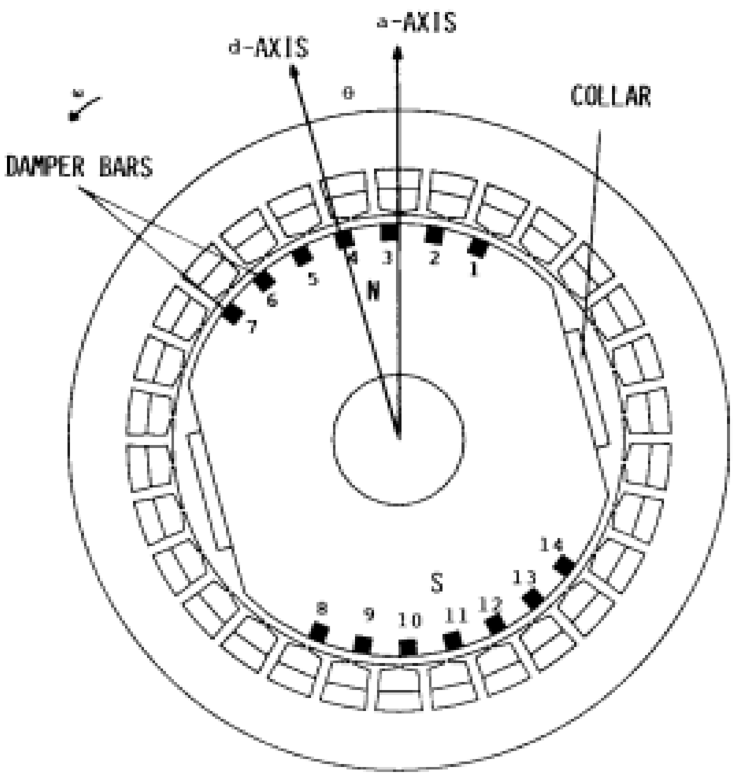
$$T=(\ e_a i_a + e_b i_{b+} e_c i_c)/\omega_m \qquad (24)$$

The angular mechanical speed, ω_m, is related in a *p*-pole machine to ω_e as:

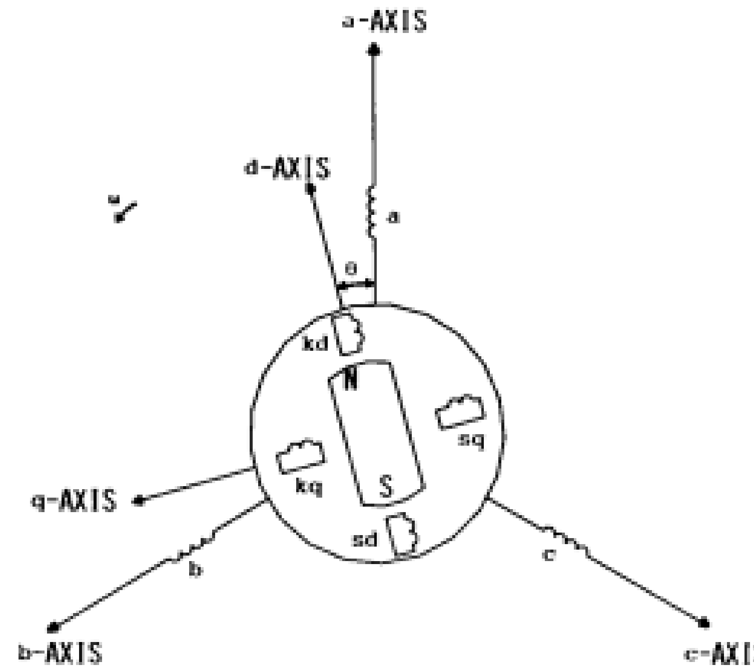
$$\omega_m = (2/p)\ \omega_e \qquad (25)$$

A Case Study

Consider the case of a Permanent Magnet (PM) generator, whose schematic diagram is shown. The field permanent magnets as well as two sets of equivalent damper windings are mounted on the rotor, while the 3-phase armature is mounted on the stator. For full details on a case study involving this PM generator, check references [1-3], given next.



MACHINE CROSS-SECTION



MACHINE WINDINGS SCHEMATIC DIAGRAM

References

1. Arkadan, A.A., Demerdash, N.A., Vaidya, J.G., and Shah, M.J., “Impact of Load on Winding Inductances of Permanent Magnet Generators with Multiple Damping Circuits Using Energy Perturbation,” IEEE Trans. on Energy Conversion, Vol. EC-3, No. 4, pp. 880-889, Dec. 1988.

2. Arkadan, A.A., and Demerdash, N.A., “Modeling of Transients in Permanent Magnet Generators with Multiple Damping Circuits Using the Natural abc Frame of Reference,” IEEE Trans. on Energy Conversion, Vol. EC-3, No. 3, pp. 722-731, Sep. 1988.

3. Arkadan, A.A., Hijazi, T.M., and Demerdash, N.A., “Computer-Aided Modeling of a Rectified DC Load-Permanent Magnet Generator System with Multiple Damper Windings in the Natural abc Frame of Reference,” IEEE Trans. on Energy Conversion, Vol. EC-4, No. 3, pp. 518-525, Sep. 1989.