EENG 577 W7 M5 - Induction Motor

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Part 1)

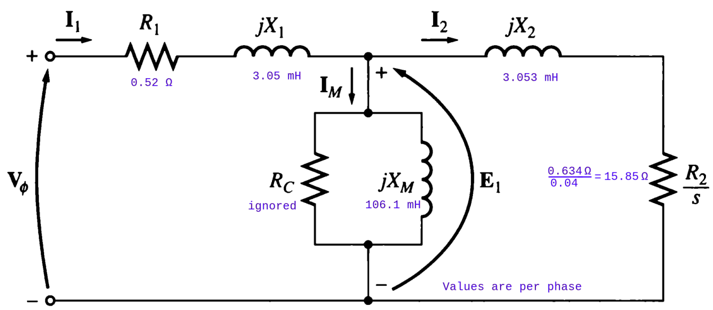


Figure 1: Equivalent circuit of the motor

The state space model in compact matrix is given as [1].

$$\begin{bmatrix} \underline{V}_{ABC} \\ \underline{V}_{abc} \end{bmatrix} = \begin{bmatrix} \underline{R}_{ss} & \underline{0} \\ \underline{0} & \underline{R}_{rr} \end{bmatrix} \begin{bmatrix} \underline{I}_{ABC} \\ \underline{I}_{abc} \end{bmatrix} + \begin{bmatrix} \underline{0} & \dot{\underline{L}}_{sr} \\ \dot{\underline{L}}_{sr}^{t} & \underline{0} \end{bmatrix} \cdot \begin{bmatrix} \underline{I}_{ABC} \\ \underline{I}_{abc} \end{bmatrix} + \begin{bmatrix} \underline{L}_{ss} & \underline{L}_{sr} \\ \underline{L}_{sr}^{t} & \underline{L}_{rr} \end{bmatrix} \begin{bmatrix} \dot{\underline{I}}_{ABC} \\ \dot{\underline{I}}_{abc} \end{bmatrix} \tag{1}$$

Rearranging the equation into general from $\dot{X} = AX + BU$ gives us

$$\frac{d}{dt} i = (-L^{-1}R - L^{-1}\dot{L})i + L^{-1}v$$

Where

$$A = -L^{-1}R - L^{-1}\dot{L} \; and \; B = L^{-1}$$

Putting this into compact matrix form we get

$$\begin{bmatrix} \dot{\underline{I}}_{ABC} \\ \dot{\underline{I}}_{abc} \end{bmatrix} = \left(- \begin{bmatrix} \underline{L}_{ss} & \underline{L}_{sr} \\ \underline{L}_{sr}^{t} & \underline{L}_{rr} \end{bmatrix}^{-1} \begin{bmatrix} \underline{R}_{ss} & \underline{0} \\ \underline{0} & \underline{R}_{rr} \end{bmatrix} - \begin{bmatrix} \underline{L}_{ss} & \underline{L}_{sr} \\ \underline{L}_{sr}^{t} & \underline{L}_{rr} \end{bmatrix}^{-1} \begin{bmatrix} \underline{0} & \dot{\underline{L}}_{sr} \\ \dot{\underline{L}}_{sr}^{t} & \underline{0} \end{bmatrix} \right) \begin{bmatrix} \underline{I}_{ABC} \\ \underline{I}_{abc} \end{bmatrix} + \begin{bmatrix} \underline{L}_{ss} & \underline{L}_{sr} \\ \underline{L}_{sr}^{t} & \underline{L}_{rr} \end{bmatrix}^{-1} \begin{bmatrix} \underline{V}_{ABC} \\ \underline{V}_{abc} \end{bmatrix} \tag{2}$$

Part 2)

The Simulink model represents a 3 phase induction motor drive system with a conduction period of 180 degrees and a DC input voltage of 546 V. This is connected to a state-space model of a three-phase 15hp induction motor with a slip speed of 0.04.

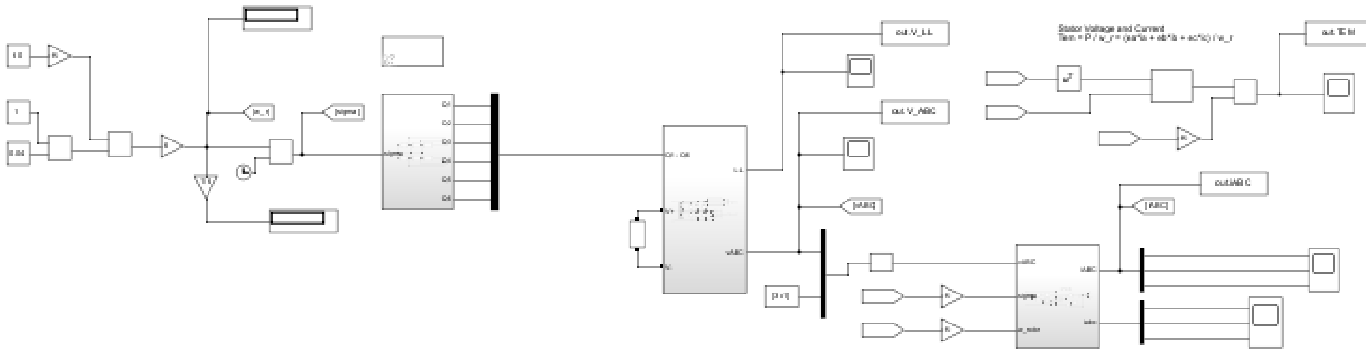


Figure 1: Simulink Overall

The rotor speed and position subsystem calculates the speed and position of the induction motor based on electrical frequency and slip. Using the formula $n_s = \frac{120 f_e}{P}$ the 8 pole 60 Hz motor has a synchronous speed of 900 RPM. With a slip of 0.04 the resulting rotor speed is 864 RMP. The rotor speed is converted to angular velocity using the relationship 1 RPM = 6 deg/s resulting in an angular velocity of 5184 deg/s. The rotor position is found by multiplying the angular velocity by time. Rotor position is fed as an input parameter into the Inverter Switching Sequence model in Figure 2.

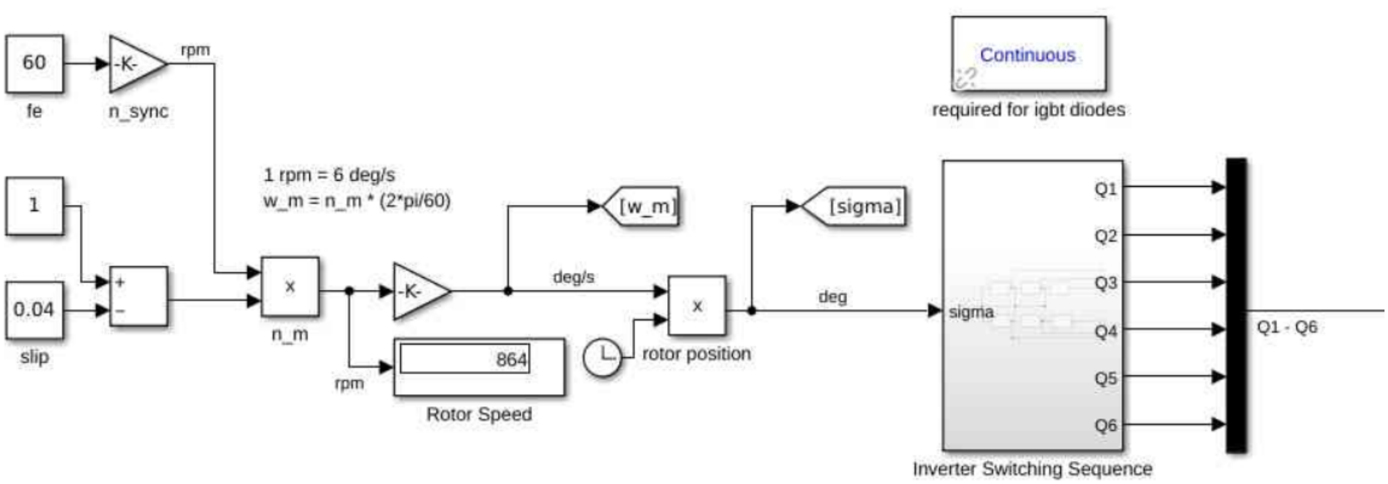


Figure 2: Rotor Speed

The switching sequence in the model uses six sub models that takes in the rotor position and returns a DC voltage used as a driver on the gate of the insulated-gate bipolar transistors (IGBT). Figures 3 and 4 show the models used to determine the phase switching sequence. Typical low level voltage control signals would be on the order of 25 to 48 volts DC which draws lower levels of power generating less heat but is still sufficient to turn on and off the IGBTs. We used an ideal level of $1V$.

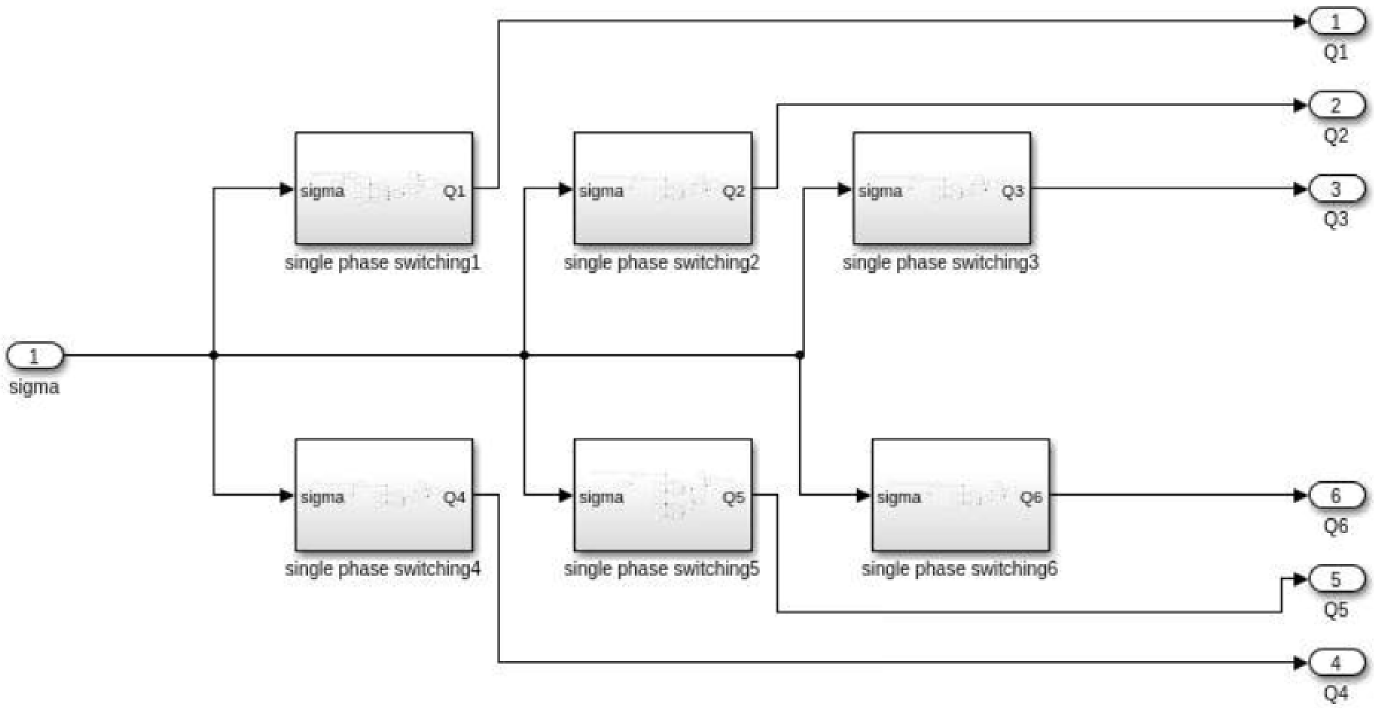


Figure 3: Switching Sequence Logic

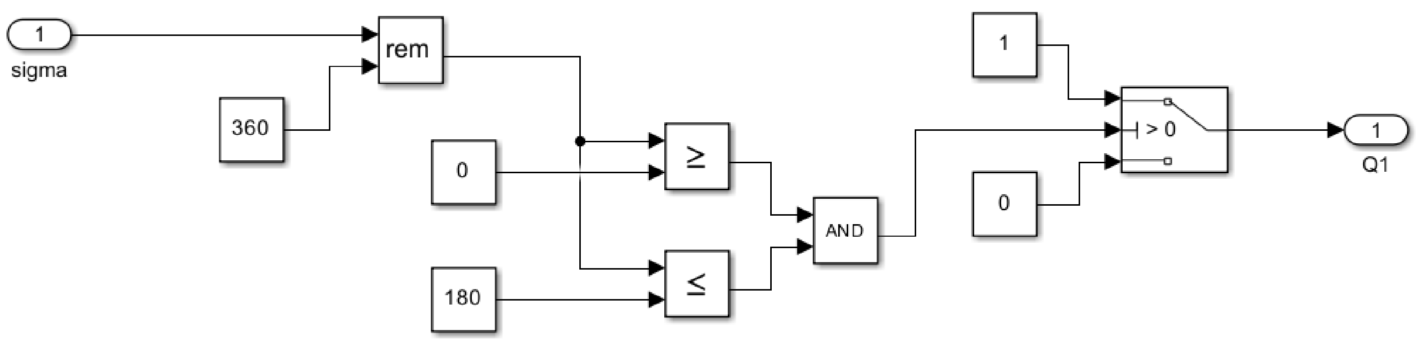


Figure 4: Q1 Switching Logic for 180 Degrees Electrical Conduction

The stator phase voltages used in the state space model are fed from a DC source inverter bridge that takes signals Q1 to Q6 from the switching model. The induced torque on the induction machine is calculated from the output phase voltage from the DC inverter source and the output phase currents from the induction motor. Figure 5 shows the layout of the inverter, induction motor, and in the induced torque calculation.

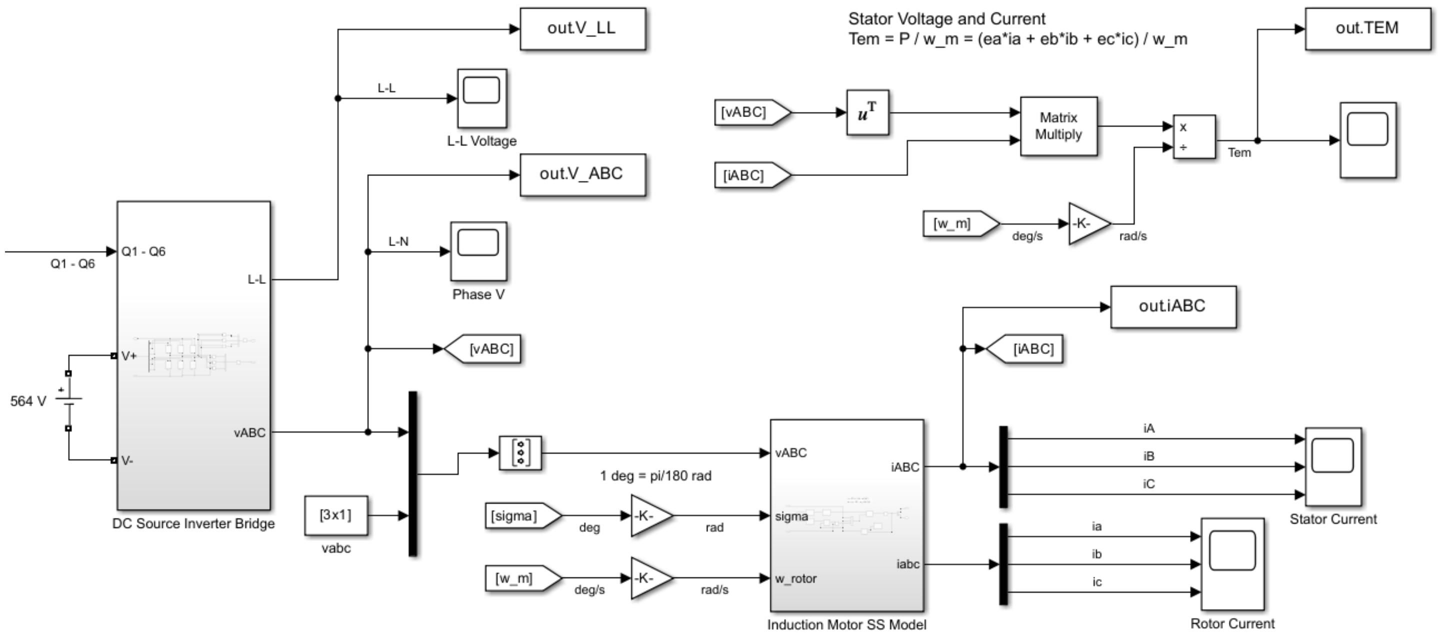


Figure 5: DC Inverter, Induction Motor, and Induced Torque

Figure 6 shows the full bridge converter using IGBT diodes and voltage meters. Phase voltage is measured from the center tap of the each half-bridge to the negative DC source voltage.

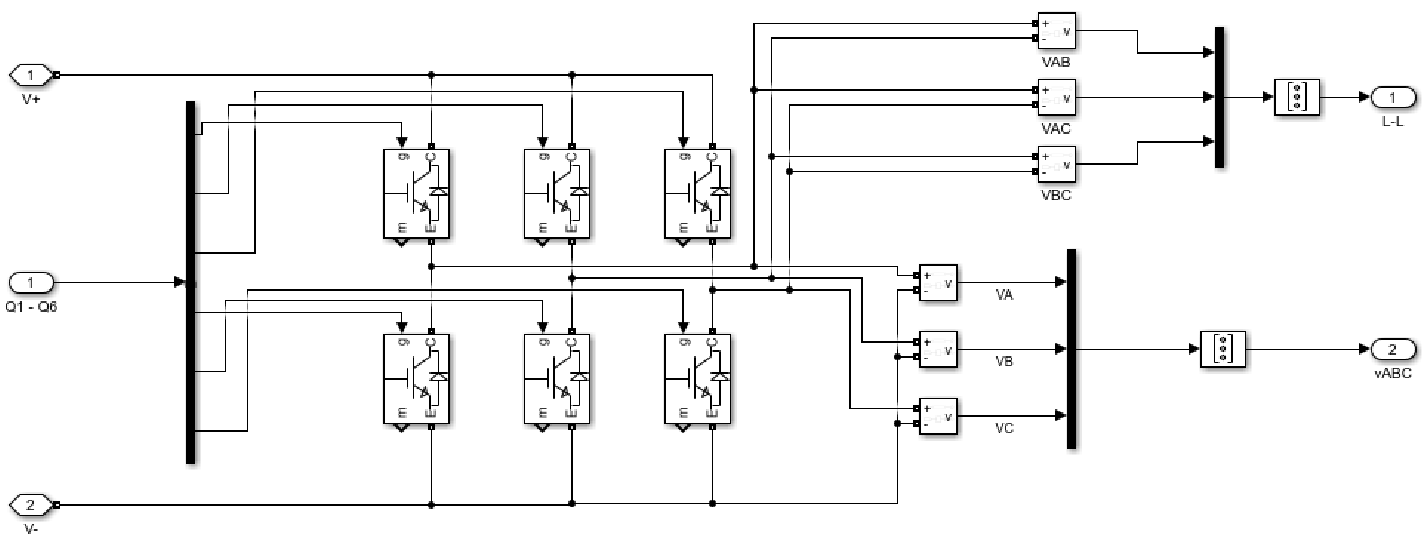


Figure 6: DC Inverter

The state space model uses three primary inputs phase voltage, rotor position, and angular speed to represent the stator and rotor currents using the natural abc reference frame. The subsystem computes the self and mutual inductances (L) as a function of rotor position. The matrix (dL) is used to account for the time variation in inductance due to rotor movement. Using these matrices the system calculates the voltage drop and electromotive force caused by the rotor movement. These are subtracted from the input voltage and multiplied by L^{-1} to calculate the rate of change of the current. This output is then passed through an integrator to give us the stator and rotor currents.

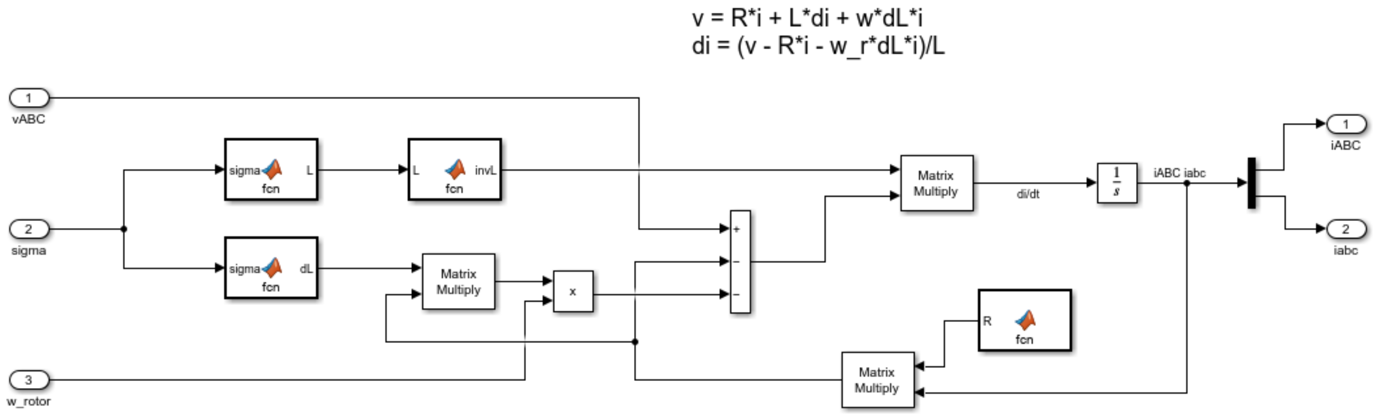
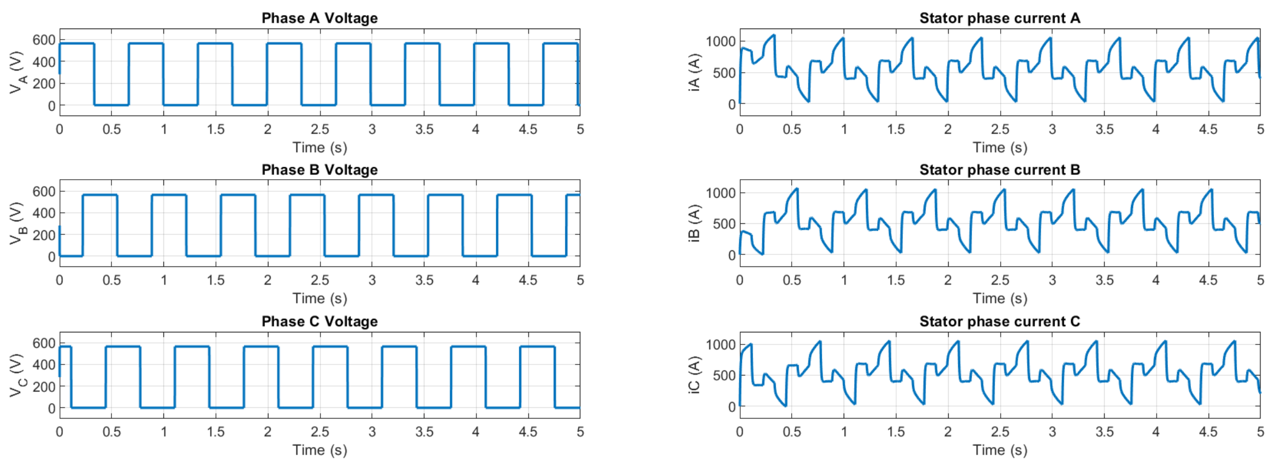


Figure 7: State Space Model for Induction Motor in ABC Reference Frame

Figure 7a and 7b show the phase voltage and phase current given a 180 degrees electrical switching sequence. The IGBT on voltage level is $564V$. The peak steady-state phase current is $1.058kA$. Figure 9a and 9b show the line to line voltage across the stator terminals and the induced torque on the induction motor. The line to line voltage is $1.128kV$ and the peak induced torque is $103.7kNm$.



High current and torque values -5

Figure 8: a) Stator Phase Voltage b) Stator Phase Current

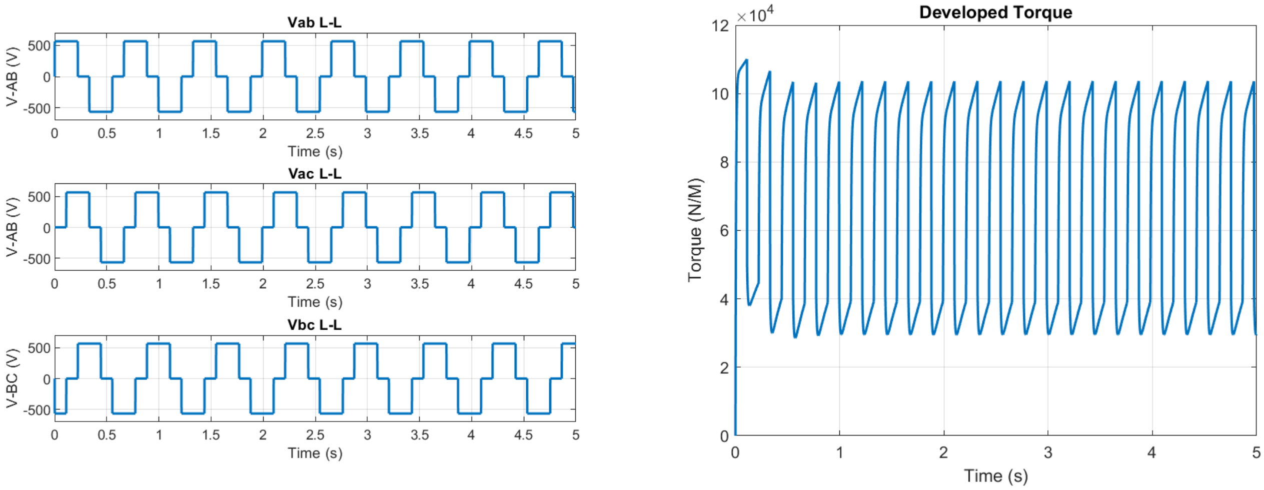


Figure 9: a) Stator Line Voltage b) Induced Torque

High torque values

Part 3)

If the machine drive system is operated with a 120° conduction period, and if it is needed to supply the same torque, then the dc voltage supplied to the inverter would have a different value. The difference is introduced to obtain an inverter output voltage whose fundamental ac component, Vrms, is equal in both cases. That is, for the motor inverter system to deliver an output power for the 120°e conduction period similar to that for the 180°e conduction period, the dc voltage level should be changed. The dc voltage can be related to the following expressions:

If we only cared about the RMS voltage for one phase, we could use the following equations:

$$V_{\text{rms},1\phi} = \sqrt{\frac{1}{T}\int_T V(t)^2 \mathrm{d}t}$$

The RMS of the Fundamental is the first coefficient of the Fourier series, divided by $\sqrt{2}$:

$$V_{1,rms} = \frac{1}{\sqrt{2}\pi}\int v(t)\sin(t)\mathrm{d}t$$

- For 180° conduction period:

$$V(t) = \begin{cases} V_{\text{dc}}, & t = \{\pi/6, 5\pi/6\} \\ -V_{\text{dc}}, & t = \{7\pi/6, 11\pi/6\} \\ 0, & t = \text{otherwise} \end{cases} \tag{3}$$

$$V_{1,\text{rms}} = \frac{1}{\sqrt{2}\pi}\left[\int_{\pi/6}^{5\pi/6} V_{\text{dc}}\sin(t)\mathrm{d}t + \int_{7\pi/6}^{11\pi/6} -V_{\text{dc}}\sin(t)\mathrm{d}t\right] \tag{4}$$

$$V_{1,\text{rms}} = \frac{2\sqrt{3}V_{\text{dc}}}{\sqrt{2}\pi} \tag{5}$$

$$V_{\text{dc}} = \frac{\sqrt{2}\pi}{2\sqrt{3}}V_{\text{dc}} \tag{6}$$

- For the 120° conduction period the voltage applied to the 3 phase windings is:

$$V(t) = \begin{cases} V_{\rm dc}/2, & t = \{0, \frac{\pi}{3}\} \\ V_{\rm dc}, & t = \{\frac{\pi}{3}, \frac{2\pi}{3}\} \\ V_{\rm dc}/2, & t = \{\frac{2\pi}{3}, \pi\} \\ -V_{\rm dc}/2, & t = \{\pi, \frac{4\pi}{3}\} \\ -V_{\rm dc}, & t = \{\frac{4\pi}{3}, \frac{5\pi}{3}\} \\ -V_{\rm dc}/2, & t = \{\frac{5\pi}{3}, 2\pi\} \end{cases} \tag{7}$$

$$V_{1,\rm rms} = \frac{1}{\sqrt{2\pi}} \begin{pmatrix} \int_0^{\pi/3} \frac{V_{\rm dc}}{2} \sin(t){\rm d}t + \int_{\pi/3}^{2\pi/3} V_{\rm dc} \sin(t){\rm d}t + \int_{2\pi/3}^{\pi} \frac{V_{\rm dc}}{2} \sin(t){\rm d}t \\ + \int_{\pi}^{4\pi/3} \frac{V_{\rm dc}}{2} \sin(t){\rm d}t + \int_{4\pi/3}^{5\pi/3} V_{\rm dc} \sin(t){\rm d}t + \int_{5\pi/3}^{2\pi} \frac{V_{\rm dc}}{2} \sin(t){\rm d}t \end{pmatrix} \tag{8}$$

$$V_{1,\rm rms} = \frac{3V_{\rm dc}}{\sqrt{2}\pi} \tag{9}$$

$$V_{\rm dc} = \frac{\sqrt{2}\pi}{3} V_{\rm dc} \tag{10}$$

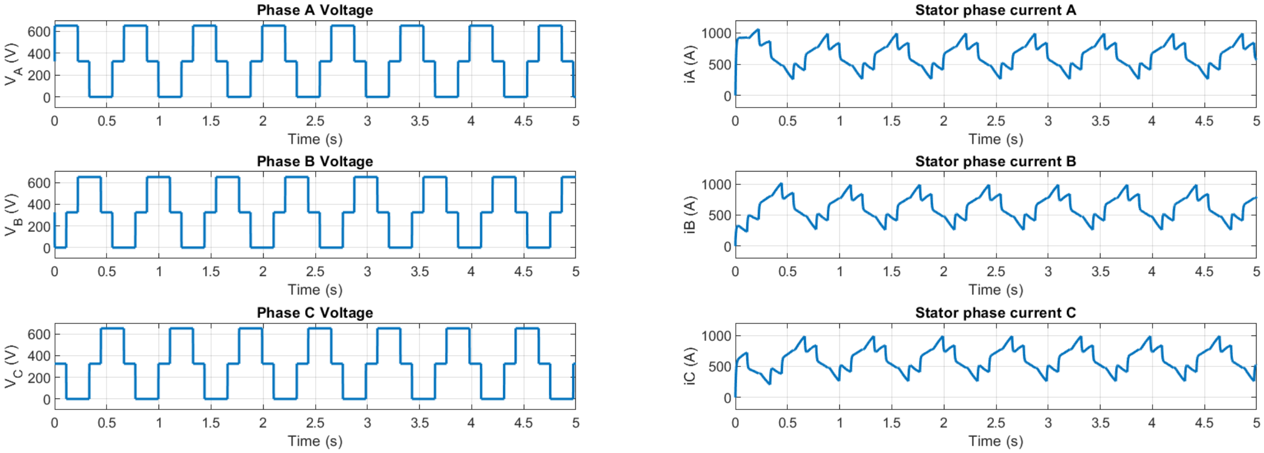
This means to maintain the same fundamental, the 120° conduction period must have a different voltage:

$$V_{\rm dc,120} = 564 \cdot \frac{\frac{\sqrt{2}\pi}{3}}{\frac{\sqrt{2}\pi}{2\sqrt{3}}} = 651.25 \text{ V}$$

Well done

Part 4)

Figures 10 and 11 shows the phase voltage, phase current, line to line voltage, and induced torque on the induction motor when the conduction period is reduced to 120 degrees.



High current and torque

Figure 10: a) Stator Phase Voltage b) Stator Phase Current

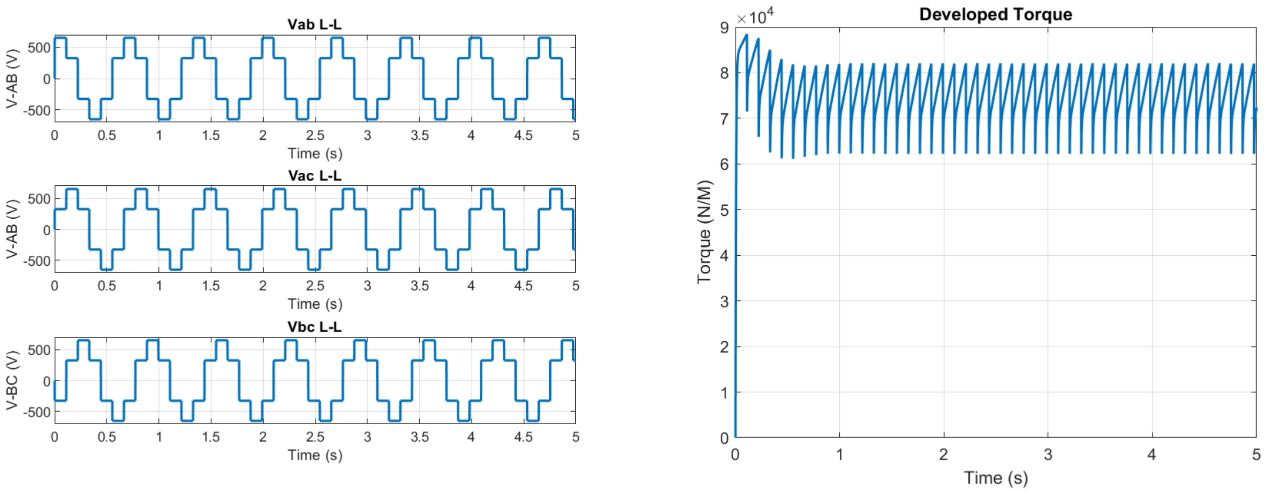


Figure 11: a) Stator Line Voltage b) Induced Torque

A reduction in the conduction period on the driver to the IGBT diode full bridge converter modulates the duration each bridge is turned on which results in a greater number of intervals of each line to line voltage. The corresponding phase currents on the induction motor are closer approximations to sinusoidal waveforms. Also, reducing the conduction period decreases the peak phase current on the stator windings. Phase currents on the windings generate the stator internal magnetic field which is connected to the rotor windings through the flux linkage and induces torque on the output shaft of the motor. If the stator windings have less current, we expect to see less output torque on the machine as well. The table below details the difference between line to line voltage and induced torque between 180 degree conduction period and 120 degree conduction period given the input voltage on the inverter is maintained at 564V and 651V respectively. The 120 degree conduction period also has less variation in the torque, which means less vibration.

Results	180° Conduction	120° Conduction
Peak Line-to-Line Voltage	564V	651V
Peak Phase Current Peak	1.058kA	987.9A
Peak Induced Torque	103.7kNm	98.79kNm

Contributions

Tasks	Eric Hildenbrandt	Devon Davis	Joseph Brownlee
Formulations & Calculations	15 %	40 %	45 %
MATLAB/Simulink Coding	55 %	20 %	25 %
Report Writing	30 %	40 %	30 %
Overall % Contribution/Member	100 %	100 %	100 %

References

- [1] A.A. Arkadan EENG577 Class Notes, Colorado School of Mines.
- [2] Stephen J. Chapman. (2005). Electric Machinery Fundamentals. McGraw-Hill.
- [3] A.A. Arkadan, and B.W. Kielgas, "Switched Reluctance Drive System and Dynamic Performance Prediction and Experimental Verification," IEEE Trans. On Energy Conversion, Vol. 9, No. 1, pp. 36-43, March 1994.
- [4] A.A. Arkadan, and B.W. Kielgas, "Switched Reluctance Drive System and Dynamic Performance Prediction Under Internal and External Fault Conditions," IEEE Trans. On Energy Conversion, Vol. 9, No. 1, pp. 45-52, March 1994.
- [5] A.A. Arkadan, and B.W. Kielgas, "Effects of Force Fitting on the Inductance Profile of a Switched Reluctance Motor," IEEE Trans. On Magnetics, Vol. 29, No. 2, pp. 2006-2009, March 1993.

Appendix: Matlab

Function L

```
function L = fcn(sigma)

Lls = 3.05e-3;
Lm = 106.1e-3;
Llr = 3.053e-3;

lss = Lls + (2/3)*Lm;
lsm = -(1/3)*Lm;
lrr = Llr + (2/3)*Lm;
lrm = -(1/3)*Lm;
lsrm = (2/3)*Lm;

l1 = lsrm*cos(sigma);
l2 = lsrm*cos(sigma + 2*pi/3);
l3 = lsrm*cos(sigma + 4*pi/3);

Lss = [lss, lsm, lsm;
    lsm, lss, lsm;
    lsm, lsm, lss];

Lrr = [lrr, lrm, lrm;
    lrm, lrr, lrm;
    lrm, lrm, lrr];

Lsr = [l1, l2, l3;
       l3, l1, l2;
       l2, l3, l1];

Lrs = Lsr';

L = [Lss, Lsr;
    Lrs, Lrr];
```

Function L^{-1}

```
    function invL = fcn(L)

invL = inv(L);
```

Function dL

```
    function dL = fcn(sigma)

Lm = 106.1e-3;
lsrm = (2/3)*Lm;

dl1 = -lsrm*sin(sigma);
dl2 = -lsrm*sin(sigma + 2*pi/3);
dl3 = -lsrm*sin(sigma + 4*pi/3);

dLsr = [dl1, dl2, dl3;
    dl3, dl1, dl2;
    dl2, dl3, dl1];

dLrs = dLsr';

dL = [zeros(3,3), dLsr;
    dLrs, zeros(3,3)];
```

Matlab Figure Generator

```
mkdir figs

time = out.V_ABC.Time;
V_abc = out.V_ABC.Data;
size(time)
size(V_abc)
V_abc = squeeze(V_abc);
V_abc = V_abc.';
%-------------------------------------------------------------------
figure(1)
subplot(3,1,1);
plot(time, V_abc(:,1));
xlabel('Time (s)');
ylabel('V_A (V)');
title('Phase A Voltage');
grid on;
subplot(3,1,2);
plot(time, V_abc(:,2));
xlabel('Time (s)');
ylabel('V_B (V)');
title('Phase B Voltage');
grid on;
subplot(3,1,3);
plot(time, V_abc(:,3));
xlabel('Time (s)');
ylabel('V_C (V)');
title('Phase C Voltage');
grid on;
saveas (gcf,'figs/phase_voltage.png')
v_LL = out.V_LL.Data;
size(v_LL);
v_LL = squeeze(v_LL);
v_LL = v_LL.';
%-------------------------------------------------------------------
figure(2)
subplot(3,1,1);
plot(time, v_LL(:,1));
xlabel('Time (s)');
ylabel('V-AB (V)');
title('Vab L-L');
grid on;
subplot(3,1,2);
plot(time, v_LL(:,2));
xlabel('Time (s)');
ylabel('V-AB (V)');
title('Vac L-L');
grid on;
subplot(3,1,3);
plot(time, v_LL(:,3));
xlabel('Time (s)');
ylabel('V-BC (V)');
title('Vbc L-L');
grid on;
saveas (gcf,'figs/L-L voltage.png')
IABC = out.iABC.Data;
size(IABC);
IABC = squeeze(IABC);
IABC = IABC.';
```

```
%--------------------------------------------------------------
figure(3)
subplot(3,1,1);
plot(time, IABC(:,1));
xlabel('Time (s)');
ylabel('iA (A)');
title('Stator phase current A');
grid on;
subplot(3,1,2);
plot(time, IABC(:,2));
xlabel('Time (s)');
ylabel('iB (A)');
title('Stator phase current B');
grid on;
subplot(3,1,3);
plot(time, IABC(:,3));
xlabel('Time (s)');
ylabel('iC (A)');
title('Stator phase current C');
grid on;
saveas (gcf,'figs/Phase_currents.png')
TEM = out.TEM.Data;
TEM = squeeze(TEM);
%--------------------------------------------------------------
figure(4)
plot(time, TEM);
xlabel('Time (s)');
ylabel('Torque (N/M)');
title('Developed Torque');
grid on;
saveas (gcf,'figs/developed_torque.png')
```