

# EE 577 - M3A1

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## 4-2

A 13.8-kV, 50-MVA, 0.9-power-factor-lagging, 60-Hz, four-pole Y-connected synchronous generator has a synchronous reactance of  $2.5 \Omega$  and an armature resistance of  $0.2 \Omega$ . At 60 Hz, its friction and windage losses are 1 MW, and its core losses are 1.5 MW. The field circuit has a dc voltage of 120 V, and the maximum  $I_F$  is 10 A. The current of the field circuit is adjustable over the range from 0 to 10 A. The OCC of this generator is shown in Figure P4-1.

a) How much field current is required to make the terminal voltage V (or line voltage ) equal to 13.8 kV when the generator is running at no load?

Looking at the OCC curve, the OCV of 13.8 kV is about **3.5A**.

b) What is the internal generated voltage  $E_A$  of this machine at rated conditions?

At rated conditions,  $I = \frac{50\text{MVA}}{13.8\text{kV, line line}} = \frac{50\text{MVA}}{\sqrt{3}13.8\text{kV, phase}} = 2.09\text{kA}$  on each output line.

$$V_{\text{phase}} = V_{\text{line line}}/\sqrt{3} = 7.967\text{kV}$$

The generator has phase reactance  $X = 2.5\Omega$  and resistance  $R = 0.2\Omega$ . To make the terminal voltage 13.8 kV (7.967 kV phase) the internal voltage needs to be higher:  $E = V_T + I \cdot (R + jX) = 7.967\text{kV} + 2.09\angle 25.8^\circ\text{kA}(0.2 + j2.5)\Omega = 11.547\text{kV}_{\text{phase, internal}} = 20\text{kV}_{\text{line-line, internal}}$

c) The phase voltage  $V_\Phi$  of this generator at rated conditions, assuming the field current is adjusted properly, is  $V_\Phi = V_{\text{line-line}}/\sqrt{3} = 11.547\text{kV}$ .

d) To make the internal OCV 20 kV, and terminal voltage 13.8 kV according to the OCC requires a the maximum field current of **10A**.

## 4-6

The internal generated voltage  $E_A$  of a 2-pole  $\Delta$ -connected, 60 Hz, three phase synchronous generator is 14.4 kV, and the terminal voltage  $V_T$  is 12.8 kV. The synchronous reactance of this machine is  $4 \Omega$ , and the armature resistance can be ignored.

a) If the torque angle of the generator  $\delta = 18^\circ$ , how much power is being supplied by this generator at the current time?

$$P = VI = 3V_\Phi(E/X) \sin(\delta) = 3 \cdot 12.8 \cdot (14.4/4) \sin(18^\circ) = 42.72\text{MW}$$

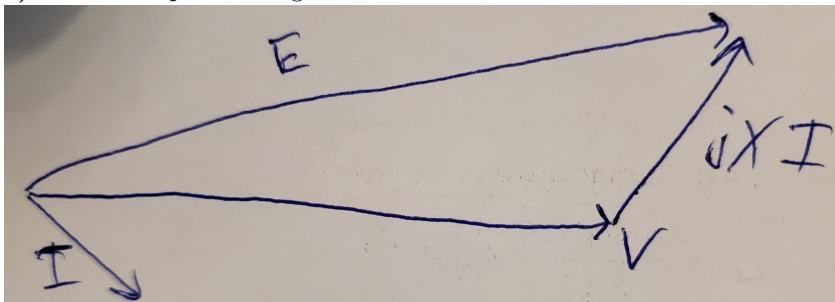
b) What is the power factor of the generator at this time?

$$pf = \cos(\angle V - \angle I) = \cos(\angle I)$$

$$I = \frac{14.4\angle 18^\circ - 12.8\text{kV}}{j4 \Omega} = 1.1348\angle \{-11.375^\circ\}\text{kA}$$

$$pf = \cos(-11.375^\circ) = 0.9804 \text{ lagging}$$

c) Sketch the phasor diagram under these circumstances



d) Ignoring losses in this generator, what torque must be applied to its shaft by the prime mover at these conditions?

$$P = \omega T$$

$$T = \frac{P}{\omega} = \frac{42.72 \text{ MW}}{2\pi 60} = 113.3 \text{ kN} \cdot \text{m}$$

## 4-7

A 100-MVA, 14.4-kV, 0.8-PF-lagging, 50-Hz, two-pole, Y-connected synchronous generator has a per-unit synchronous reactance of 1.1 and a per-unit armature resistance of 0.011

$$V_{\Phi} = 14.4 \text{ kV} / \sqrt{3} = 8.3138 \text{ kV}$$

a) What are its synchronous reactance and armature resistance in ohms?

$$R = \frac{R_{pu}}{Z_{base}} = 0.011 \cdot (14.4 \text{ kV})^2 / 100 \text{ MVA} = 22.81 \text{ m}\Omega$$

$$X = \frac{X_{pu}}{Z_{base}} = 1.1 \cdot (14.4 \text{ kV})^2 / 100 \text{ MVA} = 2.281 \Omega$$

b) What is the magnitude of the internal generated voltage E at the rated conditions? What is its torque angle  $\delta$  at these conditions?

$$\theta = \arccos 0.8 = 36.87^\circ$$

$$E = V + I \cdot Z = \frac{14.4}{\sqrt{3}} \text{ kV} + \left( \frac{100 \text{ MVA}}{\sqrt{3} \cdot 14.4 \text{ kV}} \angle -36.87^\circ \right) \cdot (0.02281 + j2.281)$$

$$E = 15.66 \angle 27.626^\circ \text{ kV}$$

c) Ignoring losses in the generator, what torque must be applied to its shaft by the prime mover at full load?

$$P = VI \cos \theta = 0.8 \cdot 1000 = 80 \text{ MW}$$

$$\text{speed } n_{\text{sync}} = \frac{120 f_{\text{sync}}}{P_{\text{poles}}} = \frac{120 \cdot 50 \text{ Hz}}{2} = 3000 \text{ rev/min} = 50 \text{ rev/s}$$

$$\text{torque applied } \tau = \frac{P}{\omega} = \frac{80 \text{ MW}}{2\pi \cdot 50 \text{ rad/s}} = 254,650 \text{ N} \cdot \text{m}$$

## 4-8

A 200-MVA, 12-kV, 0.85-PF-lagging, 50-Hz, 20-pole, Y-connected water turbine generator has a per-unit synchronous reactance of 0.9 and a per-unit armature resistance of 0.1. This generator is operating in parallel with a large power system (infinite bus).

a) What is the speed of rotation of this generator's shaft?

$$n_{\text{sync}} = \frac{120 f_{\text{sync}}}{P_{\text{poles}}} = \frac{120 \cdot 50 \text{ Hz}}{20} = 300 \text{ rev/min}$$

b) What is the magnitude of the internal generated voltage E at rated conditions?

$$\theta = \arccos 0.85 = 31.788^\circ$$

$$E_{pu} = V_{pu} + Z_{pu} \cdot I_{pu} = 1 + (0.1 + j0.9) \cdot 1 \angle 31.788^\circ = 1.7141 \angle 24.555^\circ \text{ V}_{pu}$$

$$E_A = 1.7141 \angle 24.555^\circ \text{ V}_{pu} \cdot 12 \text{ kV} / \sqrt{3} = 11.876 \angle 24.555^\circ \text{ kV per phase internal voltage}$$

c) What is the torque angle of the generator at rated conditions?

The torque angle is the angle  $\delta$  between the internal and external voltage.

$$\delta = 24.555^\circ$$

d) What are the values of the generator's synchronous reactance and armature resistance in ohms?

We are given the per unit, so we can just multiply by the base impedance.

$$X_s = 0.9 \cdot \frac{(12 \text{ kV})^2}{200 \text{ MVA}} = 0.648 \Omega$$

$$R = 0.1 \cdot \frac{(12 \text{ kV})^2}{200 \text{ MVA}} = 0.072 \Omega$$

e) If the field current is held constant, what is the maximum power possible out of this generator?

How much reserve power or torque does this generator have at full load?

$$P \approx \frac{3}{2} V_{pu} E_{pu} \sin \delta$$

$$\text{The maximum occurs at } \delta = 90^\circ: P_{\text{max}} = 3 \frac{12 / \sqrt{3} \cdot 11.876}{0.648} = 380.92 \text{ kW}$$

$$\text{The generator has } 380.92 - 200 \cdot 0.85 = 210.92 \text{ kW reserve power.}$$