EENG 577
M3-A1 Solution
Spring 2025

P4-2 (a)-(d) (20 pts)

4–2. A 13.8-kV, 50-MVA, 0.9-power-factor-lagging, 60-Hz, four-pole Y-connected synchronous generator has a synchronous reactance of 2.5 Ω and an armature resistance of 0.2 Ω. At 60 Hz, its friction and windage losses are 1 MW, and its core losses are 1.5 MW. The field circuit has a dc voltage of 120 V, and the maximum I_F is 10 A. The current of the field circuit is adjustable over the range from 0 to 10 A. The OCC of this generator is shown in Figure P4–1.

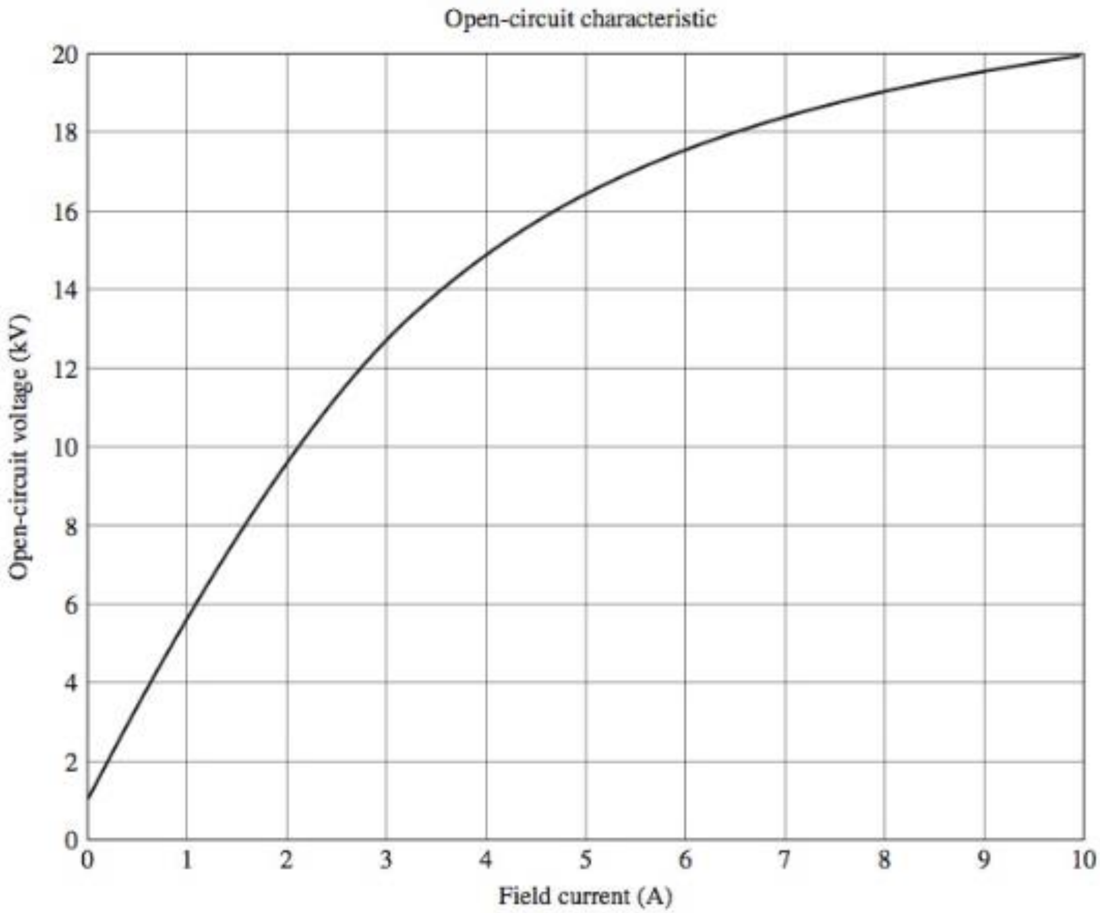


FIGURE P4–1
Open-circuit characteristic curve for the generator in Problem 4–2.

(a) How much field current is required to make the terminal voltage V_T (or line voltage V_L) equal to 13.8 kV when the generator is running at no load?
(b) What is the internal generated voltage E_A of this machine at rated conditions?

(c) What is the phase voltage V_ϕ of this generator at rated conditions?
(d) How much field current is required to make the terminal voltage V_T equal to 13.8 kV when the generator is running at rated conditions?

(a) If the no-load terminal voltage is 13.8 kV, the required field current can be read directly from the open-circuit characteristic. It is 3.50 A.

(b) This generator is Y-connected, so $I_L = I_A$. At rated conditions, the line and phase current in this generator is

$$I_A = I_L = \frac{S}{\sqrt{3}\,V_L} = \frac{50\text{ MVA}}{\sqrt{3}(13800\text{ V})} = 2092\text{ A at an angle of } -25.8°$$

The phase voltage of this machine is $V_\phi = V_T / \sqrt{3} = 7967\text{ V}$. The internal generated voltage of the machine is

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A\mathbf{I}_A + jX_S\mathbf{I}_A$$
$$\mathbf{E}_A = 7967\angle 0° + (0.20\ \Omega)(2092\angle -25.8°\text{ A}) + j(2.5\ \Omega)(2092\angle -25.8°\text{ A})$$
$$\mathbf{E}_A = 11544\angle 23.1°\text{ V}$$

(c) The phase voltage of the machine at rated conditions is $V_\phi = 7967\text{ V}$

(d) The equivalent open-circuit terminal voltage corresponding to an E_A of 11544 volts is

$$V_{T,oc} = \sqrt{3}(11544\text{ V}) = 20\text{ kV}$$

From the OCC, the required field current is 10 A.

(20 pts)

4-6. The internal generated voltage E_A of a **2-pole, Δ-connected, 60 Hz,** three phase synchronous generator is 14.4 kV, and the terminal voltage V_T is 12.8 kV. The synchronous reactance of this machine is 4 Ω, and the armature resistance can be ignored.

(a) If the torque angle of the generator $\delta = 18°$, how much power is being supplied by this generator at the current time?

(b) What is the power factor of the generator at this time?

(c) Sketch the phasor diagram under these circumstances.

(d) Ignoring losses in this generator, what torque must be applied to its shaft by the prime mover at these conditions?

SOLUTION

(a) If resistance is ignored, the output power from this generator is given by

$$P = \frac{3V_\phi E_A}{X_S}\sin\delta = \frac{3(12.8\text{ kV})(14.4\text{ kV})}{4\ \Omega}\sin 18° = 42.7\text{ MW}$$

(b) The phase current flowing in this generator can be calculated from

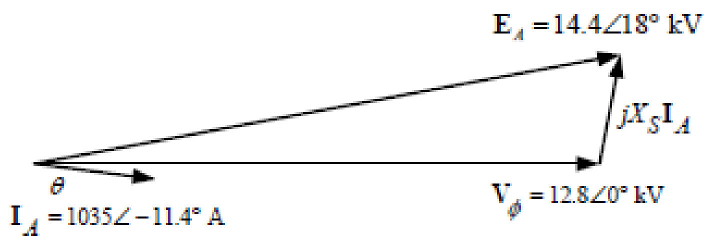
$$\mathbf{E}_A = \mathbf{V}_\phi + jX_S\mathbf{I}_A$$

$$\mathbf{I}_A = \frac{\mathbf{E}_A - \mathbf{V}_\phi}{jX_S}$$

$$\mathbf{I}_A = \frac{14.4\angle 18°\text{ kV} - 12.8\angle 0°\text{ kV}}{j4\ \Omega} = 1135\angle -11.4°\text{ A}$$

Therefore the impedance angle $\theta = 11.4°$, and the power factor is $\cos(11.4°) = 0.98\text{ lagging}$.

(c) The phasor diagram is



(d) The induced torque is given by the equation

$$P_{\text{conv}} = \tau_{\text{ind}}\omega_m$$

With no losses,

$$\tau_{\text{app}} = \tau_{\text{ind}} = \frac{P_{\text{conv}}}{\omega_m} = \frac{42.7\text{ MW}}{2\pi(60\text{ hz})} = 113{,}300\text{ N}\bullet\text{m}$$

(15 pts)

4-7. A 100-MVA, 14.4-kV, 0.8-PF-lagging, 50-Hz, two-pole, Y-connected synchronous generator has a per-unit synchronous reactance of 1.1 and a per-unit armature resistance of 0.011.

(a) What are its synchronous reactance and armature resistance in ohms?

(b) What is the magnitude of the internal generated voltage E_A at the rated conditions? What is its torque angle δ at these conditions?

(c) Ignoring losses in this generator, what torque must be applied to its shaft by the prime mover at full load?

SOLUTION

SOLUTION The base phase voltage of this generator is $V_{\phi,\text{base}} = 14,400/\sqrt{3} = 8314\text{ V}$. Therefore, the base impedance of the generator is

$$Z_{\text{base}} = \frac{3\,V_{\phi,\text{base}}{}^2}{S_{\text{base}}} = \frac{3(8314\text{ V})^2}{100,000,000\text{ VA}} = 2.074\ \Omega$$

(a) The generator impedance in ohms are:

$$R_A = (0.011)(2.074\ \Omega) = 0.0228\ \Omega$$

$$X_S = (1.1)(2.074\ \Omega) = 2.281\ \Omega$$

(b) The rated armature current is

$$I_A = I_L = \frac{S}{\sqrt{3}\,V_T} = \frac{100\text{ MVA}}{\sqrt{3}(14.4\text{ kV})} = 4009\text{ A}$$

The power factor is 0.8 lagging, so $\mathbf{I}_A = 4009\angle -36.87°\text{ A}$. Therefore, the internal generated voltage is

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A\mathbf{I}_A + jX_S\mathbf{I}_A$$
$$\mathbf{E}_A = 8314\angle 0° + (0.0228\ \Omega)(4009\angle -36.87°\text{ A}) + j(2.281\ \Omega)(4009\angle -36.87°\text{ A})$$
$$\mathbf{E}_A = 15,660\angle 27.6°\text{ V}$$

Therefore, the magnitude of the internal generated voltage E_A = 15,660 V, and the torque angle δ = 27.6°.

(c) Ignoring losses, the input power would equal the output power. Since

$$P_{\text{OUT}} = (0.8)(100\text{ MVA}) = 80\text{ MW}$$

and

$$n_{\text{sync}} = \frac{120 f_{se}}{P} = \frac{120(50\text{ Hz})}{2} = 3000\text{ r/min}$$

the applied torque would be

$$\tau_{\text{app}} = \tau_{\text{ind}} = \frac{80,000,000\text{ W}}{(3000\text{ r/min})(2\pi\text{ rad/r})(1\text{ min}/60\text{ s})} = 254,700\text{ N}\cdot\text{m}$$

P4-8 (a)-(e) (25 pts)

4-8. A 200-MVA, 12-kV, 0.85-PF-lagging, 50-Hz, 20-pole, Y-connected water turbine generator has a per-unit synchronous reactance of 0.9 and a per-unit armature resistance of 0.1. This generator is operating in parallel with a large power system (infinite bus).

(a) What is the speed of rotation of this generator's shaft?

(b) What is the magnitude of the internal generated voltage E_A at rated conditions?

(c) What is the torque angle of the generator at rated conditions?

(d) What are the values of the generator's synchronous reactance and armature resistance in ohms?

(e) If the field current is held constant, what is the maximum power possible out of this generator? How much reserve power or torque does this generator have at full load?

SOLUTION

(a) The speed of rotation of this generator's shaft is

$$n_{\text{sync}} = \frac{120 f_{se}}{P} = \frac{120(50\text{ Hz})}{20} = 300\text{ r/min}$$

(b) The per-unit phase voltage at rated conditions is $\mathbf{V}_\phi = 1.0\angle 0°$ and the per-unit phase current at rated conditions is $\mathbf{I}_A = 1.0\angle -31.79$ (since the power factor is 0.85lagging), so the per-unit internal generated voltage is

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A\mathbf{I}_A + jX_S\mathbf{I}_A$$

$$\mathbf{E}_A = 1\angle 0° + (0.1)(1\angle -31.79) + j(0.9)(1\angle -31.79)$$

$$\mathbf{E}_A = 1.71\angle 24.55\text{ pu}$$

The base phase voltage is

$$V_{\phi,\text{base}} = 12\text{ kV}/\sqrt{3} = 6928\text{ V}$$

so the internal generated voltage is

$$\mathbf{E}_A = (1.71\angle 24.55\text{ pu})(6928\text{ V}) = 11{,}876\angle 24.55\text{V}$$

(c) The torque angle of the generator is δ = 24.55

(d) The base impedance of the generator is

$$Z_{\text{base}} = \frac{3\, V_{\phi,\text{base}}{}^2}{S_{\text{base}}} = \frac{3(6928\text{ V})^2}{200{,}000{,}000\text{ VA}} = 0.72\ \Omega$$

Therefore the synchronous reactance is

$$X_S = (0.9)(0.72\ \Omega) = 0.648\ \Omega$$

and the armature resistance is

$$R_A = (0.1)(0.72\ \Omega) = 0.072\ \Omega$$

(e) If the field current is held constant (and the armature resistance is ignored), the power out of this generator is given by

$$P = \frac{3V_\phi E_A}{X_S} \sin \delta$$

The max power is given by

$$P_{max} = \frac{3V_\phi E_A}{X_S} \sin 90° = \frac{3(6928 \text{ V})(11{,}876 \text{ V})}{0.648\ \Omega} = 381 \text{ MW}$$

Since the full load power is $P = (200 \text{ MVA})(0.85) = 170 \text{ MW}$, this generator is supplying 45% of the maximum possible power at full load conditions.