EENG 577 M3 Synchronous Generator

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PART 1

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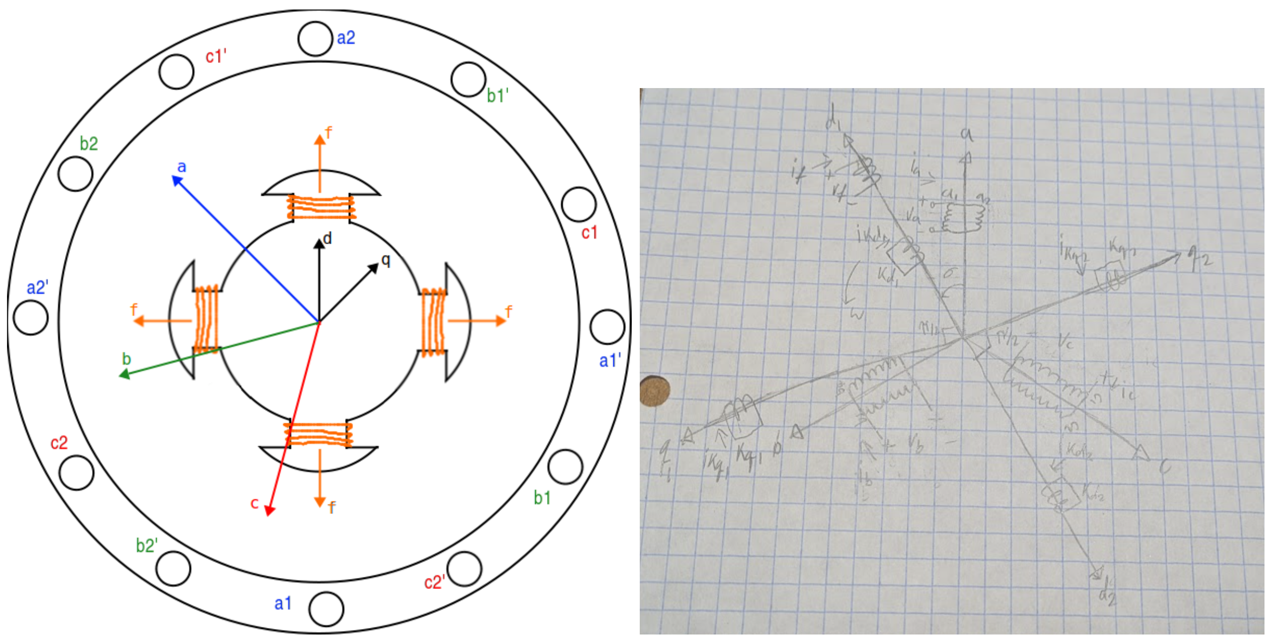

Fig. 1. Cross-section of generator, not to scale. The quadrature (q) axis is 90° electrically from the direct (d) axis. Because this is a 4-pole generator, there are 720° electrical for every 360° mechanical. The analysis would be the same if a, b, c, d, and q were rotated a multiple of 90° mechanical (180° electrical).

2.1: State Space, Case 1 (no damping)

Assuming the generator is balanced and rotating at f_s=60Hz:

$$\mathrm{L_{aa}} \cong \mathrm{L_{bb}} \cong \mathrm{L_{cc}} \cong \mathrm{L_{sa}} + \mathrm{L_{sv}} \cos(2\theta + \phi)$$
$$\text{where } [\phi_a,\ \phi_b,\ \phi_c] = [0, -\tfrac{4\pi}{3}, -\tfrac{2\pi}{3}] \text{ and } \theta = \omega t = 377 \cdot t$$

Ignoring the damping windings the state space equations can be written as such:

$$\begin{bmatrix} v_a \\ v_b \\ v_c \\ v_f \\ v_{kd} \\ v_{kq} \end{bmatrix} = \begin{bmatrix} r_s + r_l & 0 & 0 & 0 & 0 & 0 \\ 0 & r_s + r_l & 0 & 0 & 0 & 0 \\ 0 & 0 & r_s + r_l & 0 & 0 & 0 \\ 0 & 0 & 0 & r_f & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_f \\ i_{kd} \\ i_{kq} \end{bmatrix} +$$
$$\frac{d}{dt} \begin{bmatrix} L_{sa} + L_{sv}\cos(2\theta) + L_l & -L_{ma} + L_{mv}\cos(2\theta - 2\pi/3) & -L_{ma} + L_{mv}\cos(2\theta - 4\pi/3) & L_{afm}\cos(2\theta) & 0 & 0 \\ -L_{ma} + L_{mv}\cos(2\theta - 2\pi/3) & L_{sa} + L_{sv}\cos(2\theta - 4\pi/3) + L_l & -L_{ma} + L_{mv}\cos(2\theta) & L_{afm}\cos(2\theta - 2\pi/3) & 0 & 0 \\ -L_{ma} + L_{mv}\cos(2\theta - 4\pi/3) & -L_{ma} + L_{mv}\cos(2\theta) & L_{sa} + L_{sv}\cos(2\theta - 2\pi/3) + L_l & L_{afm}\cos(2\theta - 4\pi/3) & 0 & 0 \\ L_{afm}\cos(2\theta) & L_{afm}\cos(2\theta - 2\pi/3) & L_{afm}\cos(2\theta - 4\pi/3) & L_{ff} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_f \\ i_{kd} \\ i_{kq} \end{bmatrix}$$

2.2: State Space, Case 2

$$\begin{bmatrix} v_a \\ v_b \\ v_c \\ v_f \\ v_{kd} \\ v_{kq} \end{bmatrix} = \begin{bmatrix} r_s + r_l & 0 & 0 & 0 & 0 & 0 \\ 0 & r_s + r_l & 0 & 0 & 0 & 0 \\ 0 & 0 & r_s + r_l & 0 & 0 & 0 \\ 0 & 0 & 0 & r_f & 0 & 0 \\ 0 & 0 & 0 & 0 & r_k d & 0 \\ 0 & 0 & 0 & 0 & 0 & r_k q \end{bmatrix} \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_f \\ i_{kd} \\ i_{kq} \end{bmatrix} +$$
$$\frac{d}{dt} \begin{bmatrix} L_{sa} + L_{sv}\cos(2\theta) + L_l & -L_{ma} + L_{mv}\cos(2\theta - 2\pi/3) & -L_{ma} + L_{mv}\cos(2\theta - 4\pi/3) & L_{afm}\cos(2\theta) & L_{adkm}\cos(2\theta) & -L_{akqm1}\cos(2\theta) \\ -L_{ma} + L_{mv}\cos(2\theta - 2\pi/3) & L_{sa} + L_{sv}\cos(2\theta - 4\pi/3) + L_l & -L_{ma} + L_{mv}\cos(2\theta) & L_{afm}\cos(2\theta - 2\pi/3) & L_{akdm}\cos(2\theta - 2\pi/3) & -L_{akqm1}\cos(2\theta - 2\pi/3) \\ -L_{ma} + L_{mv}\cos(2\theta - 4\pi/3) & -L_{ma} + L_{mv}\cos(2\theta) & L_{sa} + L_{sv}\cos(2\theta - 2\pi/3) + L_l & L_{afm}\cos(2\theta - 4\pi/3) & L_{akdm}\cos(2\theta - 4\pi/3) & -L_{akqm1}\cos(2\theta - 4\pi/3) \\ L_{afm}\cos(2\theta) & L_{afm}\cos(2\theta - 2\pi/3) & L_{afm}\cos(2\theta - 4\pi/3) & L_{ff} & L_{kdf} & L_{kqf} \\ L_{adkm}\cos(2\theta) & L_{akdm}\cos(2\theta - 2\pi/3) & L_{akdm}\cos(2\theta - 4\pi/3) & L_{kdf} & L_{kd1kd1} & 0 \\ -L_{akqm1}\cos(2\theta) & -L_{akqm1}\cos(2\theta - 2\pi/3) & -L_{akqm1}\cos(2\theta - 4\pi/3) & L_{kqf} & 0 & L_{kq1kq1} \end{bmatrix} \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_f \\ i_{kd} \\ i_{kq} \end{bmatrix}$$

3.1

$$V = RI + \frac{d}{dt}LI \tag{1}$$
$$A = -L^{-1}R \tag{2}$$
$$B = L^{-1} \tag{3}$$

In compact matrix from.

$$X' = AX + BU$$

Where without damping

$$A = -\begin{bmatrix} 1.95 + 0.05\cos(2\theta) + L_l & -0.80 - 0.05\cos(2\theta - \frac{2\pi}{3}) & -0.80 - 0.05\cos(2\theta - \frac{4\pi}{3}) & 26.1\cos(2\theta) \\ -0.80 - 0.05\cos(2\theta - \frac{2\pi}{3}) & 1.95 + 0.05\cos(2\theta) + L_l & -0.80 - 0.05\cos(2\theta - \frac{4\pi}{3}) & 26.1\cos(2\theta - \frac{2\pi}{3}) \\ -0.80 - 0.05\cos(2\theta - \frac{4\pi}{3}) & -0.80 - 0.05\cos(2\theta - \frac{2\pi}{3}) & 1.95 + 0.05\cos(2\theta) + L_l & 26.1\cos(2\theta - \frac{4\pi}{3}) \\ 26.1\cos(2\theta) & 26.1\cos(2\theta - \frac{2\pi}{3}) & 26.1\cos(2\theta - \frac{4\pi}{3}) & 444 \end{bmatrix}^{-1} \cdot$$

$$\left(\begin{bmatrix} 7.4\cdot 10^{-4} + r_l & 0 & 0 & 0 \\ 0 & 7.4\cdot 10^{-4} + r_l & 0 & 0 \\ 0 & 0 & 7.4\cdot 10^{-4} + r_l & 0 \\ 0 & 0 & 0 & 0.086 \end{bmatrix} + \begin{bmatrix} -0.1\sin(2\theta) + L_l & -0.1\sin(2\theta - \frac{2\pi}{3}) & -0.1\sin(2\theta - \frac{4\pi}{3}) & -52.2\sin(2\theta) \\ -0.1\sin(2\theta - \frac{2\pi}{3}) & -0.1\sin(2\theta) + L_l & -0.1\sin(2\theta - \frac{4\pi}{3}) & -52.2\sin(2\theta - \frac{2\pi}{3}) \\ -0.1\sin(2\theta - \frac{4\pi}{3}) & -0.1\sin(2\theta - \frac{2\pi}{3}) & -0.1\sin(2\theta) & -52.2\sin(2\theta - \frac{4\pi}{3}) \\ -52.2\sin(2\theta) & -52.2\sin(2\theta - \frac{2\pi}{3}) & -52.2\sin(2\theta - \frac{4\pi}{3}) & 0 \end{bmatrix}\right) \cdot 10^{-3} H^{-1}$$

and

$$B = \begin{bmatrix} 1.95 + 0.05\cos(2\theta) + L_l & -0.80 - 0.05\cos(2\theta - \frac{2\pi}{3}) & -0.80 - 0.05\cos(2\theta - \frac{4\pi}{3}) & 26.1\cos(2\theta) \\ -0.80 - 0.05\cos(2\theta - \frac{2\pi}{3}) & 1.95 + 0.05\cos(2\theta) + L_l & -0.80 - 0.05\cos(2\theta - \frac{4\pi}{3}) & 26.1\cos(2\theta - \frac{2\pi}{3}) \\ -0.80 - 0.05\cos(2\theta - \frac{4\pi}{3}) & -0.80 - 0.05\cos(2\theta - \frac{2\pi}{3}) & 1.95 + 0.05\cos(2\theta) + L_l & 26.1\cos(2\theta - \frac{4\pi}{3}) \\ 26.1\cos(2\theta) & 26.1\cos(2\theta - \frac{2\pi}{3}) & 26.1\cos(2\theta - \frac{4\pi}{3}) & 444 \end{bmatrix}^{-1} \cdot 10^{-3} H^{-1}$$

With damping

$$A = -\begin{bmatrix} 7.4\cdot 10^{-4} + r_l & 0 & 0 & 0 & 0 & 0 \\ 0 & 7.4\cdot 10^{-4} + r_l & 0 & 0 & 0 & 0 \\ 0 & 0 & 7.4\cdot 10^{-4} + r_l & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.086 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.58\cdot 10^{-4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.227\cdot 10^{-4} \end{bmatrix}$$

$$\begin{bmatrix} -0.1\sin(2\theta) + L_l & -0.1\sin(2\theta - \frac{2\pi}{3}) & -0.1\sin(2\theta - \frac{4\pi}{3}) & -52.2\sin(2\theta) & -0.894\sin(2\theta) & 0.894\sin(2\theta) \\ -0.1\sin(2\theta - \frac{2\pi}{3}) & -0.1\sin(2\theta) + L_l & -0.1\sin(2\theta - \frac{4\pi}{3}) & -52.2\sin(2\theta - \frac{2\pi}{3}) & -0.894\sin(2\theta - \frac{2\pi}{3}) & 0.894\sin(2\theta - \frac{2\pi}{3}) \\ -0.1\sin(2\theta - \frac{4\pi}{3}) & -0.1\sin(2\theta - \frac{2\pi}{3}) & -0.1\sin(2\theta) + L_l & -52.2\sin(2\theta - \frac{4\pi}{3}) & -0.894\sin(2\theta - \frac{4\pi}{3}) & 0.894\sin(2\theta - \frac{4\pi}{3}) \\ -52.2\sin(2\theta) & -52.2\sin(2\theta - \frac{2\pi}{3}) & -52.2\sin(2\theta - \frac{4\pi}{3}) & 0 & 0 & 0 \\ -0.894\sin(2\theta) & -0.894\sin(2\theta - \frac{2\pi}{3}) & -0.894\sin(2\theta - \frac{4\pi}{3}) & 0 & 0 & 0 \\ 0.894\sin(2\theta) & 0.894\sin(2\theta - \frac{2\pi}{3}) & 0.894\sin(2\theta - \frac{4\pi}{3}) & 0 & 0 & 0 \end{bmatrix}$$

$$\times 10^{-3}\mathrm{H}^{-1}.$$

and

$$b = \begin{bmatrix} 1.95 + 0.05\cos(2\theta) + L_l & -0.80 - 0.05\cos(2\theta - 2\pi/3) & -0.08 - 0.05\cos(2\theta - 4\pi/3) & 26.1\cos(2\theta) & 0.447\cos(2\theta) & -0.670\cos(2\theta) \\ -0.80 - 0.05\cos(2\theta - 2\pi/3) & 1.95 + 0.05\cos(2\theta - 4\pi/3) + L_l & -0.80 - 0.05\cos(2\theta) & 26.1\cos(2\theta - 2\pi/3) & 0.447\cos(2\theta - 2\pi/3) & -0.670\cos(2\theta - 2\pi/3) \\ -0.80 - 0.05\cos(2\theta - 4\pi/3) & 1.95 + 0.05\cos(2\theta) & -0.80 - 0.05\cos(2\theta - 2\pi/3) + L_l & 26.1\cos(2\theta - 4\pi/3) & 0.447\cos(2\theta - 4\pi/3) & -0.670\cos(2\theta - 4\pi/3) \\ 26.1\cos(2\theta) & 26.1\cos(2\theta - 2\pi/3) & 26.1\cos(2\theta - 4\pi/3) & 444 & 6.29 & 5.05 \\ 0.447\cos(2\theta) & 0.447\cos(2\theta - 2\pi/3) & 0.447\cos(2\theta - 4\pi/3) & 6.29 & 0.1254 & 0 \\ -0.670\cos(2\theta) & -0.670\cos(2\theta - 2\pi/3) & -0.670\cos(2\theta - 4\pi/3) & 5.05 & 0 & 0.3762 \end{bmatrix}^{-1} \times 10^{-3}\mathrm{H}^{-1}$$

PART 2

1.1

The necessary excitation field current is given by $i_f = \frac{V_f}{r_f} = \frac{338}{0.0860} = 3930[A]$. The initial conditions at time $t = 0$ can be calculated by dividing the given phase voltages on the bus bar with the magnitude of the load resistance and reactance.

1.2

$$I_a = \frac{S}{\sqrt{3}V_{phi}} = 32950[A] \qquad V_a = \frac{20 * 10^3}{\sqrt(3)}\sqrt(2)cos(\omega t) = V_m cos(0) \qquad i_a = \frac{V_a}{\sqrt{R_l^2 + X_l^2}}$$

$$Rl = \frac{S \cdot pf}{I_A^2} = 0.5463[\Omega] \qquad V_b = V_m cos(0 - 2\pi/3) \qquad i_b = \frac{V_b}{\sqrt{R_l^2 + X_l^2}}$$

$$Xl = \frac{S \cdot \sqrt{1 - pf^2}}{I_a^2} = 0.2646[\Omega] \qquad V_c = V_m cos(0 - 4\pi/3) \qquad i_c = \frac{V_c}{\sqrt{R_l^2 + X_l^2}}$$

The initial conditions are put into a column vector as follows. $\begin{bmatrix} 26904 \\ -13452 \\ -13452 \\ 3930 \\ 0 \\ 0 \end{bmatrix}$

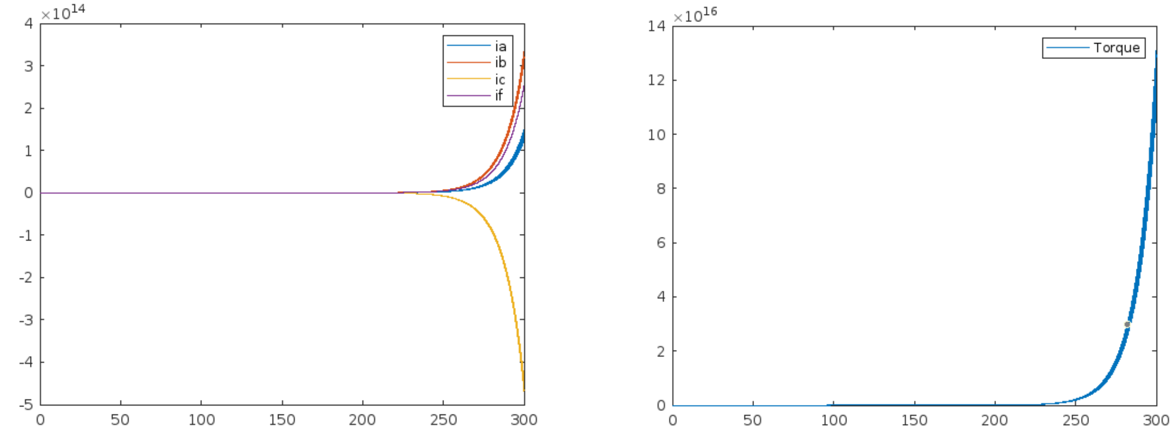

Fig. 2. Current values for i_a, i_b, i_c, and i_f (left) and the torque (right)

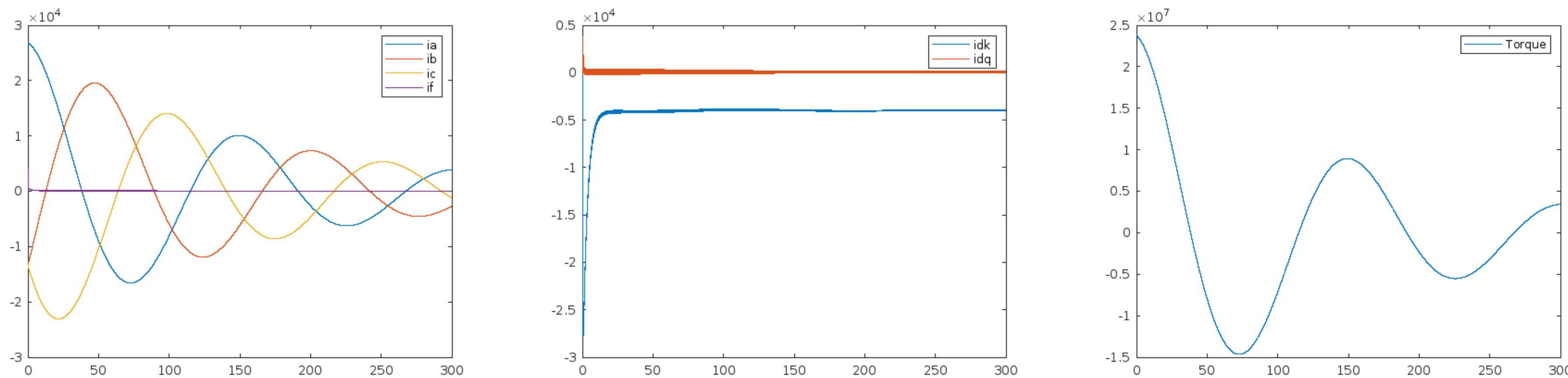

Fig. 2. Current values for i_a, i_b, i_c, and i_f (left) i_{kd} and i_{kq} (center) and the torque (right)

We found that without the damping windings, the generator speeds out of control which would effectively grenade itself once the mechanical limit of the structural components is reached. This is likely because of undamped resonance in the system causing instability. But with the damping windings the generator stays at a reasonable current. The Matlab code for both models can be viewed at the end of this document.

1 Appendix: MATLAB Code

```
%-----------------------------------------------------------------
% Code for SS Model without dampers
%-----------------------------------------------------------------
% define synch gen values
fq = 60;
vf = 377;
nrpm = fq*120/4; % speed of rotor in rev/min
n = nrpm * 0.10472; % speed of rotor in rad/s

S = 659*10^6;
vline = 20*10^3;
vphi = vline/sqrt(3);
pf = 0.9;
w = 377;

Ia = S/sqrt(3)/vphi;
P = pf*S;
R = P/(Ia^2);
Q = S*sqrt(1-pf^2);
Xl = Q/(Ia^2);
L = Xl / w;

Vm = (20*10^3/sqrt(3))*sqrt(2);
iao = Vm/(sqrt(R^2 + Xl^2));
ibo = Vm*cos(-2*pi/3)/(sqrt(R^2 + Xl^2));
ico = Vm*cos(-4*pi/3)/(sqrt(R^2 + Xl^2));

ifo = 338 / 0.0860;
% ikdo = 0;
% ikqo = 0;

iniCon = [iao;ibo;ico;ifo]
tspan = [0 300];
[t,x] = ode45(@myOde, tspan, iniCon);


plot(t,x(:,1),t,x(:,2),t,x(:,3),t,x(:,4));
legend('ia','ib','ic','if')

plot(t,((x(:,1).*(3*Vm*pf))./(n*Xl)))
legend('Torque')

%plot(t,((3.*x(:,1).*Vm)./(n*Xl)))
```

```
function dx = myOde(t,x)

    w = 377;
    S = 659*10^6;
    pf = 0.9;
    Ia = S/ 20*10^3;
    P = 0.9*S;
    Rl = P/(Ia^2);
    Q = S*sqrt(1-pf^2);
    Xl = Q/(Ia^2);
    L = Xl / w;

    % angle given time t

    theta = w*t;                % (2 theta)
    a = -1*(2*pi/3);       % (2 theta - 2pi/3)
    b = -1*(4*pi/3);       % (2 theta - 4pi/3)
    % define values for resistance matrix
    rs = 7.4*10^(-4);
    rf = 0.0860;
    rkd1 = 1.58*10^(-4);
    rkq1 = 1.227*10^(-4);

     % R = [rs+Rl, 0,      0, 0,  0,  0;
     %      0,     rs+Rl, 0, 0,  0,  0;
     %      0,     0,      rs+Rl, 0,  0,  0;
     %      0,     0,      0,     rf, 0,  0;
     %      0,     0,      0,     0,  0,  0;
     %      0,     0,      0,     0,  0,  0];
    R = diag([rs+Rl rs+Rl rs+Rl rf]);

    % define values for inductance matrix
    pwr = 10^(-3);
    Lsa = 1.95*pwr;
    Lma = 0.80*pwr;
    Lsv = 0.05*pwr;
    Lmv = 0.05*pwr;

    Lafm = 26.1*pwr;
    % Lakdm1 = 0.447*pwr;
    % Lakqm1 = 0.670*pwr;
    %
    % Lfkd1 = 5.05*pwr;
    % Lkd1f = 6.29*pwr;
```

```
    Lff = 444*pwr;
    % Lkd1kd1 = 0.1254*pwr;
    % Lkq1kq1 = 0.3762*pwr;

    %------------------------------
    Laa = Lsa + Lsv*cos(2*theta);
    Lab = -Lma + Lmv*cos(2*theta + a);
    Lac = -Lma + Lmv*cos(2*theta + b);
    Laf = Lafm*cos(2*theta);
    % Lakd = Lakdm1*cos(2*theta1);
    % Lakq = -Lakqm1*cos(2*theta1);

    % derivative values
    dLaa = -2*Lsv*sin(2*theta);
    dLab = -2*Lmv*sin(2*theta + a);
    dLac = -2*Lmv*sin(2*theta + b);
    dLaf = -2*Lafm*sin(2*theta);
    % dLakd = -2*Lakdm1*sin(2*theta1);
    % dLakq = 2*Lakqm1*sin(2*theta1);

    %-----------------------------------
    Lba = -Lma + Lmv*cos(2*theta + a);
    Lbb = Lsa + Lsv*cos(2*theta + b);
    Lbc = -Lma + Lmv*cos(2*theta);
    Lbf = Lafm*cos(2*theta + a);
    % Lbkd = Lakdm1*cos(2*theta2);
    % Lbkq = -Lakqm1*cos(2*theta2);

    % derivative
    dLba = -2*Lmv*sin(2*theta + a);
    dLbb = -2*Lsv*sin(2*theta + b);
    dLbc = -2*Lmv*sin(2*theta);
    dLbf = -2*Lafm*sin(2*theta + a);
    % dLbkd = -2*Lakdm1*sin(2*theta2);
    % dLbkq = 2*Lakqm1*sin(2*theta2);

    %-------------------------------
    Lca = -Lma + Lmv*cos(2*theta + b);
    Lcb = -Lma + Lmv*cos(2*theta);
    Lcc = Lsa + Lsv*cos(2*theta + a);
    Lcf = Lafm*cos(2*theta + b);
    % Lckd = Lakdm1*cos(2*theta3);
    % Lckq = -Lakqm1*cos(2*theta3);

```

```
    % derivative
    dLca = -2*Lmv*sin(2*theta + b);
    dLcb = -2*Lmv*sin(2*theta);
    dLcc = -2*Lsv*sin(2*theta + a);
    dLcf = -2*Lafm*sin(2*theta + b);
    % dLckd = -2*Lakdm1*sin(2*theta3);
    % dLckq = 2*Lakqm1*sin(2*theta3);

    %---------------------------
    Lfa = Lafm*cos(2*theta);
    Lfb = Lafm*cos(2*theta + a);
    Lfc = Lafm*cos(2*theta + b);

    % derivative
    dLfa = -2*Lafm*sin(2*theta);
    dLfb = -2*Lafm*sin(2*theta + a);
    dLfc = -2*Lafm*sin(2*theta + b);

    % %------------------------------
    % Lkda = Lakdm1*cos(2*theta1);
    % Lkdb = Lakdm1*cos(2*theta2);
    % Lkdc = Lakdm1*cos(2*theta3);
    %
    % % derivative
    % dLkda = -2*Lakdm1*sin(2*theta1);
    % dLkdb = -2*Lakdm1*sin(2*theta2);
    % dLkdc = -2*Lakdm1*sin(2*theta3);
    %
    % %------------------------------
    % Lkqa = -Lakqm1*cos(2*theta1);
    % Lkqb = -Lakqm1*cos(2*theta2);
    % Lkqc = -Lakqm1*cos(2*theta3);
    %
    % % derivative
    % dLkqa = 2*Lakqm1*sin(2*theta1);
    % dLkqb = 2*Lakqm1*sin(2*theta2);
    % dLkqc = 2*Lakqm1*sin(2*theta3);

    % phew that was a lot

    % define inductance matrix and derivative inductance matrix

    L = [Laa+L, Lab, Lac, Laf;
        Lba, Lbb+L, Lbc, Lbf;
        Lca, Lcb, Lcc+L, Lcf;
```

```
        Lfa, Lfb, Lfc,   Lff];

    dL = [dLaa, dLab, dLac, dLaf;
          dLba, dLbb, dLbc, dLbf;
          dLca, dLcb, dLcc, dLcf;
          dLfa, dLfb, dLfc, 0];

    % V = RI + dL*I + L*dI
    % L*dI = V - RI - dL*I
    % dI = -invL(R + dL)*I + invL*V

    A = -1*(R+dL)\L;
    B = 1\L;
    Vm = (20*10^3/sqrt(3))*sqrt(2);
    u = [Vm*sin(w.*t+pi/2); Vm*sin(w.*t + pi/2 - 2*pi/3); Vm*sin(w.*t + pi/2 - 4*pi/3); 338];
    %K = [Vm*cos(w.*t);Vm*cos(w.*t - (2*pi/3));Vm*cos(w.*t - (4*pi/3));338];
    %K = [0;0;0;377];
    %u = K.*sin(w*t + pi/2);

    dx = A*x + B*u;
end
```

```
%------------------------------------------------------------------
% Code for SS Model with dampers
%------------------------------------------------------------------
% define synch gen values
fq = 60;
vf = 377;
nrpm = fq*120/4; % speed of rotor in rev/min
n = nrpm * 0.10472; % speed of rotor in rad/s

S = 659*10^6;
vline = 20*10^3;
vphi = vline/sqrt(3);
pf = 0.9;
w = 377;

Ia = S/(sqrt(3)*vphi)
P = pf*S;
R = P/(Ia^2)
Q = S*sqrt(1-pf^2);
Xl = Q/(Ia^2)
L = Xl / w;

Vm = (20*10^(3)/sqrt(3))*sqrt(2);
iao = Vm/(sqrt(R^2 + Xl^2));
ibo = Vm*cos(-2*pi/3)/(sqrt(R^2 + Xl^2));
ico = Vm*cos(-4*pi/3)/(sqrt(R^2 + Xl^2));
ifo = 338 / 0.0860;
ikdo = 0;
ikqo = 0;

iniCon = [iao;ibo;ico;ifo;ikdo;ikqo]
tspan = [0 300];
[t,x] = ode45(@myOde, tspan, iniCon);

plot(t,x(:,1),t,x(:,2),t,x(:,3),t,x(:,4));
legend('ia','ib','ic','if')

plot(t,x(:,5),t,x(:,6));
legend('idk','idq')

plot(t,((x(:,1).*(3*Vm*pf))./(n*Xl)))
legend('Torque')

function dx = myOde(t,x)
```

```
w = 377;
S = 659*10^6;
vline = 20*10^3;
vphi = vline / sqrt(3);
pf = 0.9;
Ia = S/(sqrt(3) * vphi);
P = 0.9*S;
Rl = P/(Ia^2);
Q = S*sqrt(1-pf^2);
Xl = Q/(Ia^2);
L = Xl / w;

% angle given time t

theta = w*t;                % (2 theta)
a = -1*(2*pi/3);       % (2 theta - 2pi/3)
b = -1*(4*pi/3);       % (2 theta - 4pi/3)
% define values for resistance matrix
rs = 7.4*10^(-4);
rf = 0.0860;
rkd1 = 1.58*10^(-4);
rkq1 = 1.227*10^(-4);

% R = [rs+R,0,0,0,0,0;
%      0,rs+R,0,0,0,0;
%      0,0,rs+R,0,0,0;
%      0,0,0,rf,0,0;
%      0,0,0,0,rkd1,0;
%      0,0,0,0,0,rkq1];
R = diag([rs+Rl, rs+Rl, rs+Rl, rf, rkd1, rkq1]);

% define values for inductance matrix
pwr = 10^(-3);
Lsa = 1.95*pwr;
Lma = 0.80*pwr;
Lsv = 0.05*pwr;
Lmv = 0.05*pwr;

Lafm = 26.1*pwr;
Lakdm1 = 0.447*pwr;
Lakqm1 = 0.670*pwr;

Lfkd1 = 5.05*pwr;
Lkd1f = 6.29*pwr;
```

```matlab

Lff = 444*pwr;
Lkd1kd1 = 0.1254*pwr;
Lkq1kq1 = 0.3762*pwr;

%------------------------------
Laa = Lsa + Lsv*cos(2*theta);
Lab = -Lma + Lmv*cos(2*theta + a);
Lac = -Lma + Lmv*cos(2*theta + b);
Laf = Lafm*cos(2*theta);
Lakd = Lakdm1*cos(2*theta);
Lakq = -Lakqm1*cos(2*theta);

% derivative values
dLaa = -2*Lsv*sin(2*theta);
dLab = -2*Lmv*sin(2*theta + a);
dLac = -2*Lmv*sin(2*theta + b);
dLaf = -2*Lafm*sin(2*theta);
dLakd = -2*Lakdm1*sin(2*theta);
dLakq = 2*Lakqm1*sin(2*theta);

%-----------------------------------
Lba = -Lma + Lmv*cos(2*theta + a);
Lbb = Lsa + Lsv*cos(2*theta + b);
Lbc = -Lma + Lmv*cos(2*theta);
Lbf = Lafm*cos(2*theta + a);
Lbkd = Lakdm1*cos(2*theta + a);
Lbkq = -Lakqm1*cos(2*theta + a);

% derivative
dLba = -2*Lmv*sin(2*theta + a);
dLbb = -2*Lsv*sin(2*theta + b);
dLbc = -2*Lmv*sin(2*theta);
dLbf = -2*Lafm*sin(2*theta + a);
dLbkd = -2*Lakdm1*sin(2*theta + a);
dLbkq = 2*Lakqm1*sin(2*theta + a);

%-------------------------------
Lca = -Lma + Lmv*cos(2*theta + b);
Lcb = -Lma + Lmv*cos(2*theta);
Lcc = Lsa + Lsv*cos(2*theta + a);
Lcf = Lafm*cos(2*theta + b);
Lckd = Lakdm1*cos(2*theta + b);
Lckq = -Lakqm1*cos(2*theta + b);

```

```
% derivative
dLca = -2*Lmv*sin(2*theta + b);
dLcb = -2*Lmv*sin(2*theta);
dLcc = -2*Lsv*sin(2*theta + a);
dLcf = -2*Lafm*sin(2*theta + b);
dLckd = -2*Lakdm1*sin(2*theta + b);
dLckq = 2*Lakqm1*sin(2*theta + b);

%----------------------------
Lfa = Lafm*cos(2*theta);
Lfb = Lafm*cos(2*theta + a);
Lfc = Lafm*cos(2*theta + b);

% derivative
dLfa = -2*Lafm*sin(2*theta);
dLfb = -2*Lafm*sin(2*theta + a);
dLfc = -2*Lafm*sin(2*theta + b);

%--------------------------------
Lkda = Lakdm1*cos(2*theta);
Lkdb = Lakdm1*cos(2*theta + a);
Lkdc = Lakdm1*cos(2*theta + b);

% derivative
dLkda = -2*Lakdm1*sin(2*theta);
dLkdb = -2*Lakdm1*sin(2*theta + a);
dLkdc = -2*Lakdm1*sin(2*theta + b);

%--------------------------------
Lkqa = -Lakqm1*cos(2*theta);
Lkqb = -Lakqm1*cos(2*theta + a);
Lkqc = -Lakqm1*cos(2*theta + b);

% derivative
dLkqa = 2*Lakqm1*sin(2*theta);
dLkqb = 2*Lakqm1*sin(2*theta + a);
dLkqc = 2*Lakqm1*sin(2*theta + b);

% phew that was a lot

% define inductance matrix and derivative inductance matrix

L = [Laa+L, Lab, Lac, Laf, Lakd, Lakq;
    Lba, Lbb+L, Lbc, Lbf, Lbkd, Lbkq;
    Lca, Lcb, Lcc+L, Lcf, Lckd, Lckq;
```

```
        Lfa, Lfb, Lfc,       Lff,   Lfkd1,   0;
        Lkda, Lkdb, Lkdc,    Lkd1f, Lkd1kd1, Lkq1kq1;
        Lkqa, Lkqb, Lkqc,    0,     0,       Lkq1kq1;];

    dL = [dLaa, dLab, dLac, dLaf, dLakd, dLakq;
         dLba, dLbb, dLbc, dLbf, dLbkd, dLbkq;
         dLca, dLcb, dLcc, dLcf, dLckd, dLckq;
         dLfa, dLfb, dLfc,    0,0,0;
         dLkda, dLkdb, dLkdc, 0,0,0;
         dLkqa, dLkqb, dLkqc, 0,0,0;];

    % V = RI + dL*I + L*dI
    % L*dI = V - RI - dL*I
    % dI = -invL(R + dL)*I + invL*V

    A = -1*(R+dL)\L;
    B = 1\L;
    Vm = (20*10^(3)/sqrt(3))*sqrt(2);
    u = [Vm*sin(w.*t+pi/2), Vm*sin(w.*t + pi/2 - 2*pi/3), Vm*sin(w.*t + pi/2 - 4*pi/3), 338, 0, 0]';
    %u = [Vm;Vm;Vm;377;0;0];
    %u = K*sin(377.*t + pi/2);
    dx = A*x + B*u;
end
```